

Coin Toss

Goals: Understand the use and application of measurement and error. Use relevant equations to measure and propagate statistical errors. Create equations in Word and Excel for use in a report.

PROBLEM

The bell curve is a familiar shape. It shows up in many places and is used throughout science, where it takes on the formal name normal distribution. It can take different forms as illustrated in Figure 1. In this case it is a graph of the heights of 100 men and an ideal distribution. We'll be looking at some other measurements, both alone and in combination to see whether they too form a normal distribution.

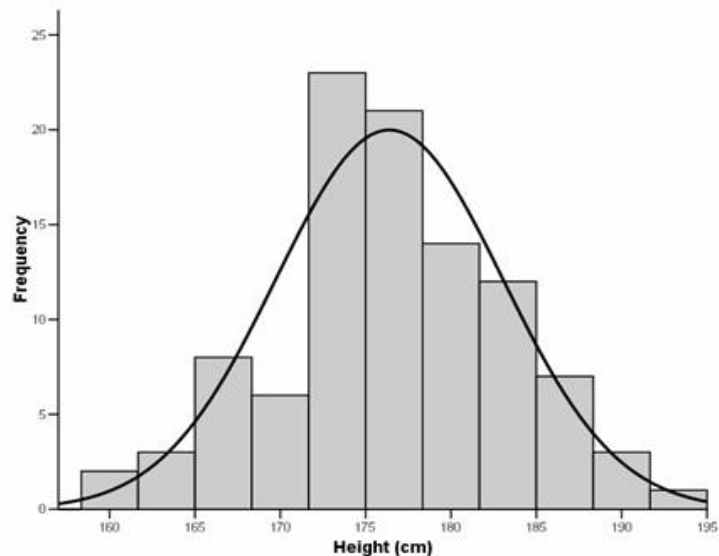


Figure 1. Frequency distribution for adult males. Image from the Sheffield University School of Health Science.

PROBLEM SKILLS

The set of data in the previous figure is called a distribution, and has been placed in a type of graph called a histogram, often called a bar chart. The x -axis is used to show the value of a particular measurement. In many cases, like that above, a range of measurements are grouped together for counting and each bar corresponds to a range called the bin. The y -axis shows frequency, which is the number of times that measurement occurred. In this case a curve is displaced on top of the measurements for comparison, but the curve is not part of the data.

In a histogram there are three ways to define the central value of the distribution. The *mode* represents the bin with the greatest frequency, 172-175 cm in our example. The *median* represents the value such that an equal number of occurrences are on either side, about 176 cm. The *mean* is the average of all the values and is 176.5 cm. These three values may be different, and we will use the mean for our measurements. For a histogram with a number of bins H , each with value x_i and frequency n_i the mean is

$$\bar{x} = \frac{\sum_{i=1}^H n_i x_i}{\sum_{i=1}^H n_i} = \frac{\sum_{i=1}^H n_i x_i}{N} \quad (\text{EQ 1})$$

where N is the total number of measurements.

The spread of values in a distribution tells us how likely the mean describes the measurement. One useful measure of the spread is the standard deviation symbolized by σ . For a measurement that fits a normal distribution the measured value will be within 1σ of the mean about $2/3$ of the time and within 2σ of the mean about 95% of the time. The standard deviation for data in a histogram in terms of the mean is

$$\sigma = \sqrt{\frac{\sum_{i=1}^H n_i (x_i - \bar{x})^2}{N - 1}} \quad (\text{EQ 2})$$

The standard deviation refers to the spread of measurements, but it doesn't say how accurate the mean is compared to the expected value if an infinite number of measurements were made. The standard error of the mean σ_m is related to the standard deviation by

$$\sigma_m = \frac{\sigma}{\sqrt{N}} \quad (\text{EQ 3})$$

So making repeated measurements improves the measurement of the mean, but the additional reduction is less as more measurements are made. Making 4 measurements reduces the error of the mean by a factor of 2, but making 16 measurements only reduces the error of the mean by a factor of 4.

BACKGROUND INFORMATION

The standard error of the mean is an example of a *statistical error*. This isn't an error in the sense of a mistake, but it is a way of describing the uncertainty in a value. A measurement value can be off due to other factors such as bad calibration or biased techniques, and these are called *systematic errors*. Statistical errors cannot be avoided, but that can be handled mathematically.

We've seen how to get the standard error of the mean from a statistical sample, but sometimes we don't have a sample to rely on. Significant figures are another way of getting the measurement error. For a typical measuring device the 1 unit of most significant figure is the measurement error. A stopwatch reading 12.07 s would have a measurement error of 0.01 s.

In laboratory experiments we often have to use measurements in formulas to compute other quantities. What is the error on the resulting computation? The process to find that is called the propagation of errors. If a value with an error is multiplied by a number precisely known, then multiply the error by the same amount. If two values a and b are added or subtracted and they have errors σ_a and σ_b the error on the result σ_y is given by

$$\sigma_y^2 = \sigma_a^2 + \sigma_b^2 \quad (\text{EQ 4})$$

This is known as adding errors in quadrature. If two values a and b are multiplied or divided and they have errors σ_a and σ_b the error on the result σ_y is given by

$$\left(\frac{\sigma_y}{y}\right)^2 = \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 \quad (\text{EQ 5})$$

In this case the fractional errors are added in quadrature. Finally, if a value is raised to a power then the fractional error is multiplied by that power.

PROBLEM SOLVING**Part A. Coin Toss**

1. Assign group members to be persons A, B (and C). The assignments should be different than in previous oral presentation labs.
2. Toss 10 coins and count the numbers of heads that occur.
3. Repeat step 1 nine additional times and record the ten trials in a histogram.
4. Calculate the mean number of heads from the data in steps 1 and 2.
5. Find the standard deviation of the number of heads using EQ 2.
6. Find the standard deviation of the mean using EQ 3.
7. Share your data from steps 1 and 2 with other groups to make a single histogram.
8. Find the mean and standard deviation for the data in step 6 using EQ 1 and EQ 2.
9. Share your results from steps 4, 5, and 7 with other groups (person A); Record your results and those of the other groups (person B); Ask questions of other presenters (person C).
10. Find the mean and standard deviation of the mean from all groups using EQ 3.

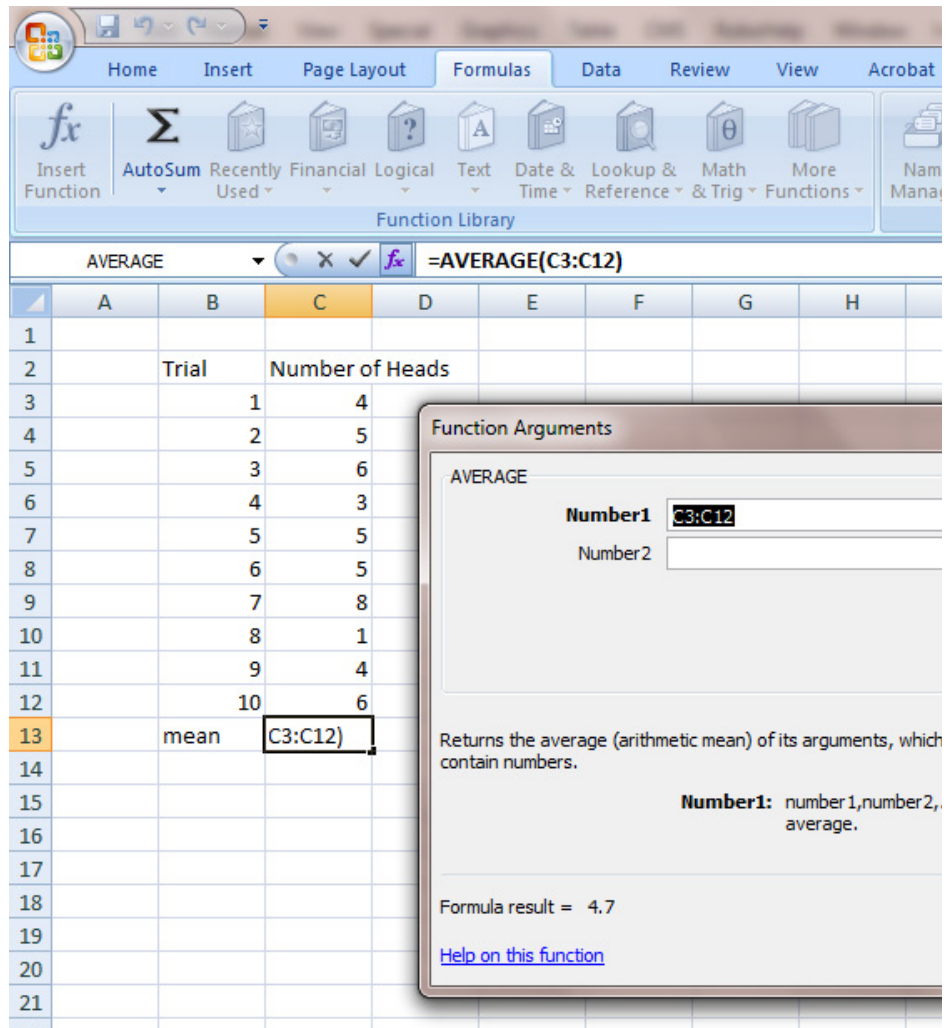
Part B. Table Size

11. Note the smallest unit on the meterstick and assume that is the measurement error.
12. Have each group member use the meter stick to separately measure the length and width of your table.
13. Use the length and width to calculate the perimeter of the table for each member's measurement.
14. Find the error on the measurement of the perimeter using EQ 4.
15. Compare the range of calculated perimeters to the error.
16. Use the length and width to calculate the area of the table for each group member's measurement.
17. Find the error on the measurement of the area using EQ 5.
18. Compare the range of calculated areas to the error.
19. Make a presentation of the steps and results (person B); Record your results and those of the other groups (person A); Ask questions of other presenters (person C).

COMPUTER SKILLS

The lab computers have both Word and Excel available. There is no internet in the lab, so bring a USB drive to store your work. Each person in the group should separately do this exercise in the lab. Let the TA know when it is complete, so that you can get credit and answer any questions from the TA.

1. Open a blank document in Word.
2. Type the title, date, and name with lab partners for the lab with the "Title" and "Sub-title" styles. Use the style "Title" in the home tab for this.
3. Write a paragraph describing your work in part A (person A, C) of the problem solving, or in part B (person B)
4. Select the **Insert** tab, then click on **Equation** in the *Symbols* section.
5. In the empty box type EQ 3. Use the **Script** menu in the *Structures* section to enter a subscript, and options in the *Symbols* section to type Greek characters like σ . Use the **Fraction** and **Radical** menus in the *Structures* section to enter those parts of the equation.
6. Create a table in Excel of either the 10 coin toss trials for your group (person A), the average number of heads for each group (person B), or the error on the measurement for each group (person C). Include a column to note the trial number or group on the left and label the top of each column.
7. Click on the blank cell below the bottom of the column with the trial/group numbers and type "mean".
8. Click on the blank cell below the bottom of the column with the data.
9. Select the **Formulas** tab and then click on **Insert Function**. In the dialog box select the "Statistical" category from the drop down menu, and the AVERAGE function. Click OK. In the **Function Arguments** dialog box confirm that the values in the **Number1** field correspond to the first and last cells that have your data, such as C3:C12 in the example that follows.



10. Click OK and the value of the mean will appear in the cell.
11. In the cell below “mean” type “std. dev.”, and in the cell to its right repeat steps 7 and 8, but select the STDEV function. Make sure that the range of cells does not include the cell where you put the mean.
12. Place a border around the table and paste it into your Word document with a caption.
13. Let the TA check you on your work (each member must do this separately).
14. Save the file and turn it in for grading.