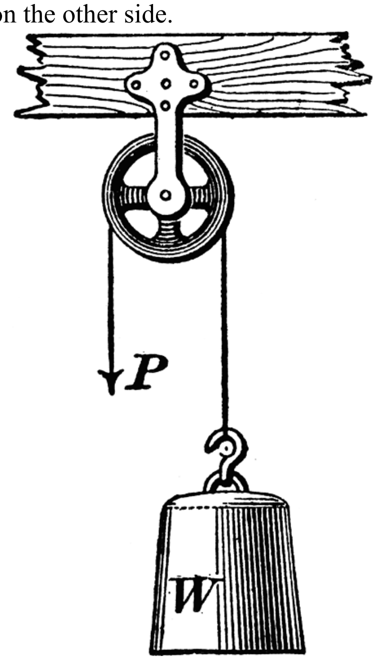
Pulley

Goals: Use measurements to find a linear relationship between variables. Use a linear fit to find the slope and intercept of a line. Estimate errors on a linear fit.

APPARATUS

The pulley is a device that can lift and hold weight, transmit power, and redirect force. Archimedes recognized the pulley as one of three simple machines in the 3rd century BC, and it remains a building block for complex machines in the present day. In its simplest form the pulley consists of a wheel and an axle with a rope wrapped around part of the wheel. In this lab we will be using the pulley to redirect force. The tension in the rope allows the force to be redirected from the direction of the rope on one side of the pulley to the direction on the other side.



In 1784 George Atwood used a pulley to transfer the downward force of weight from a mass to work upward against another weight. With his Atwood machine he was able to test the mechanics of constant accelerations with a value less than g. We will use his idea to measure the value of g. At the same time we will measure friction in the pulley.

Our Atwood machine consists of two unequal masses that are connected by a cord run over a pulley. The larger mass (m_2) is suspended above the smaller mass (m_1), and sits upon a platform that is equipped with a mechanical trip release. Upon release, the heavier mass accelerates down to a platform a measured distance (h) below. Each group will measure the time (t) required for m_2 to move from the upper to the lower platform.

THEORY

The masses in the Atwood machine are subject to a number of forces, all of which are constant. The net constant force will produce a constant acceleration. We know that a constant acceleration (a) will cause an object to move a distance equal to (1/2) $a\,t^2$. If we measure the distance and time we can solve for the acceleration.

$$a = \frac{2h}{t^2} \qquad \textbf{(EQ 1)}$$

A perfect Atwood machine would have a massless and frictionless pulley. In that simplified case the force acting to pull down the heavy mass is $F_2 = m_2 g$, and the force pulling up due to the lighter mass being pulled down is $F_1 = m_1 g$. The net force pulling down on the heavy mass would be $F = F_2 - F_1 = (m_2 - m_1)g$. Both masses are accelerated together so Newton's second law is $F = (m_2 + m_1)a$. Combining these two equal expressions for force allows us to solve for the ideal acceleration $a = (m_2 - m_1)\,g \,/\, (m_2 + m_1)$.

Our real Atwood machine has a pulley with some mass that resists acceleration and some kinetic friction on the axle of the pulley that reduces the net force pulling down the heavy mass. The mass of the pulley m_p is being accelerated by the pull of the string at its rim. It turn out that a rotating disk provides an equivalent additional inertia at its rim equal to half of its mass: m_p / 2. For this experiment we can use m_p / 2 as if it were an additional mass to be accelerated with m_1 and m_2.

How should we deal with the frictional force? If it acts like kinetic friction it should be a constant force (f). If we include this as an additional force opposing the downward motion of m_2 and we also include the inertia of the spinning pulley then the equation for Newton's second law becomes:

$$(m_2 - m_1)g - f = \left(m_2 + m_1 + \frac{m_p}{2}\right)a \qquad \textbf{(EQ 2)}$$

In our experiment we will determine the acceleration (a) by EQ 1 as a function of the mass difference ($\Delta m = m_2 - m_1$). These are the two variables in our experiment. We can rearrange EQ 2 to solve for a as we separate out Δm.

$$a = \frac{g}{\left(m_2 + m_1 + \frac{m_p}{2}\right)}\Delta m + \frac{-f}{\left(m_2 + m_1 + \frac{m_p}{2}\right)} \qquad \textbf{(EQ 3)}$$

The equation for a straight line is $y = mx + b$, where m is the slope and b is the y-intercept. EQ 3 has a similar form if we consider a as y and Δm as x. By thinking of the relationship as the graph of a straight line we can collect data, plot it and derive the unknown quantities. So, we will plot a on the y-axis and Δm on the x-axis the result should be a straight line with a slope equal to $g / (m_2 + m_1 + m_p / 2)$, and a y-intercept at $-f / (m_2 + m_1 + m_p / 2)$.

Note that the y-intercept is a negative value. If it were 0 then the y-intercept would be exactly at the origin. That would be the case if the friction (f) was 0.

DATA COLLECTION

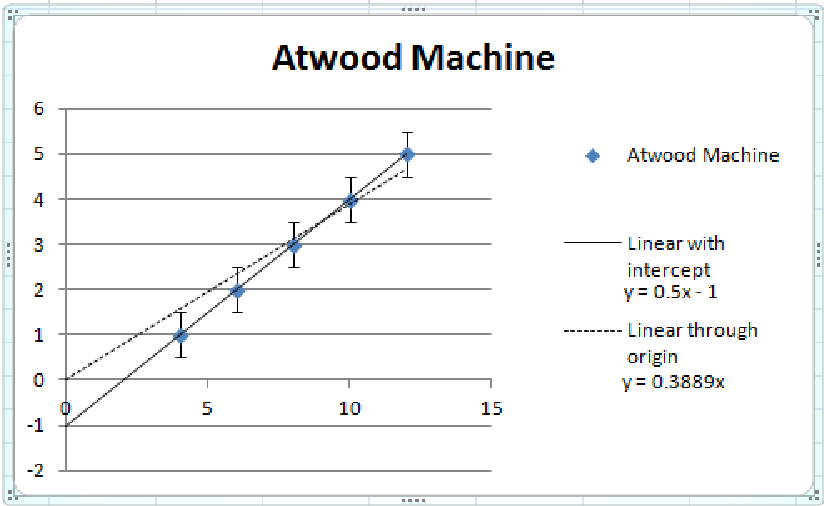
1. Start a report with the proper title, date, and participants in your group. Add a brief description of your apparatus with a simple diagram, and include a reference to this lab handout. As you go along you should definitely record any steps or activities that vary from this procedure.
2. Read and record the mass of the pulley m_p from the side of the apparatus. DO NOT REMOVE THE PULLEY FROM THE APPARATUS.
3. Attach a piece of tape as a label on each weight (cylinders and disks), then measure and record the mass of each weight and the uncertainty. Exercise caution not to confuse weights that may look the same, since they probably have different masses.
4. Measure and record the distance from the upper to the lower weight platform (h).
5. Place 12 grams (one 5 g, three 2 g, and one 1 g) on one cylinder the total of which is m_2. The other cylinder is m_1. Using the measurements in step 1 and step 2, record the total mass ($m_2 + m_1 + m_p / 2$) and the mass difference ($\Delta m = m_2 - m_1$) for this first data set in grams.
6. Time the fall of m_2 from the upper to the lower platform and record this value. This should be repeated for a total of 5 trials without changing the masses.
7. Find the average time (t) for the 5 trials and the uncertainty on the average using the standard error.
8. Calculate the acceleration (a) using EQ 1 above and calculate the uncertainty on a using the propagation of errors. Remember that t^2 is the same as $t \times t$.
9. Move the 1 gram weight from m_2 to m_1 so that the mass difference is about 10 grams, and record your new value for the mass difference (Δm) in grams. Repeat steps 6 through 8 with this mass difference.
10. Repeat step 9 with mass differences of 8, 6 and 4 grams. You should have a total of five sets of data, with mass differences of about 12, 10, 8, 6, and 4 grams.

DATA ANALYSIS

11. Use Excel to make a table of your five sets of data and place that in your report.
12. Use Excel to plot the acceleration (a) in cm/s^2 versus the mass difference (Δm) in grams for the five data sets. Note that the acceleration should be on the dependent y-axis.
13. Find the average standard error from the data sets in step 8. Use that value for vertical error bars on your graph.
14. Use the **Trendline** tool to draw a straight line through your data. Use the **Linear** option and set the name to “Linear with intercept”. Set the **Forecast Backward** to

the smallest value of Δm on the graph, which should be 4. That will cause the line to extend back to the y-axis. Remember to display the equation, too.

15. Use the **Trendline** tool a second time to draw another straight line with the Linear option. This time name it "Linear through origin" but also check the **Set Intercept =** box with the value 0. Use the menu on the left side of the *Format Trendline* box to select **Line Style** and then set **Dash type** to **Square Dot**. Display that equation, too so that the graph looks something like the following.



16. Add labels to the axes and insert the graph into your report with a caption.
17. Note in the report the value of the slope and the *y*-intercept of the line. The y-intercept should be in cm/s^2 and the slope should be in cm/g s^2.
18. Find the frictional force (f) on the pulley by multiplying the value of the *y*-intercept times the total mass. Convert the value of the force to newtons. Calculate the experimental error on f using the average standard error on a as the error on the *y*-intercept.
19. Estimate the error on the slope by comparing the slope of the "linear with intercept" line to the "linear through origin" line. The difference in the slopes of the two lines is one way to judge the error on the slope. A better method is to estimate the amount the slope changes if the "linear with intercept" line is shifted but still passes within the error bars to the same degree as the fit line.
20. Find the gravitational acceleration (g) by multiplying the value of the slope times the total mass. Convert the value of the acceleration to m/s^2. Calculate the experimental error on g using the estimated error on the slope from step 19.
21. Compute the difference between the g determined in step 20 and the accepted value 9.8 m/s^2. Divide this difference by the accepted value and multiply by 100 to get the percent error.
22. Your TA will assign an addition question or two to answer in the report. This work should be done by each group member individually.
23. Each student should assemble a single report from the group data report and the additional individual question. This report will be turned in for grading.