Name ______________________________

Section ______________ Date ______________

exercise

29

parallax and absolute magnitude

1 materials

~~Logarithm table or~~ calculator.

2 purpose

Astronomers measure the distances to the nearer stars by measuring a small angle called the parallax. **Parallax** is expressed in seconds of arc, and the reciprocal of the parallax is the distance to the star in **parsecs**. This exercise will introduce you to parallax, and will demonstrate how the distance and apparent magnitude are related to the magnitude of the star.

3 trigonometric parallax

The parallax of a star is the angle p, expressed in seconds of arc, subtended by 1 A.U. at the star's distance, as in Figure 29–1. With the exception of the Sun, parallaxes of all stars are less than 1 second ($1''$) of arc. It is not feasible to draw a diagram to scale because, if p were $1''$ and 1 A.U. were represented by a 1 cm length, the distance in the diagram from the Earth to the star would be 206,265 cm or about 2 km.

The skinnny triangle in Figure 29–1 for all prac tical purposes may be replaced by a skinny sector of a circle as in Figure 29–2 in which p is the central angle of the sector and the 1 A.U. distance is the arc of a circle of radius D. For circular sectors we recall that arc divided by radius gives the central angle in radians. To convert degrees to radians we multiply degrees by $\pi/180$. It follows that

$$1'' = \frac{1^\circ}{3600} = \frac{1^\circ}{3600} \times \frac{\pi \text{ radians}}{180^\circ} = \frac{1}{206{,}265} \text{ radian}$$

Definition: A parsec (pc) is the distance to an object whose parallax is 1″ of arc.

It is a simple matter to show that 1pc is approximately equal to 206,265 A.U.

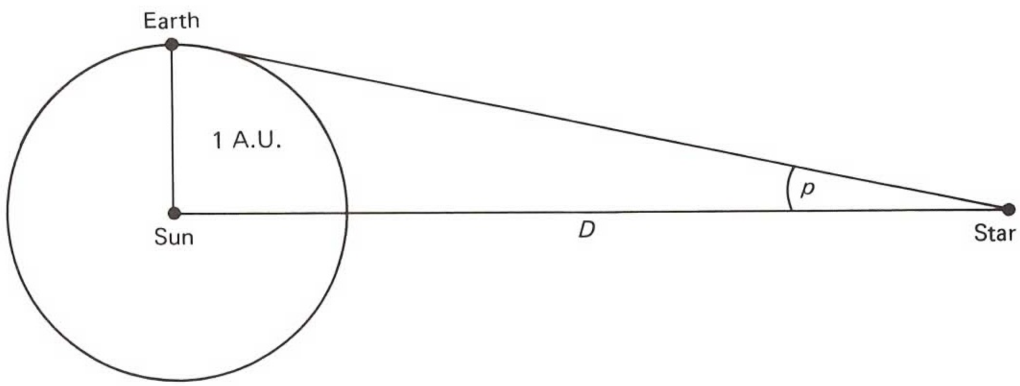

Figure 29-1. A triangle illustrating the relationship between distance and parallax.

In Figure 29–2 if $p = 1''$, then

$$\frac{1\,(\text{A.U.})}{D\,(\text{A.U.})} = \frac{1}{206{,}265} \text{ radian}$$

and $D = 206{,}265$ A.U. or 1pc.

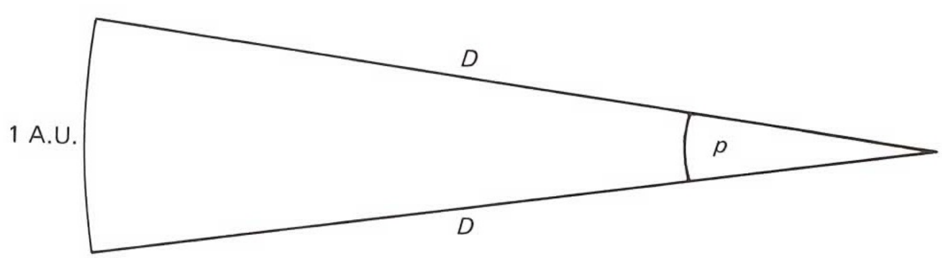

Figure 29-2. A sector that can be used in place of the trinagle in Figure 29-1.

problem

If 1 year is 3.156×10^7 seconds, 1 A.U. is 1.5×10^8 cm, and the velocity of light is 300,000 km/sec, express 1 pc in light years. 1 pc = ________ l.y.

4 parallax and distance

It may be shown that distance varies inversely as parallax, for example, doubling the distance halves the parallax. In Figure 29–3, consider two stars S and S' with parallaxes p and p' respectively. Since arcs x and x' for all practical purposes are equal, let arc x = arc $x' = 1$ A.U. Then

$$\frac{x'}{D'} = \frac{p'}{206{,}265} \text{ radians}$$

and

$$\frac{x}{D} = \frac{p}{206{,}265} \text{ radians.}$$

Using the relation arc x = arc x' we have

$$\frac{D}{D'} = \frac{p'}{p} \qquad \text{or} \qquad D = \frac{p'D'}{p}$$

If $p' = 1''$, then $D' = 1$ pc, and it follows that

$$D = \frac{1''}{p} \text{ parsecs,}$$

where p is the parallax angle measured in seconds of arc.

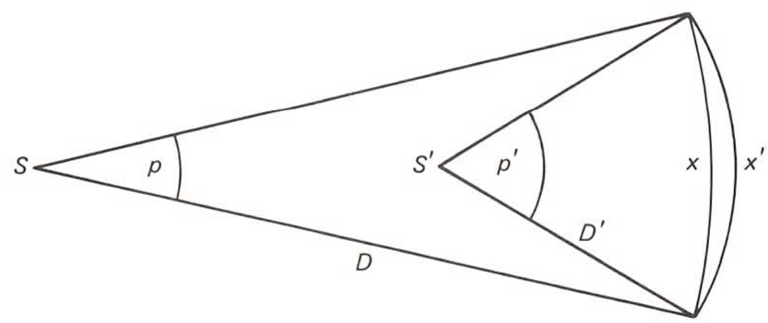

Figure 29-3. Illustration of the inverse relationship between parallax and distance.

Example 29–1. The parallax of a star is $0.05''$. Its distance is $\dfrac{1'' \text{ parsecs}}{0.05''} = 20$ pc.

problems

1. A star is at a distance of 250 parsecs. Its parallax is ________________.

2. A star has parallax $0.025''$. Its distance is ____________________.

3. The nearest star has a parallax of $0.76''$.

(a) Its distance in parsecs is ______________________________.

(b) Its distance in light years is ____________________________.

5 absolute magnitude

The absolute magnitude of a star is the apparent magnitude that star would have if it were at a standard distance of 10 parsecs. Recall that the apparent brightness of an object varies inversely as the square of the distance to that object. Hence $L(m) = k/D^2$ where D is the distance in parsecs and $L(m)$ is the brightness of the object having apparent magnitude m. We are assuming that space is transparent.

you don't need to understand the magnitude scale for exams, I'll convert below for you.

When the distance D is specified to be 10 pc, the apparent magnitude m is equal to the absolute magnitude M. Hence $L(M) = k/10^2$. Now we get

$$\frac{L(m)}{L(M)} = \frac{10^2}{D^2} = (2.512)^{M-m}.$$

$L(M) = A\, 2.5^{-M}$

Taking the logarithm of both sides, we have

$$\log 10^2 - \log D^2 = \log (2.512)^{M-m}$$

or

$$2 - 2 \log D = 0.4\,(M - m)$$

and finally

$$M = m + 5 - 5 \log D,$$

where D is measured in parsecs.

If we know any two quantities in the above equation, we can solve for the third quantity. Generally m is known and if the distance D of a star is known, we can find the absolute magnitude M of the star. If we wish to find D, then m and M must be available. In some cases M may be determined by use of the Hertzsprung-Russell diagram, as we shall see in a later exercise.

Example 29-2. Find the absolute magnitude of the Sun given that the Sun is at a distance of 1 A.U. = 1/206,265 pc and the apparent magnitude of the Sun is -26.7. Substituting these values into the formula, we have

$$\begin{aligned} M &= -26.7 + 5 - 5 \log 1/206{,}265 \\ &= -26.7 + 5 - 5(\log 1 - \log 206{,}265) \\ &= -26.7 + 5 + 5 \log 206{,}265 \\ &= 4.9 \end{aligned}$$

This is the apparent magnitude the Sun would have if it were at a distance of 10 pc from us.

IF $M = 4.9$ and $m = 8.0$ what is D?

$\mathcal{L}_{10}(\text{Sun}) = 2.5^{-4.9} = 0.0112$

$\mathcal{L}_{D}(\text{Sun}) = 2.5^{-8.0} = 0.00066$

$\dfrac{.00066}{.0112} = \dfrac{10^2}{D^2} \Rightarrow D^2 = \dfrac{10^2 * .0112}{.00066} = 1709$

$D = \sqrt{1709} = 41$ pc

problems

1. ~~If the Milky Way contains 100 billion stars (10^{11} stars) each as luminous as the Sun, what would be the absolute magnitude M of the Milky Way?~~

SKIP

2. If $m = 13.5$ and $M = 10$, find the distance D. Follow bottom p182

$\mathcal{L}(M=10) = 2.5^{-10} = .00010$

$\mathcal{L}(m=13.5) = 2.5^{-13.5} = .0000042$

3. What would be the apparent magnitude of Rigel if it were at a distance of 1 A.U.? Assume that M for Rigel is -7.1. SKIP

$\mathcal{L}(M=-7.1) = 2.5^{7.1} = 669$

$\mathcal{L}(M=1.3) = 2.5^{-1.3} = 0.30$

4. The luminosity of a star is its total brightness compared with that of the Sun at the same distance, 10 pc. If we take absolute magnitude of the Sun to be 4.9 and the luminosity of the Sun as 1, then an absolute magnitude of 2.4 for Altair implies that the luminosity of Altair is 10.0. If the absolute magnitude of Sirius is 1.3 what is its luminosity? If the absolute magnitude of Rigel is -7.1 what is its luminosity?

Name ______________________________

Section ______________ Date ______________

exercise

39

stars in Orion

1 materials

None.

2 purpose

The sky is not eternal, not fixed. It is continuously changing as the stars change, grow old, die, as new stars are born, and as the stars move a cross the sky. This exercise will acquaint you with how a familiar constellation is changing.

3 proper motions

The regular change in position of a star per year across the sky is called its proper motion and is represented by the symbol μ. Proper motion is measured in seconds of arc per year and is determined by measuring the position of the star on two dates separated by many years.

Just as we might measure the motion of a ship at sea by the number of miles it travels north or south per day and the number of miles it travels east or west per day, we measure the motion of a star by the distance in seconds of arc it travels north or south per year and the distance in seconds of arc it travels east or west per year. The north-south motion represents a change in declination, and this component of μ is called the proper motion in declination, μ_δ. The component of μ in the east-west direction is called the proper motion in right ascension, μ_α. If the star moves northward, μ_δ is positive. If it travels eastward, μ_α is positive. If it travels in the other directions then the corresponding proper motions are negative.

The proper motions of most bright stars are much less than 1″ of arc per year. This is a very small angle. A sheet of paper viewed edge on at arms length is about 30″ of arc thick. It would take most stars 30 to 3000 years to move that far across the sky.

3600 seconds of arc = 1 degree

Table 39–1 The Stars in Orion   seconds of arc/year

Star	*R.A.*	*Dec.*	μ_α	μ_δ	*Spectral Type*
Alpha	$5^h\ 54^m$	$+7°\ 24'$	0.027	0.007	
Beta	5 13	−8 14	0.001	0.000	
Gamma	5 24	+6 19	−0.006	−0.014	
Delta	5 30	−0 19	0.001	−0.001	
Epsilon	5 35	−1 13	0.000	0.000	
Zeta	5 39	−1 57	0.004	−0.002	
Kappa	5 46	−9 41	0.004	−0.002	
Iota	5 34	−5 56	0.003	0.004	
Theta	5 34	−5 25	0.003	0.003	

In Table 39-1 only the components of the proper motion are given because the total proper motion μ may be determined from the components, μ_α and μ_δ. See Figure 39–1.

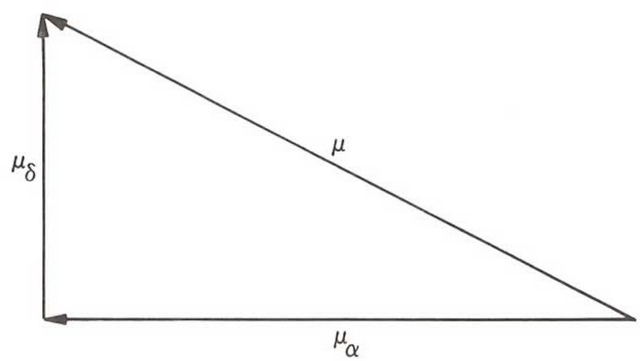

Figure 39-1. The relation between the proper motion in right ascension μ_α and declination μ_δ, and total proper motion μ of a star near the celestial equator.

4 Orion

Plot in Figure 39–2 the positions of the stars in Orion by consulting the right ascensions and declinations listed in Table 39–1. The result will be a star chart showing the constellation Orion as it appears in our sky. Compare this with the star chart ~~following the appendices.~~ in your book or star wheel,

Trace the scale shown in Figure 39–2 and use it to draw in lightly arrows representing μ_α and μ_δ for each star as shown in Figure 39-1. Since μ_α and μ_δ are so small, plot the distance the star would travel in 1,000,000 years instead of in 1 year. Thus, Alpha Orionis has a μ_α of 0.027″ of arc per year. In 1,000,000 years it will travel 27,000″ of arc. Since μ_α is positive we conclude that the star is traveling eastward (to the left) in the constellation.

Now draw in the hypotenuse of the triangle for each star, showing its total motion across the sky for a period of 1,000,000 years.

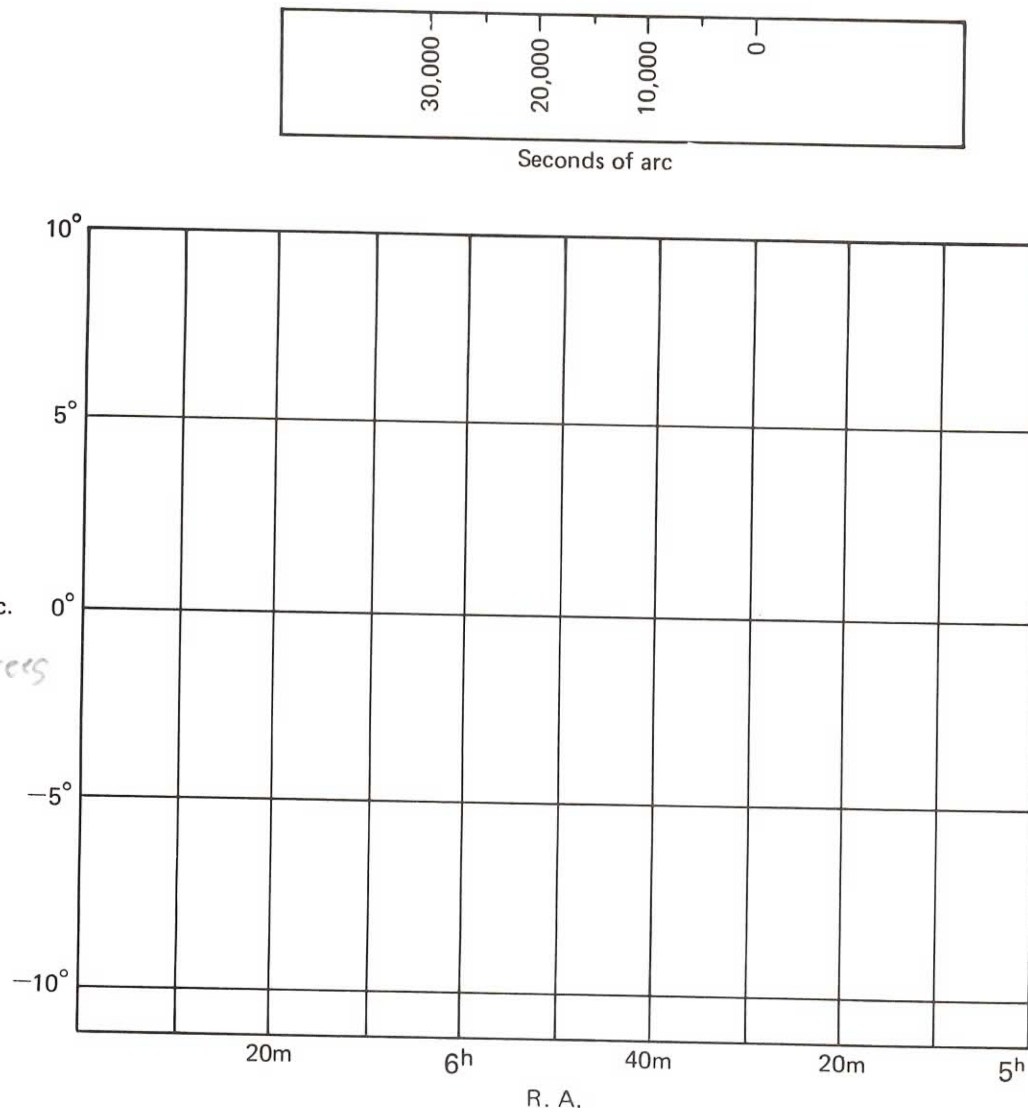

Figure 39-2. Area of sky near Orion. Plot star positions according to right ascension and declination. Scale for measurement of proper motion is in seconds of arc.

24 hours (RA) = 360 degrees
1 hour = 15 degrees   20 m = 5 degrees
= 18000 seconds of arc

Orion in 1,000,000 years

Place a blank piece of paper over your diagram and draw dots or stars at the tip of each arrow, μ, to show where the star will be 1,000,000 years from now. Would Orion be recognizable?   use different symbol

Orion 1,000,000 years ago

Extend the arrows you have drawn backwards to show where these stars were 1,000,000 years ago. Again use a black piece of paper to make a map of
Use different symbol
stars in Orion

Orion as it appeared then. Humans have been on Earth for at least 2 million years, and so would have seen quite a different constellation pattern from the Orion we know.

5 assumptions

We have made some assumptions in making these maps. We have assumed that the stars are moving in straight lines. We know that each of these stars is moving in a curved orbit about the center of our galaxy, but in just 1,000,000 years they will not have moved very far along in their orbits and the curve in their paths should not be apparent. In the life of the galaxy, 1,000,000 years is not a very long time.

We have also assumed that the proper motions in Table 39–1 are correct. However, these quantities are very difficult to measure and there are probably some small errors in the values listed. Suppose the components of the proper motion of Alpha Orionis given in the table are too small by 0.001″ of arc per year. Increase the table values for this star by 0.001 and replot the motion of the star. How much of a difference will this small error make in the position of the star 1,000,000 years from now?

optional exercise

SKIP We will discuss

We have also assumed that these stars will still be bright stars in 1,000,000 years, and that they were bright stars 1,000,000 years ago. Look up the spectral types of these stars and estimate their ages and life expectancies. Consult your observer's handbook for spectral types and your text book to estimate life expectancies. Now write a paragraph or two describing how the brightness and the color of the stars in Orion might change in the future and how they might have looked in the past.