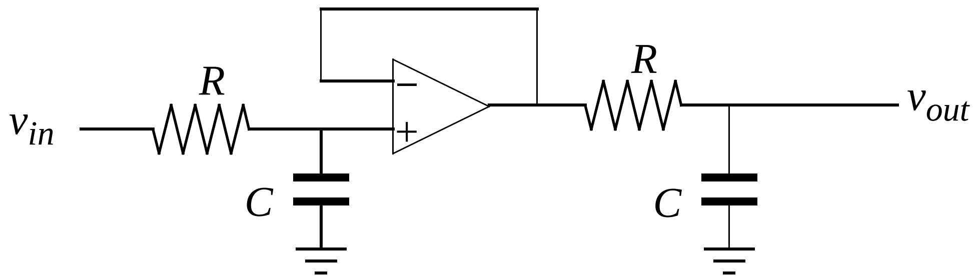
Active Filters

- An active filters means using amplifiers to improve the filter.
- An acive second-order RC low-pass filter still has two RC components in series.



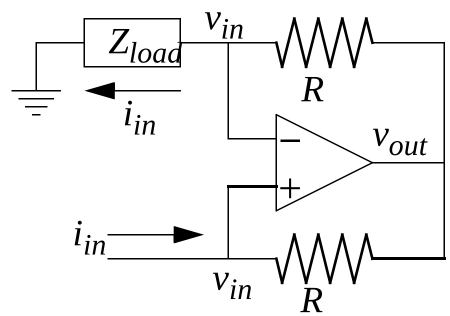
$$H(j\omega) = \left(\frac{1}{1 + j\omega/\omega_B}\right)^2 = \frac{1}{1 + 2j\omega/\omega_B - (\omega/\omega_B)^2}$$

$$G(\omega) = \frac{1}{\sqrt{1 + 2(\omega/\omega_B)^2 + (\omega/\omega_B)^4}} = \frac{1}{1 + (\omega/\omega_B)^2}$$

- The amplifier buffer changes the impedance.
- This filter is sharper than the non-buffered filter.

Gyrator

- Active feedback alters the effective impedance of an element.
- The gyrator alters the apparent impedance of the load element.



- The op-amp rules are used to find the effective input impedance.

  On the non-inverting input:

$$v_{in} = v_{out} + i_{in}R$$

On the inverting input:

$$v_{in} = \left[\frac{Z_{load}}{Z_{load} + R}\right]v_{out} = \left[\frac{Z_{load}}{Z_{load} + R}\right](v_{in} - i_{in}R)$$

$$i_{in} = \frac{1}{R}\left[1 - \frac{Z_{load} + R}{Z_{load}}\right]v_{in} = \left[-\frac{1}{Z_{load}}\right]v_{in}$$

So and the impedance is the negative of the load. $Z_{in} = \frac{v_{in}}{i_{in}} = -Z_{load}$

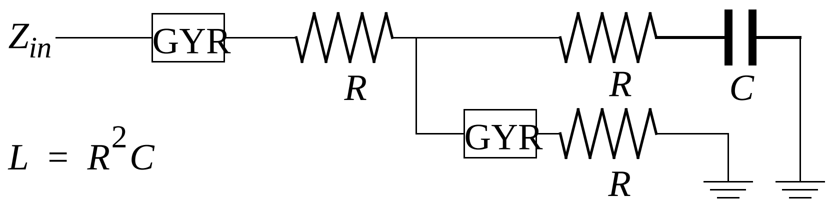
Artificial Inductor

- If the load on a gyrator is a capacitor the phase behaves like an inductor.

$$Z_{in} = -\frac{1}{j\omega C} = \frac{j}{\omega C}$$

- This would not have the frequency dependence of an inductor.

- With two gyrators a capacitor can act as an inductor in phase and frequency.



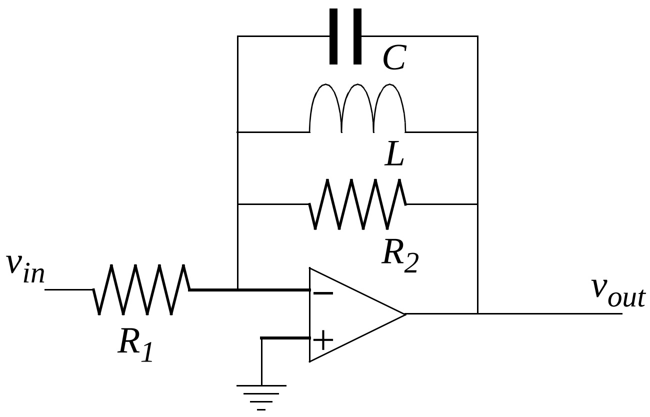
$$L = R^2 C$$

$$Z_{in} = -\left[R + \frac{-R(R + 1/j\omega C)}{1/j\omega C}\right] = j\omega R^2 C$$

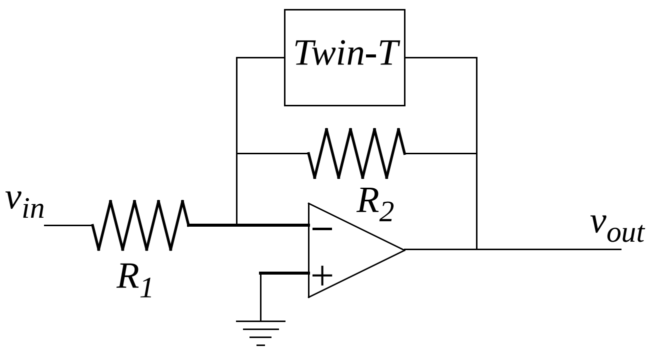
- Inductors are difficult to miniaturize and have intrinsic resistance that may be undesirable.
- The inductor can be replaced by a gyrator circuit.

Active Bandpass

- A single pole filter uses a parallel RLC circuit as the feedback network.



- Use a notch filter in a inverting amplifier also makes a bandpass filter.

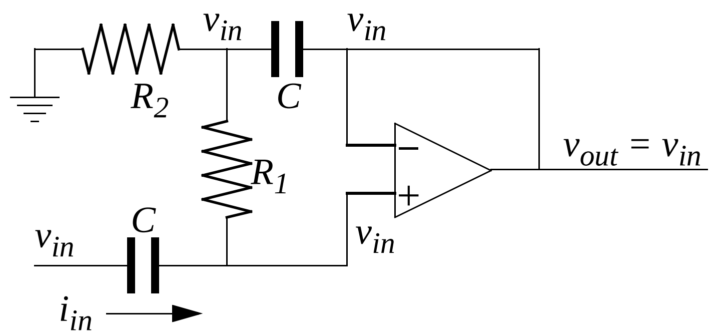


At the notch frequency the Twin-T has infinite impedance, $A = -R_2/R_1$.

If $R_1 >> Z_{TT}$, for other frequencies $A = -R_{TT}/R_1 = 0$.

Bootstrapping

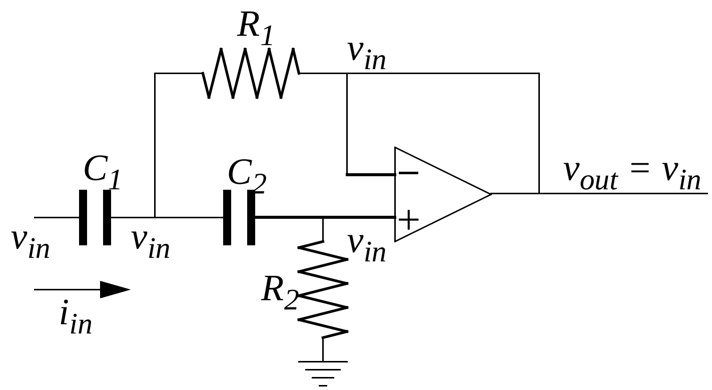
- Bootstrapping refers to the use of feedback to make a very large input impedance by setting $i_{in} = 0$ through a coupling capacitor for AC frequencies of interest.



- The circuit is set for an AC input signal with capacitive coupling to the non-inverting input.
- The feedback circuit will give unity gain of the AC part of the signal.
- If the AC part of the signal can pass through the coupling capacitor, it can also pass through the capacitor at the inverting input.
- The signal is the same on both sides of resristor R_1.
- Since the voltage drop across $R_1 = 0$ and no current enters the opamp, $i_{in} = 0$.
- The input impedance $Z_{in} = v_{in}/i_{in}$, and should be very large.

Sallen-Key Filter

- The Sallen-Key filter looks like two RC filters (two pole) and a x1 amplifier (buffer).
- There is a bootstrap to create a large input impedance.
- For example, a high-pass Sallen-Key filter uses a resistor as the bootstrap.



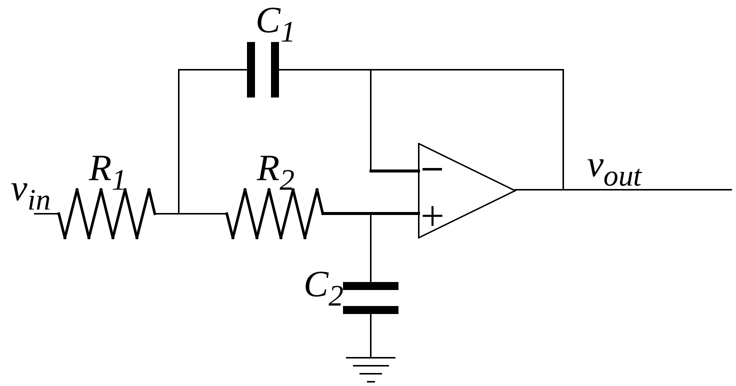
- The breakpoint frequency is

$$\omega_b = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

- The roll-off for very low frequencies is 40 dB/decade or 12 dB/octave.

Low Pass Sallen-Key

- The low-pass Sallen-Key filter swaps the resistors and capacitors.



- The circuit behavior is equivalent to a damped driven mechanical oscillator.
- The driving force is v_{in}.
- The damping factor is

$$d_0 = \frac{1}{Q} = (R_1 + R_2)C_2\omega_b$$

- As a passive network, the oscillator can be overdamped, underdamped or critically damped:

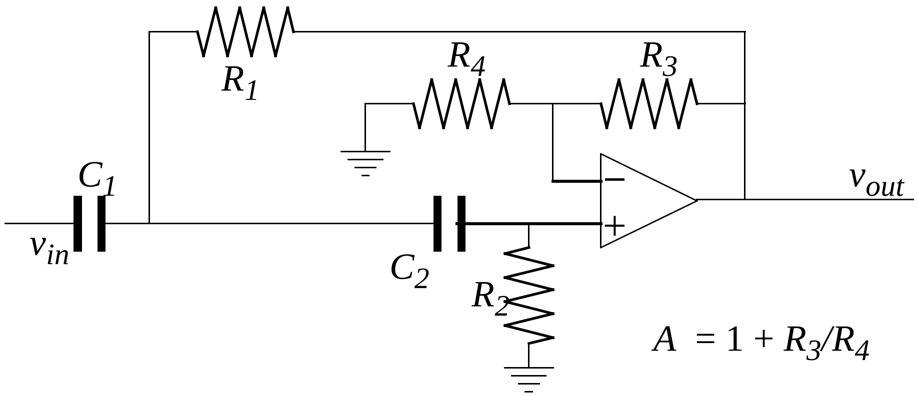
  overdamped ($d_0^2 > 2$),

  underdamped ($d_0^2 < 2$),

  critically damped ($d_0^2 = 2$).

Voltage-Controlled Voltage Source

- A voltage-controlled voltage source (VCVS) replaces the Sallen-Key unity gain buffer with a non-inverting amplifier.
- The high-pass VCVS has two additional resistors to create a gain A.

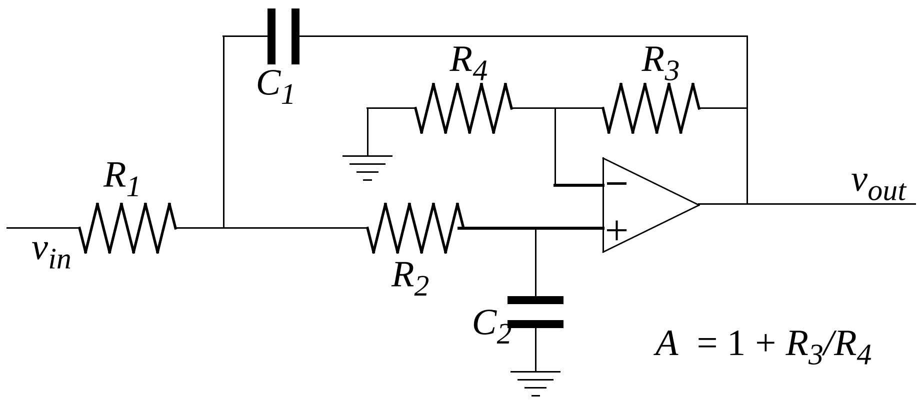


- The gain is usually expressed as a factor K.

$$K = A = 1 + \frac{R_3}{R_4}$$

VCVS Damping

- As with the Sallen-Key, the low-pass VCVS swaps pairs of resistors and capacitors.



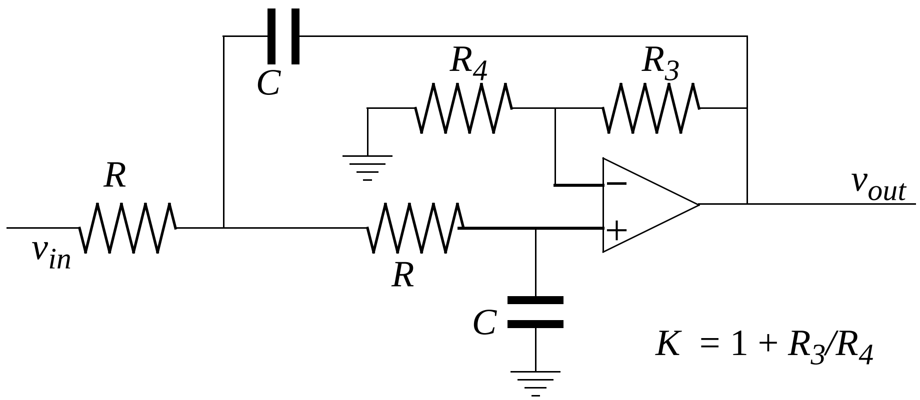
- With a variable gain for negative feedback and matching $R_1 = R_2 = R$, $C_1 = C_2 = C$, the gain and damping are independent of the break frequency $\omega_B = \frac{1}{RC}$.

- The gain remains $A_0 = 1 + \frac{R_3}{R_4}$.

- The damping is $d_0 = 3 - A_0 = 2 - \frac{R_3}{R_4}$

Butterworth Low-Pass

- The VCVS can be used to make an active filter version of the Butterworth filter.



- The desired damping is $d_0 = 1.414$, so $K = 3 - d_0 = 1.586$.

  The ratio $R_3 / R_4 = 2 - d_0 = 0.586$.

- For a cutoff frequency at 1 KHz, $\omega_B = 6.28 \times 10^3\ s^{-1}$.

  Resistors are best in range from 1 KΩ to 10 KΩ

  So typical $C = 1/R\omega_B$; about 0.1 μF at 1.5 KΩ.

- High pass just inverts R and C in the circuit.
- Higher order filters requires multiple stages, with K-values set for each stage.

Bessel and Chebyshev VCVS

- The VCVS can also be used to make other special purpose active filters.

- The Bessel filter needs a normalizing factor f_n for the frequency: $RC = 1/f_n\omega_B$.

  The gain for a Bessel filter requires $K = 1.268$, and the frequency factor is $f_n = 1.272$.

- The Chebyshev filter has an additional variable that controls the ripple band size.

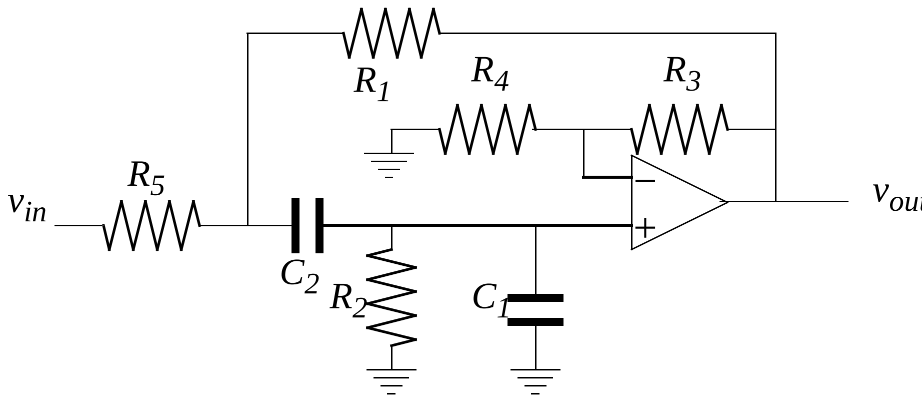
  For a ripple band of 0.5 dB: $K = 1.842$, $f_n = 1.231$.

  For a ripple band of 2.0 dB: $K = 2.114$, $f_n = 0.907$.

- Users of specific values and higher order filters rely on tanles of K and f_n values for circuit design.

Bandpass VCVS Filter

- The VCVS circuit can also be used to create a bandpass filter.
- Use one-pole for each cutoff frequency.



- The cutoff frequencies are at $1/R_1C_1$ and $1/R_2C_2$.