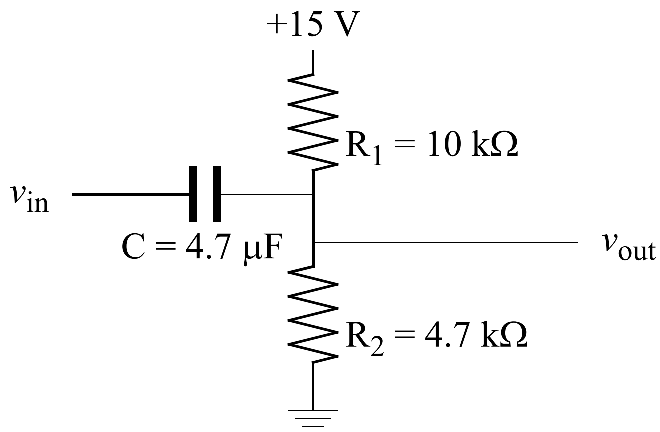
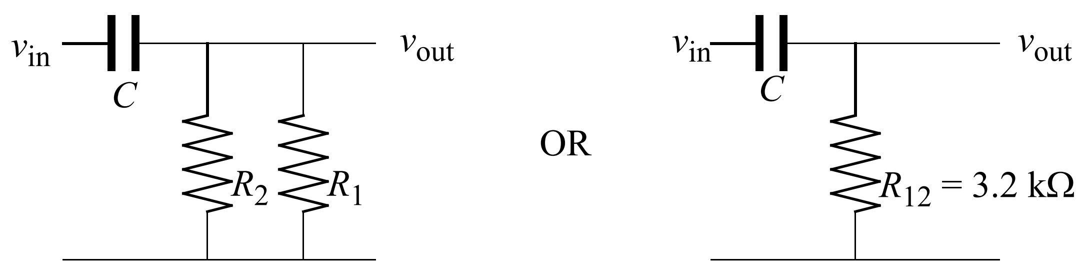
Signal Transmission

Capacitive Coupling



- For AC signals the equivalent circuit is



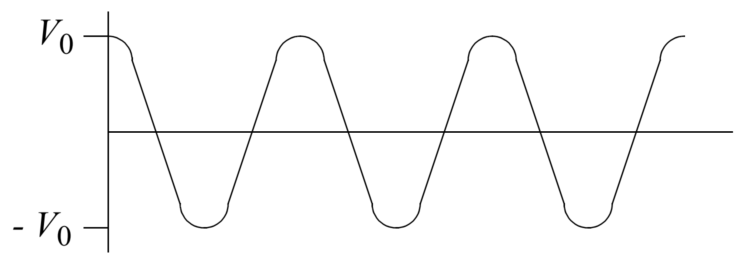
- The high pass breakpoint is at $f = 1/2\pi R_{12}C$ = 10.6 Hz.

DC Shift

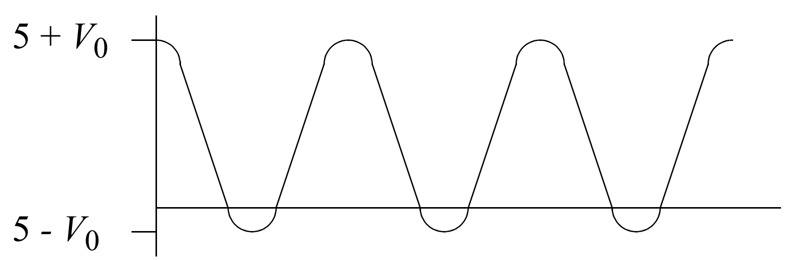
- With no input or DC input, R_1, R_2 form a resistor divider: $V_{out} = 5$ V.
- The AC part of v_{in} is added to 5 V.

$$v_{out} = (v_{in})_{AC} + 5V$$

- Input AC signal



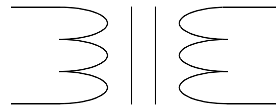
- Output AC signal



- A capacitor blocks DC signals but passes AC signals.

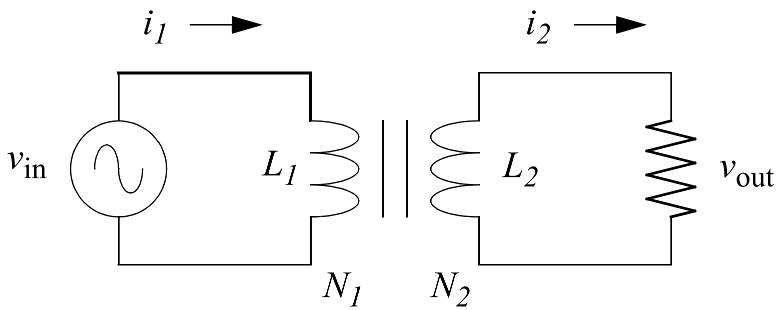
Transformer

- Schematic symbol



Transformers are a pair of inductors around a common center used to convert AC voltages to lower or higher values.

- Mutual Inductance



$$v_{in} = -\frac{d\phi_1}{dt}$$

$$v_{out} = -\frac{d\phi_2}{dt}$$

Magnetic flux in the the primary coil is carried into the secondary coil.

Induced current depends on the magnetic flux in the secondary coil

$$v_{in} = M\frac{di_2}{dt} + L_1\frac{di_1}{dt} \qquad v_{out} = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

M is the mutual inductance of the transformer (henrys)

Transformer Coupling

- Voltage and current change through a transformer.

  when $i_1 = 0$, $v_{in}/v_{out} = M/L_2$; when $i_2 = 0$, $v_{in}/v_{out} = L_1/M$.

$$\frac{[v_{in}/v_{out}]_{i_1 = 0}}{[v_{in}/v_{out}]_{i_2 = 0}} = \frac{M^2}{L_1 L_2} = K^2$$

  ideal K = 1

  ratio of turns $n = N_1/N_2$.

$$v_{out}/v_{in} = \frac{N_2}{N_1} = \frac{1}{n}$$

$$i_{out}/i_{in} = \frac{N_1}{N_2} = n$$

- Input and output impedance through a transformer

$$Z_{in} = \frac{v_{in}}{i_{in}}$$

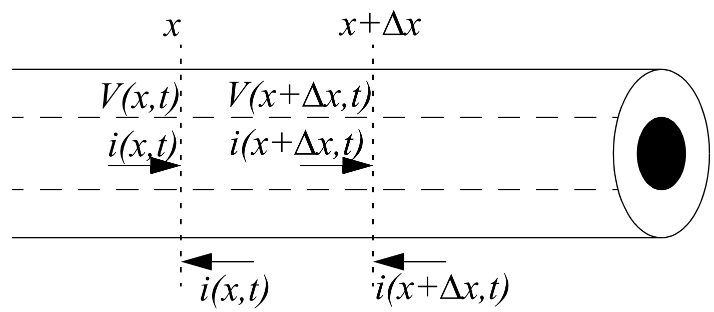
$$Z_{out} = \frac{v_{out}}{i_{out}}$$

$$Z_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{i_{in}}\left(\frac{i_{out}}{v_{out}}\frac{v_{out}}{i_{out}}\right) = n^2 Z_{out}$$

Transmission Lines

Coaxial Cable

- One conductor wrapped around another



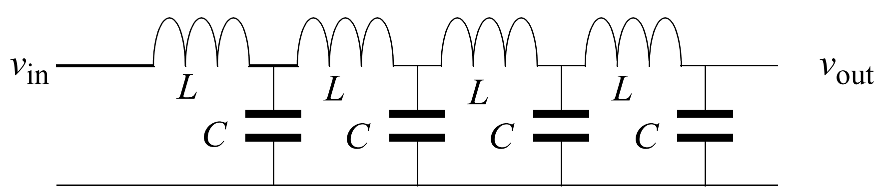
The voltage and current are both functions of position and time

For a transmission line of length l and speed v the time it takes is $t = l/v$.

For low frequencies ($\omega \ll v/l$) the voltage is the same on both ends.

For high frequencies ($\omega \gg v/l$) the voltage will be different on the two ends of the cable.

- A coaxial cable looks like a capacitance between the inner and outer conductors and an inductance (and resistance) along its length.



- This can be approximated by a capacitance per unit length c_0 and inductance per unit length l_0.

Transmission Signal

Consider the change in voltage on a length Δx of the cable and assume R=0.

$$V(x) - V(x + \Delta x) = -\Delta V = l_0 \frac{\partial i}{\partial t} \Delta x$$

$$-\frac{\partial V}{\partial x} = l_0 \frac{\partial i}{\partial t}$$

Now consider the change in current

$$i(x) - i(x + \Delta x) = -\Delta i = c_0 \frac{\partial V}{\partial t} \Delta x$$

$$-\frac{\partial i}{\partial x} = c_0 \frac{\partial V}{\partial t}$$

Differentiating the two equations by x and t permits solving them for current or voltage separately as second order differential equations.

$$\frac{\partial^2 V}{\partial x^2} = l_0 c_0 \frac{\partial^2 V}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial x^2} = l_0 c_0 \frac{\partial^2 i}{\partial t^2}$$

These are known as wave equations and $v^2 = 1/l_0 c_0$, the velocity of the wave. The solution to the wave equation for voltage is

$$V(x, t) = V_1(x + vt) + V_2(x - vt)$$

Cable Impedance

- Travelling wave on the cable

$$V(x - vt) = V_0 e^{j(x - vt)}$$

$$i(x - vt) = i_0 e^{j(x - vt)}$$

- Use the differential equation to get a relation between V and i.

$$-\frac{\partial V}{\partial x} = l_0 \frac{\partial i}{\partial t}$$

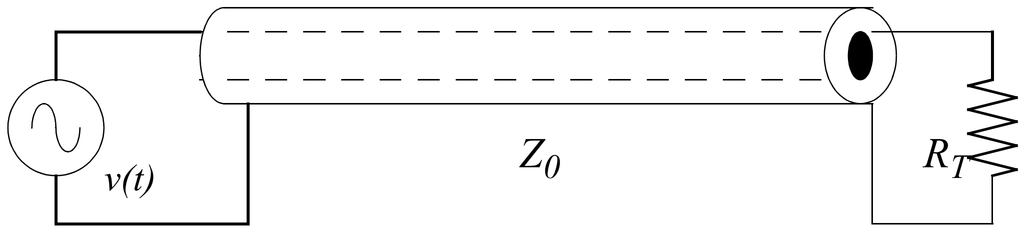
$$-jV_0 = -jvl_0 i_0 = \frac{-jl_0 i_0}{\sqrt{l_0 c_0}} = -ji_0 \sqrt{\frac{l_0}{c_0}}$$

$$V_0 = i_0 \sqrt{\frac{l_0}{c_0}}$$

- This looks like Ohm's law with the impedance $Z_0 = \sqrt{\frac{l_0}{c_0}}$.
- A cable can be treated simply as a device of fixed impedance, regardless of length.

Reflection

- A voltage source driving a cable looks like the following



- If the circuit is open, the reflected wave bounces back along the center conductor with the same magnitude as the incident wave arrived.

$$V_{ref} = V_{inc}$$

- If $R_T = 0$ (short circuit) the reflected wave comes back along the case, so at the center conductor it appears with opposite sign, canceling the incident wave.

$$V_{ref} = -V_{inc}$$

Termination

- For an arbitrary resistor $Z=R_T$ (the *terminating* resistor) the reflected and incident waves are related by the following equation.

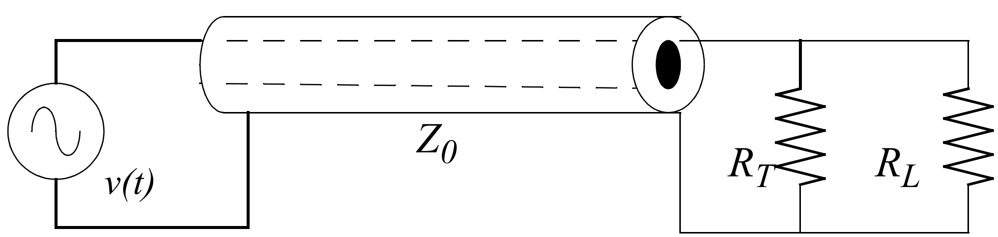
$$\frac{V_{ref}}{V_{inc}} = \frac{R_T - Z_0}{R_T + Z_0}$$

- This can be rewritten in terms of an unknown terminating impedance

$$Z = Z_0 \frac{V_{inc} + V_{ref}}{V_{inc} - V_{ref}}$$

Impedance Matching

- If $R_T = Z_0$ then there is no reflected wave $V_{ref} = 0$. In terms of the energy, the maximum energy moves out of the cable into the following circuit.
- If a circuit with a cable is attached to another circuit, the impedances should be matched to maximize transmission of electrical energy.



Noise

Voltage Noise

- Fluctuations in voltage due to thermal motion in resistors
- Depends on temperature T and the bandwidth Δf
- An approximation gives: $\Delta V = \sqrt{4RkT(\Delta f)}$

Current Noise

- Fluctuations in current, usually across a junction

Flicker Noise

- Low frequency circuit noise, goes as $1/f$

Interference

- External noise

Interference

- High frequency pick up is common in power connections
  May be eliminated with low-pass filters - “surge suppressors”

- Capacitive coupling picks up large fluctuations from nearby circuits
  Move components apart or near to a large ground, use a “ground plane”
  Add metal shielding
  Lower impedance of inputs where possible

- Inductive coupling picks up nearby large fields, especially 60 Hz
  Keep loops small, twist wire pairs together
  Add high pass filters if appropriate

- RF (radio frequency) coupling can be picked up and amplified by resonant circuits
  Shield cables, and keep unshielded leads short

Signal-to-Noise Ratio

- Signal-to-noise uses the decibel scale for power,

  S/N = $10 \log_{10} (P_{signal}/P_{noise}) = 10 \log_{10} (V^2_s / V^2_n)$
- Useful rules:

  A factor of 10 is a 10 dB measure

  A factor of 2 is about a 3 dB measure

  0 dB is equal signal and noise
- The bandwidth used in the calculation matters:

  If the signal only covers bandwidth Δf, with an S/N = A, the noise is

$$(\Delta V)^2 = 4RkT(\Delta f)$$

  If S/N is measured with $2\Delta f$, the signal is the same but the noise is

$$(\Delta V)^2 = 4RkT(2\Delta f)$$

  a factor 2 greater, so S/N drops by 3 dB