

Name \_\_\_\_\_

Section \_\_\_\_\_ Date \_\_\_\_\_

## exercise

# 21

## the orbit of Mercury

**1 materials** Protractor, millimeter rule, sharp pencil.

**2 purpose** How do we know what the orbit of a planet is like? At first glance this appears to be a difficult question, but in many cases it is surprisingly easy to derive an orbit from basic observations. In this exercise you will use a set of simple observations, which you could have made yourself, to discover the size and shape of the orbit of Mercury. You will be repeating work that Johannes Kepler did to formulate his laws of planetary motion. Kepler made his first discoveries while studying the orbit of Mars. He then extended his analysis to the six planets then known. We will see how he studied Mercury.

**3 obser-  
vations** The ancient astronomers of Greece were familiar with planet Mercury, but they did not recognize it as the same object when it appeared in the evening sky and in the morning sky. It was known as Apollo when seen in the morning sky and as Mercury when seen in the evening sky. Sixteen hundred years later the motions of the planets were observed and recorded by the Danish astronomer Tycho Brahe. Brahe's records covered a period of 20 years, and on his death in 1601 these records were taken by one of his assistants, Johannes Kepler.

From Brahe's records Kepler was able to find the maximum angular distance from the Sun that Mercury reached during each of its passages. You could make the same observations, but to save time a list of these distances, called **maximum elongations**, is given in Table 21-1. These data were taken from observer's handbook for the years 1967-69. You could obtain similar data from the handbook for the current year. These data contain the dates on which

Mercury was at its maximum elongation, the angular distance, and the direction from the Sun.

**Table 21-1** Maximum Elongations for Mercury

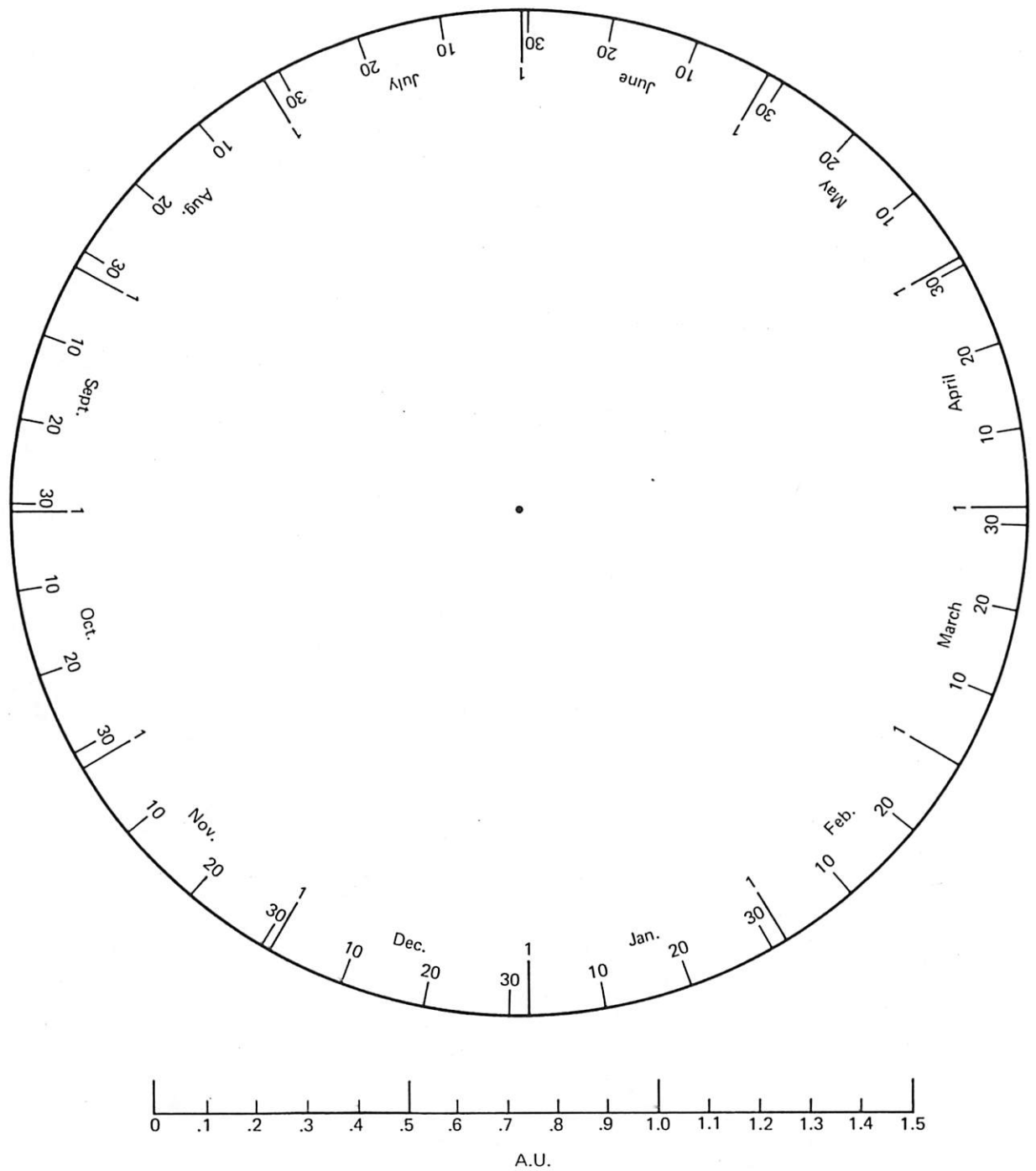
February	16,	1967	18°	east
March	31		28°	west
June	12		25°	east
July	30		20°	west
October	9		25°	east
November	18		19°	west
January	31,	1968	18°	east
March	13		27°	west
May	24		23°	east
July	11		21°	west
September	20		26°	east
October	31		18°	west
January	13,	1969	18°	east
February	23		26°	west
May	6		21°	east
June	23		23°	west
September	3		27°	east
October	15		18°	west
December	28		19°	east

#### 4 drawing the orbit

Figure 21-1 shows the orbit of the Earth with the dates marked to represent the position of the Earth in its orbit throughout the year. The scale below the orbit is marked in astronomical units and will enable you to make measurements on the diagram in astronomical units. The Sun is of course located near the center of the Earth's orbit.

As you draw on the diagram use a sharp pencil; sharp so you can be accurate, and pencil so you can make corrections as necessary. (The width of a sharp pencil line on this diagram is about 100,000/km).

Now plot each of the elongation as follows: Locate the date of the maximum elongation on the orbit of the Earth. Draw a light pencil line from the position of the Earth to the Sun. Center a protractor at the position of the Earth and construct a line so that the angle from the Earth-Sun line to this constructed line is equal to the given maximum elongation. Extend this line across the diagram well past the Sun. It represents the direction toward Mercury. Remember, facing the Sun from the position of Earth, eastern elongations are to the left as in Figure 21-2. In each case the planet Mercury will lie somewhere along the line you have drawn. As you draw more lines, you will see the orbit taking form.



**Figure 21-1.** The orbit of Earth with the Sun near the center. The position of Earth in its orbit is indicated for various dates.

After you have plotted the data you may sketch in the orbit of Mercury. The orbit must be a smooth curve that just touches each of the elongation lines you have drawn. The orbit may not cross any of these lines.

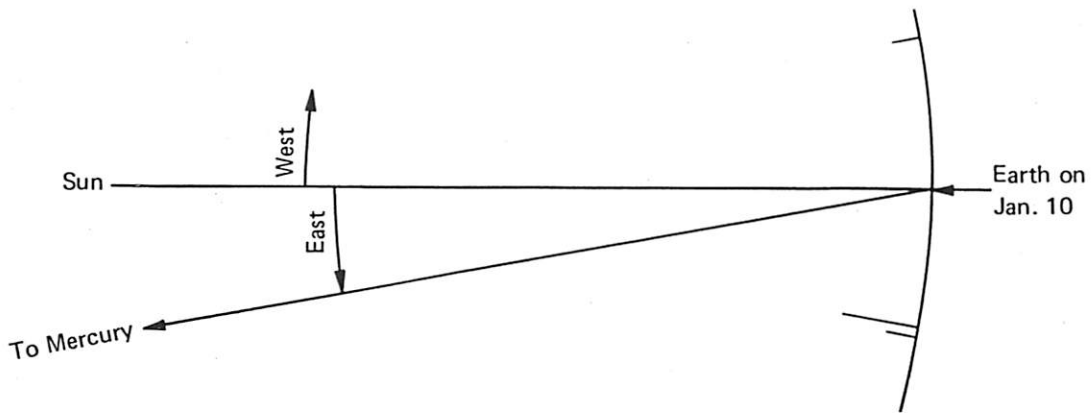


Figure 21-2. Example showing maximum eastern elongation on January 10.

**5** the shape  
of the orbit

problems

6

1. Does it appear that the curve you have drawn is not a circle? How could you be sure?

Kepler had the same problem; his circles didn't fit either. At last he hit upon the idea of using ellipses for the orbits. They fit. Through the Sun draw the longest diameter possible in the orbit of Mercury. This is the major axis of the ellipse. Measure the length of the major axis, and bisect it to find the center of the ellipse. Draw the minor axis through the center perpendicular to the major axis.

7

2. What is the semimajor axis  $a$  of Mercury's orbit in astronomical units (A.U)? Does your result agree with that given in your book?
  - (a) Measured semimajor axis:  $a =$  \_\_\_\_\_
  - (b) Book value:  $a =$  \_\_\_\_\_
3. Use your measured value of  $a$  to calculate the sidereal period  $P$  of Mercury in years. To convert to days, multiply the sidereal period in years by 365.26.
  - (a) Calculated period:  $P =$  \_\_\_\_\_ years = \_\_\_\_\_ days.
  - (b) Book value:  $P =$  \_\_\_\_\_
4. The Sun lies at one of the foci of the ellipse. Measure the distance from the

Sun to the center of the ellipse and call it  $c$ . If we divide  $c$  by the semimajor axis  $a$  we get the eccentricity  $e$  of the orbit.

Calculate  $e$  for your orbit and compare it with the value given in the book.

(a) Calculated value:  $e =$  \_\_\_\_\_

(b) Book value:  $e =$  \_\_\_\_\_

5. Look up a greatest elongation for Mercury for any year in an observer's handbook and plot it on your diagram. Is it tangent to your orbit?

**6 the distance to Mercury**

On October 15, 1969, Mercury was at greatest elongation west. Fifteen days earlier, on October 1, 1969, Mercury was  $9^\circ$  west of the Sun. Plot the position of Mercury on October 1, 1969.

**problems**

1. What was the distance from Earth to Mercury on October 1, 1969?
2. From measurements made on your diagram, determine the minimum distance from Earth to Mercury.

**7 the accuracy of the result**

This is a simple exercise and, if you were careful and used a sharp pencil, your results for the orbit of Mercury will be surprisingly accurate. The orbits of most of the other planets are much less eccentric than the orbit of Mercury, and the results from a graphical study of these orbits would not be accurate enough to measure the eccentricities of the orbits. When Kepler studied the orbits of the planets, he did not depend on a diagram but, rather, he built a mathematical model of the orbits based on the observations. His analysis of these orbits led him to a statement that has become known as **Kepler's first law of planetary motion**: *The orbits of the planets about the Sun are ellipses with the Sun at one focus.*

problems

What are some of the assumptions that we have made that affect the accuracy of our results?

**8** supplement-  
ary prob-  
lem using  
the celes-  
tial globe

Tycho's records contained the right ascension and declination of the Sun and Mercury. Kepler had to calculate the angular distance between them. We could obtain the distance between them without using spherical trigonometry by using a celestial globe.

Use a small spot of tape to mark the position of the Sun and another spot of tape to mark the position of Mercury. Place a strip of paper against the globe and mark the distance between the two objects. Move the paper so that one mark rests on the equator and the other mark rests directly north of the equator. The declination of the upper spot is equal to the angular separation of the Sun and Mercury. Determine the elongations of Mercury on the following days:

		<i>Sun</i>		<i>Mercury</i>		<i>Elongation</i>
		<i>R.A.</i>	<i>Dec.</i>	<i>R.A.</i>	<i>Dec.</i>	
February	9	21 <sup>h</sup> 30 <sup>m</sup>	-14° 48'	20 <sup>h</sup> 08 <sup>m</sup>	-17° 22'	_____
February	16	21 <sup>h</sup> 57 <sup>m</sup>	-12° 28'	20 <sup>h</sup> 15 <sup>m</sup>	-18° 24'	_____
February	23	22 <sup>h</sup> 24 <sup>m</sup>	- 9° 58'	20 <sup>h</sup> 39 <sup>m</sup>	-18° 20'	27° west
March	2	22 <sup>h</sup> 51 <sup>m</sup>	- 7° 21'	21 <sup>h</sup> 12 <sup>m</sup>	-17° 11'	_____
March	9	23 <sup>h</sup> 17 <sup>m</sup>	- 4° 39'	21 <sup>h</sup> 50 <sup>m</sup>	-14° 56'	_____

Plot the positions of Mercury in its orbit on these days. Notice that the dates are all separated by 7 days. Does it appear that Mercury is obeying Kepler's second law?