Axle

Goals: Use a computer-interfaced timing device. Compare data from two measurement techniques on the same apparatus. Combine and compare analog and digital uncertainties.

APPARATUS

The axle when used with a wheel is one of the simple machines known since ancient times. By itself an axle is a central shaft designed to rotate and transfer that rotary motion to and from attached components such as in the anemometer used to measure wind speed.

THE ROBINSON ANEMOMETER.

This experiment uses a vertical shaft that can freely rotate to spin a massive bob (m). The bob hangs by two strings from a horizontal bar with a counterweight on the other side. The counterweight helps the shaft rotate evenly. A spring can connect the bob to the shaft and provides a force to pull the hanging mass toward the shaft.

A vertical indicator is used to line up the bob. When the spring is not connected to the bob, the bob should hang directly over the indicator. A string can be connected from the bob over a pulley at the end of the apparatus to a hanging weight. The weight is used to measure the spring force when the shaft is not rotating.

When the weight is disconnected from the bob, the shaft can be spun by hand. As the shaft spins, the mass will swing away from the shaft held back by the spring. With enough rotational speed, the mass will line up directly over a vertical post. A photogate is used to measure the number of revolutions (n) in a period of time (t) for the spinning bob. The photogate counts both the entry and exit of the bob through the gate so it actually counts $2n$.

THEORY

Force, acceleration and velocity are all vectors. This means that they have both size and direction. Acceleration occurs when a velocity is changed in some length of time. Because velocity is a vector, this can change in direction and not speed. A car accelerates as it goes around a curve even though its speed is not changing.

Uniform circular motion occurs when an object moves in a circular path at a constant speed (v).The speed of the object moving in a circle can be determined by the angular frequency (ω) times the radius of the circle (r). The angular frequency is the number of times (n) an object spins around times 2π radians divided by the period of time (t). As an equation this is

$$v = \omega r = \frac{2\pi n r}{t} \qquad \textbf{(EQ 1)}$$

For uniform circular motion the acceleration is always pointed toward the center of the circle. This acceleration is called centripetal acceleration. The magnitude of the centripetal acceleration (a) is v^2/r or using EQ 1 for the speed (v):

$$a = \frac{v^2}{r} = \frac{4\pi^2 n^2 r}{t^2} \qquad \textbf{(EQ 2)}$$

A net force on an object gives rise to an acceleration on that object. The acceleration is in the same direction as the force and has a magnitude equal to the net force divided by the mass of the object. Since an object in uniform circular motion has an inward pointing acceleration, it must also have an inward pointing force. That force is called the centripetal force and from EQ 2 it is:

$$F_c = ma = \frac{4\pi^2 m n^2 r}{t^2} \qquad \textbf{(EQ 3)}$$

A stretched spring exerts a force that tends to pull the spring back together. The force exerted by the spring can be balanced by an equal and opposite gravitational force from the hanging mass (M) using the pulley. The inward-directed force of the spring is

$$F_s = Mg \qquad \textbf{(EQ 4)}$$

In our apparatus the spring provides the centripetal force for the bob. Therefore we expect that the centripetal force calculated with EQ 3 equals the spring force calculated with EQ 4.

DATA COLLECTION

1. Determine the mass of the bob (m) with a scale and the diameter of the vertical shaft (d) with a ruler. Record these values with uncertainties.
2. Slide the indicator bar to the closest position and measure the distance from the shaft to the indicator and add half the diameter ($d/2$). This total is the radius of its circular path (r). Record r and an estimated uncertainty on r.
3. Unscrew the spring from the shaft. With the spring disconnected, adjust the top screw on the horizontal bar so that the bob hangs directly over the indicator. Reattach the spring and tighten all screws.
4. Attach the string to the pulley and hanging mass.
5. Measure the mass of the weight hanger and add weight to the hanger until the bob is directly over the indicator. Record the total hanging mass (M) and its uncertainty.
6. Disconnect the string and hanging mass from the bob.
7. Open LoggerPro on the computer. From the **Experiment** pull-down menu, select **Set Up Sensors**, then select the *LabPro: 1* option. On the *LabPro: 1* dialog box, click on **DIG/SONIC 1** and choose *Photogate*. You should see three graphs appear, and they may pop up automatically when Logger Pro is started.
8. From the **Experiment** pull-down menu, select **Extend Collection**, and repeat until the time shown by **Extend Selection** is greater than 100 s.
9. One person should practice rotating the vertical shaft so that the bob moves to consistently pass over the indicator. Hold a white sheet of paper behind the indicator to see the bob and indicator tips more easily.
10. When you feel that you can do this, position the photogate so that the spinning bob passes through the photogate without hitting it.
11. Press **COLLECT** to initiate the photogate for data collection. The times will appear in the leftmost column. The times are in pairs corresponding to when the bob first enters (*GateState 1*) then leaves (*GateState 2*) the photogate. The time difference between two consecutive *GateState 1* times is the time for one revolution.
12. Press **STOP** to end data collection after times are filled to at least 100.
13. Record the time (t) it takes for the bob to make 50 revolutions (n) using the photogate detector (recall that the detector actually counts $2n$). To reset for a new run, click on the **Experiment** drop-down menu and select **Clear Latest Run**.
14. Repeat steps 10 through 13 four more times and record the results. Find the average value of the five values for t, using the standard error.
15. Slide the indicator bar to a position near the middle. Measure the radius (r) as in step 2, and repeat steps 3 through 14 for the new position finding M and t.
16. Slide the indicator bar to a position near the end. Measure the radius (r) as in step 2, and repeat steps 3 through 14 for the new position finding M and t.

DATA ANALYSIS

17. Use EQ 3 and *m*, *r*, *t*, and *n* from steps 2 through 14 to find the measured centripetal force (F_c) from the spring in N.
18. Determine the uncertainty in *Fc* using propagation of errors.
19. Use EQ 4 and *M* from step 5 to find the measured spring force (F_s) in N.
20. Determine the uncertainty in *Fs* using propagation of errors.
21. Find the difference between F_c and F_s. Divide this by the average of F_c and F_s and multiply that ratio by 100 to get the percent difference between the values.
22. Repeat steps 17 through 21 for the data collected in steps 15 and 16.
23. Your TA will assign an additional question or two to answer in the report. This work should be done by each group member individually.
24. Each student should assemble a single report from the group data report and the additional individual question. This report will be turned in for grading.