Pendulum

Goals: Use a computer-interfaced timing device. Measure the effects of different variables. Compare graphs of straight-line data to data on curves.

APPARATUS

The pendulum became an important device for time keeping after Galileo's famous measurements in the early 17th century. He found that the pendulum keeps the same period of oscillation independent of the amplitude. Later that century Huygens used the pendulum to build a mechanical clock. Huygens realized that the period of a simple pendulum did have a slight dependence on amplitude and made corrections in his clock. Ever more precise refinements were made to the pendulum clock until it was surpassed in accuracy by the quartz crystal oscillator. (Image from Science Museum of London)

A pendulum can be made by supporting a mass (*m*) at the end of string. In this experiment the mass is held by two equal length strings, supported from a horizontal rod about 10 cm apart, as shown in Figure 1. This arrangement will let the mass swing only along a line, and will prevent the mass from striking the photogate. The length (*l*) of the pendulum is the distance from the point on the rod halfway between the strings to the center of the mass.

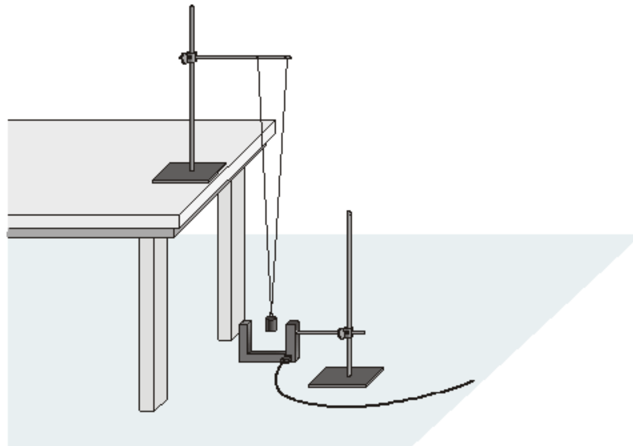

FIGURE 1. **Experimental setup for the pendulum.**

The electronic photogate will be used to measure the time (t) as the mass makes each swing at its lowest point. The photogate should always be positioned so that the mass blocks the sensor in the photogate when it hangs straight down. The photogate is attached to the graphing calculator for readout.

THEORY

A pendulum mass (m) at the end of string is subject to two forces: gravity (F_g) and tension in the string (F_T). Since the string doesn't change length, Newton's first law, the law of inertia, says that there is no net force on the string from end to end. If the string hangs vertically this means that $F_T = F_g = mg$. When the pendulum mass displaced from the vertical at an angle (θ) the tension needed to oppose gravity is $F_T = mg\cos\theta$. Part of the force of gravity is not opposed by the tension and that results in a net force, $F_{net} = mg\sin\theta$. From Newton's second law, that net force causes an acceleration toward the vertical position.

The maximum position displacement (x) from the vertical is related to the angle and the length (l) of the string in EQ 1.

$$x = l\sin\theta \qquad \text{(EQ 1)}$$

For small angles the sine of the angle is approximately equal to the angle when measured in radians. Radians measure the number of radii around the circumference of circle, so 360º = 2π radians, or 1 radian = 57.3º.

As the pendulum passes through the vertical position, the net force acts to slow the mass down and eventually cause it to reverse direction. The force alternates back and forth, and the mass moves back and forth at a regular rate. The time it takes for the mass to go back and forth in one complete cycle is called the period (T). Note that the mass will travel through the vertical position twice during each period. If the maximum angle of amplitude isn't too large the period can be approximated by EQ 2.

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{(EQ 2)}$$

Squaring both sides gives EQ 3.

$$T^2 = \left(\frac{4\pi^2}{g}\right)l \qquad \text{(EQ 3)}$$

If the period squared is plotted versus the length, the line should pass through the origin and the slope (s) of the line should be the quantity in parentheses in EQ 3. The acceleration due to gravity is then related to that slope by EQ 4.

$$g = \frac{4\pi^2}{s} \qquad \text{(EQ 4)}$$

DATA COLLECTION

1. Set the strings supporting the pendulum mass to be at least 100 cm and use a 100 g mass. Measure and record the length (l) and record the mass (m) including the hanger.
2. Open LoggerPro on the computer. From the **File** pull-down menu, select **Open**, then select the *Physics with Vernier* folder and open the file “14 Pendulum Periods”. When the **Sensor Confirmation** box appears, the *Interface and Channel:* selection should be set to ‘DG1 on LabPro:1’, and the *Sensor:* selection should be set to ‘Phtogate’.
3. Press the **Connect** button and a table with column headings ‘Time (s)’, ‘GateState’, and ‘Period (s)’ should appear on the left side of the screen, and a graph titled ‘Pendulum Periods’ should appear on the right side of the screen.
4. Press **COLLECT** to initiate the photogate for data collection.
5. Pull the mass out about 10º from vertical and release. (For a pendulum that is 100 cm long, that corresponds to pulling the bob about 15 cm to the side.) Be sure to pull the mass straight out so that the mass does not strike the photogate as it swings.
6. After the computer display indicates that the pendulum has passed back and forth five times and five trials have been recorded, press **STOP** to end data collection.
7. Calculate and record the average period for the five trials. To reset for a new run, click on the **Experiment** drop-down menu and select **Clear Latest Run**.
8. Repeat steps 4 through 7 for a 200 g and 300 g mass. Record the mass and period in a table using Excel along with the first measurement with the 100 g mass. Record the standard error for each set of measurements with a single weight.
9. Place the 200 g mass on the string. Repeat steps 4 through 7 for amplitudes of 15º, 20º, 25º and 30º. Use EQ 1 to determine the amount of displacement needed to achieve the desired angle. Include estimates of the error in the displacement and angle. Record the angle, displacement and period in a table along with the 200 g measurement made at 10º in step 8.
10. Use the 200 g mass and an amplitude of 10º, and repeat steps 4 through 7 for pendulum lengths of 90 cm, 80 cm, 70 cm, 60 cm and 50 cm. Record the length, displacement and period in a table. Remember to measure the pendulum length from the horizontal rod to the middle of the mass, and to recalculate the displacement for each length using EQ 1.

DATA ANALYSIS

11. Use the data in the table from step 8, and plot the period (T) vs. mass (m) using Excel. Scale each axis from the origin (0,0). Include appropriate error bars for the period in this and the following graphs.
12. Use the data in the table from step 9, and plot the period (T) vs. amplitude angle (θ).
13. Use the data in the table from step 10, and plot the period (T) vs. length (l).
14. Use the data in the table from step 10, and plot the period (T) vs. length squared (l^2).
15. Use the data in the table from step 10, and plot the period squared (T^2) vs. length (l). Use propagation of errors to get the error on T^2.
16. Use the **Trendline** tool in Excel to draw a straight line through your data in the graph from step 15, and check the **Set Intercept =** box with the value 0.
17. Use the slope of the line determined by the trendline in step 16 to find the acceleration of gravity (g) with EQ 4.
18. Estimate the error on the slope and use that estimate to determine the error on your measurement of g. You may want to see how much the slope changes without the Set Intercept feature as an estimate of the error on the slope.
19. Create a report that includes the tables and graphs (all from Excel). Remember to include captions that distinguish the tables and graphs from each other and enough text to help the reader follow the steps. Don't just submit the tables and graphs.
20. Your TA will assign an additional question or two to answer in the report. This work should be done by each group member individually.
21. Each student should assemble a single report from the group data report and the additional individual question. This report will be turned in for grading.