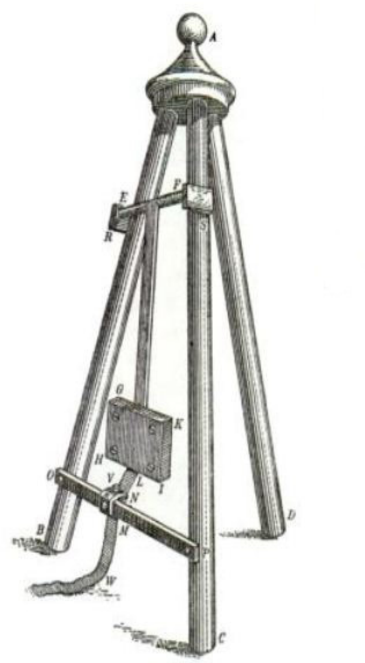
Ballistics

Goals: Use a data table for interpolation.
Propagate errors for values involving
products, sums, and square roots.

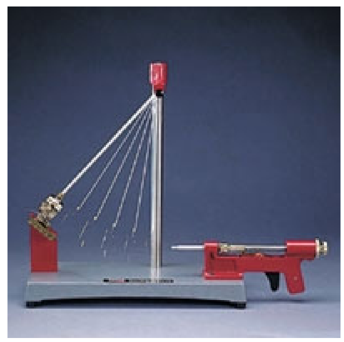
APPARATUS

Ballistics is the science that deals with the launch, flight, and performance of projectiles. One of the important parameters of ballistics is the muzzle velocity, that is the initial velocity of a projectile upon leaving a launcher. A high speed projectile like a bullet is difficult to measure, but in 1742 Benjamin Robins invented the ballistic pendulum to measure both the momentum and kinetic energy of a gun and the bullet that it fires and his original diagram is shown below. Later inventions have replaced the ballistic pendulum to measure muzzle velocity with greater accuracy, but the technique still works.



Our ballistic pendulum is a device consisting of three parts: a spring gun, a ball that can be launched from the gun, and a cup at the end of a pendulum to catch the ball. The spring gun is designed to fire a ball of mass (m_b) with an initial velocity (v_i). The pendulum and cup can be moved out of the way. This permits the ball to be fired as a projectile and the initial velocity measured by means of its trajectory.

When the pendulum is in its lower position, the cup with mass (m_c) is ready to catch the ball when fired. When the ball is caught in the cup, the energy of the combined cup and ball is used to swing the pendulum up by a height (h). A ratchet catches the cup and allows you to read a position measurement by means of a pointer that catches in a groove of the ratchet. A multiple exposure image of the apparatus is below (D. Simanek, Lock Haven University).



The ratchet does not directly measure the height, but is marked off in arbitrary units that go from 0 to 40 in steps of 10. There are marks at the halfway points (5, 15, 25, 35), and each individual ratchet position that represent steps of 1 unit. The height displacement has been measured for the labeled positions and can be found in Table 1.

TABLE 1. Conversion of ratchet measurement to height.

Ratchet Measurement	Height (mm)
0	65
10	73
20	80
30	88
40	96

To find the height for other ratchet measurements requires interpolation. Interpolation means estimating the value between two known values. We do this by assuming that the conversion function is a straight line between the two known points. Then the ratio of difference between the measured value and the lower known value (Δx) compared to the difference between the two known values (Δx_T) must be equal to the ratio of the actual height difference (Δh) to the height difference between the two points (Δx_T).

$$\frac{\Delta x}{\Delta x_T} = \frac{\Delta h}{\Delta h_T} \tag{EQ 1}$$

For our experiment let x be the position of the ratchet and we want to find the corresponding height of the ball (h). To interpolate look at your measurement x. If it's not in the table, find the measurement in the table (x_L) that is immediately below your measurement and its height (h_L) from the table. Find the measurement in the table (x_H) that is immediately above your measurement and its height (h_H). The ratchet difference is $\Delta x = x - x_L$ and the difference between the points on the table is $\Delta x_T = x_H - x_L$ which should be 10. The interpolated height difference is $\Delta h = h - h_L$ and the height difference from the table is $\Delta h_T = h_H - h_L$. By rearranging EQ 1 we can directly find the interpolated height (h) for your measurement.

$$h = \left(\frac{x - x_L}{x_H - x_L}\right)(h_H - h_L) + h_L \tag{EQ 2}$$

Like any measurement there is error associated with interpolation. The table doesn't provide errors for the values, but you can assume that the error on the ratchet position is equal to 1 unit. Since there are 10 units between points on the table you can use an error equal to 1/10 the height difference on the table Δh_T.

THEORY

In an earlier laboratory we studied projectile motion. A ball moving horizontally with initial velocity (v_i) was allowed to fall freely to the ground a distance (d) below the launch point. The range (R) beyond the launch point was derived from the kinematics of an object subject only to gravitational acceleration (g). The relation between the initial velocity, range, and vertical distance is in EQ 3.

$$v_i = R\sqrt{\frac{g}{2d}} \tag{EQ 3}$$

Kinetic energy is the energy of motion. It depends on the mass and velocity of an object. When the ball is initially launched it has an initial kinetic energy (K_i).

$$K_i = \frac{1}{2}m_b v_i^2 \tag{EQ 4}$$

When the ball is caught by the cup both the mass and velocity will have changed. The new mass will be the sum of the mass of the ball and the cup ($m_b + m_c$). The final velocity (v_f) immediately after the ball is in the cup is given by EQ 5.

$$v_f = v_i\left(\frac{m_b}{m_b + m_c}\right) \tag{EQ 5}$$

Using the new mass and the velocity in EQ 5, the kinetic energy after the capture (K_f) is

$$K_f = \frac{1}{2}(m_b + m_c)\left(\frac{m_b}{m_b + m_c}\right)^2 v_i^2 = \frac{1}{2}\frac{m_b^2 v_i^2}{(m_b + m_c)} \tag{EQ 6}$$

Potential energy is the energy of position. Objects that are higher can gain kinetic energy as they go lower. This gravitational potential energy (U) can be represented as

$$U = (m_b + m_c)gh \tag{EQ 7}$$

where we have used the total mass of the ball and cup, the gravitational acceleration (g = 9.8 m/s^2), and the height that the pendulum ended at after catching the ball. Conservation of energy is a fundamental principle of physics. In mechanical systems like the ballistic pendulum, energy conservation predicts that the kinetic energy can be fully converted into potential energy.

Propagation of errors applies to measurements in these equations as well. The basic rule for error propagation when the equation has addition or subtraction such as $y = a + b$ was given in an earlier lab.

$$\sigma_y^2 = \sigma_a^2 + \sigma_b^2 \tag{EQ 8}$$

The rule for expressions involving multiplication or division such as $y = a/b$ was given in an earlier lab.

$$\left(\frac{\sigma_y}{y}\right)^2 = \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 \tag{EQ 9}$$

The same rule applies to more complicated equations, but there are a few things to keep in mind. First is that any pure number like 2 has no error associated with it. Second is that a physical constant like g has an error that is equal to the smallest digit, 0.1 m/s^2 if you use 9.8 m/s^2. Third is that the square of the error of term inside a square root are multiplied by 1/2, since the square root is raising a value to the 1/2 power. That makes the error equation for EQ 3 look like the following expression.

$$\left(\frac{\sigma_v}{v_i}\right)^2 = \left(\frac{\sigma_R}{R}\right)^2 + \frac{1}{2}\left(\frac{\sigma_g}{g}\right)^2 + \frac{1}{2}\left(\frac{\sigma_d}{d}\right)^2 \tag{EQ 10}$$

When a quantity has the square of a variable a factor of two appears in the error equation. For example EQ 4 has the following expression to propagate errors.

$$\left(\frac{\sigma_K}{K_i}\right)^2 = \left(\frac{\sigma_m}{m_b}\right)^2 + 2\left(\frac{\sigma_v}{v_i}\right)^2 \tag{EQ 11}$$

DATA COLLECTION

Part A - Muzzle velocity

1. Make sure the apparatus is clamped to the table, a stop wall is available, and swing the pendulum arm out of the way.
2. Measure the vertical drop (d) from the end of the gun to the floor and record this value.
3. Fire the gun and note where the ball lands. Fasten sheets of carbon paper at the location the ball landed.
4. Fire the gun 10 times so that the impacts are recorded on the carbon paper.
5. Measure the horizontal distance from the spot on the floor directly beneath the tip of the gun to the edge of the paper.
6. Measure and record the distance of each mark on the paper from the edge of the paper and find the average, and the standard error.
7. Add the average distance on the paper to the distance of the paper from the gun along the floor to get the measured range (R). Find the error on the total range using the rules for the sum of values.
8. Calculate the initial velocity (v_i) using EQ 3 above. Find the error on the initial velocity using EQ 10.

Part B - Conservation of energy

9. Weigh and record the mass of the ball (m_b) and record the mass of cup (m_b) written on the apparatus. Include errors for those recorded values, and assume an error equal to the least significant digit, if no error value is given.
10. Fire the gun with the ball into the cup 10 times and record the ratchet measurement for each trial.
11. Find the average ratchet measurement (x) from step 10 and use EQ 2 to interpolate with the data in Table 1 to find the height (h). Include the error on the height.
12. Use the initial velocity from step 8 and EQ 4 to find the initial kinetic energy (K_i). Use EQ 11 to find the error on K_i.
13. Use the initial velocity from step 8 and EQ 6 to find the final kinetic energy (K_f).
14. Use the height from step 11 and EQ 7 to find the potential energy (U). First use the sum rule to get the error on the total mass, then use the product rule to get the error on U.
15. Find the difference between K_f and U, and compute a percent difference between K_f and U.
16. Your TA will assign an addition question or two to answer in the report. This work should be done by each group member individually.
17. Each student should assemble a single report from the group data report and the additional individual question. This report will be turned in for grading.