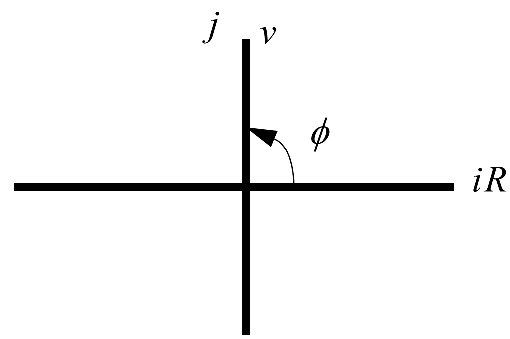
Signal Phase

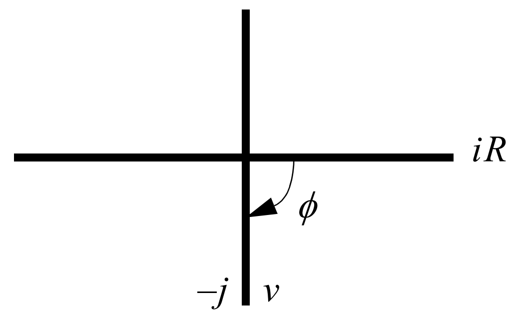
- Voltage (E) through an inductor (L) leads the current (I).

$$v = j\omega L i$$



- Current (I) through a capacitor (C) leads the voltage (E).

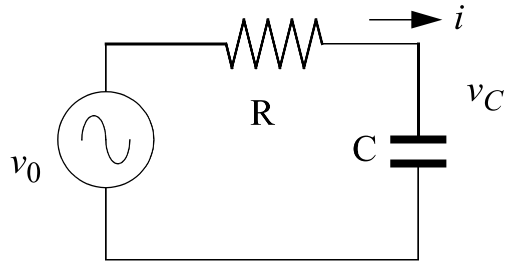
$$v = \frac{1}{j\omega C} i$$



- "ELI the ICE man."

RC Filter Phase

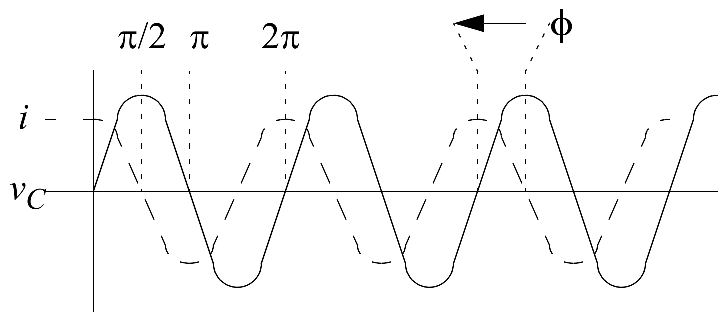
- Example: the low pass filter:



$$v_0 = Ri + \frac{1}{C}\int i\,dt$$

$$v_C = \frac{1}{C}\int i\,dt$$

If $i = \cos(\omega t)$, $v_C = \sin(\omega t) = \cos(\omega t - \pi/2)$



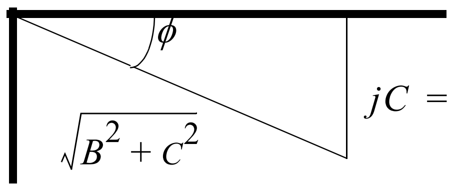
The current is peaking ahead of the voltage - hence current *leads*.

Complex Phase

- Phase in terms of complex impedance

$$\frac{v_C}{v_0} = \frac{1/j\omega C}{(1/j\omega C) + R} = \frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2}$$

$$B = \frac{1}{1 + \omega^2 R^2 C^2}$$



$$jC = \frac{-j\omega RC}{1 + \omega^2 R^2 C^2}$$

$$\phi = \operatorname{atan}\frac{-\omega RC}{1} = \operatorname{atan}(-\omega RC)$$

- The phase depends on the frequency

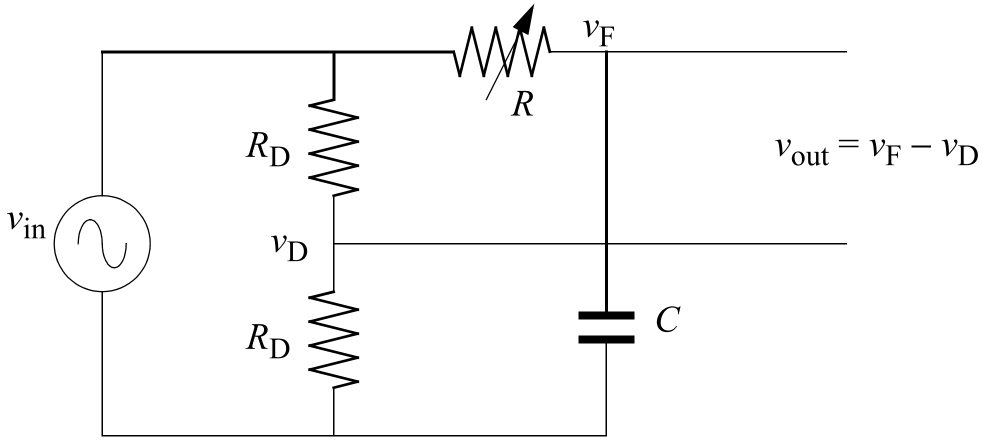
  At high frequency, ϕ -> –90

  At $\omega = 1/RC$, $\phi = -45$

  At low frequency, ϕ -> 0

Phase Shifting Network

- Divider and filter



- Consider each part separately:

$$v_D = \frac{R_D}{R_D + R_D} v_{in} = \frac{v_{in}}{2}$$

$$v_F = \frac{1/j\omega C}{(1/j\omega C) + R} v_{in} = \frac{v_{in}}{1 + j\omega RC}$$

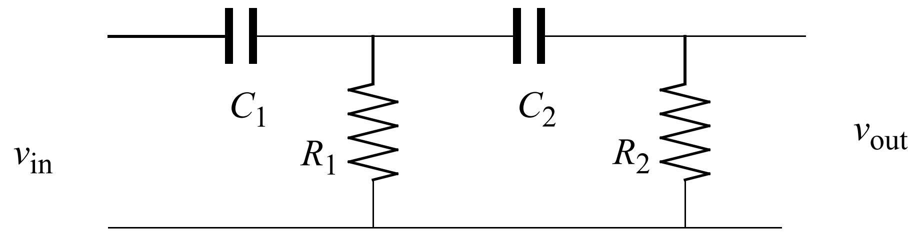
$$v_{out} = \left(\frac{1}{1 + j\omega RC} - \frac{1}{2}\right) v_{in} = \left(\frac{2 - (1 + j\omega RC)}{2 + 2j\omega RC}\right) v_{in} = \frac{1}{2}\left(\frac{1 - j\omega RC}{1 + j\omega RC}\right) v_{in}$$

$$v_{out} = \frac{1}{2} e^{-2j\phi} v_{in} \qquad \tan\phi = \omega RC$$

- Changing the resistance, *R*, changes the phase but not the amplitude.

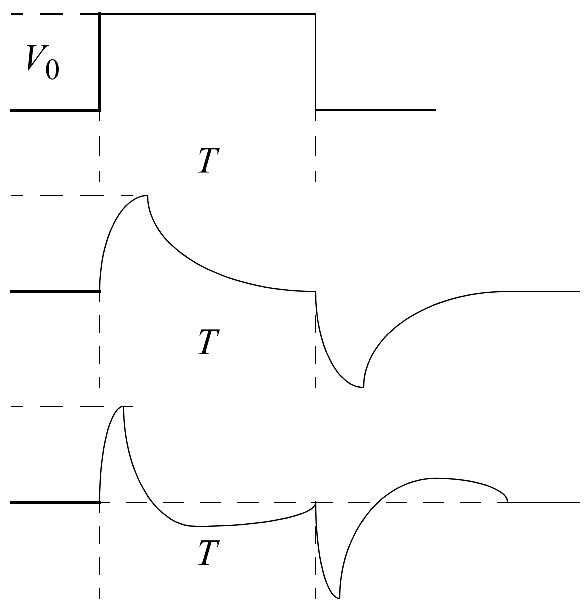
Double Differentiator

- Two high-pass filters in series

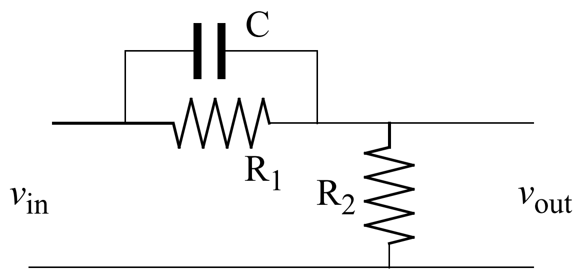


- The solution for $t << R_iC_i$ is the combination: $v_{out} = R_2C_2\frac{d\left(R_1C_1\frac{dv_{in}}{dt}\right)}{dt} = R_1C_1R_2C_2\left(\frac{d^2v_{in}}{dt^2}\right)$

- Pulse is differentiated twice



Speed-up Capacitor



$$v_{out} = \frac{R_2}{R_2 + Z_1} v_{in}$$

- A resistor divider with R_1 replaced with Z_1 which includes a capacitor.

$$Z_1 = \frac{R / j\omega C}{R + 1 / j\omega C} = \frac{R}{j\omega RC + 1}$$

- Consider that a capacitor looks like an open connection for low *f* and a short circuit at high *f*.
- The expectation is that at high *f*, the divider has $Z_1 = 0$.

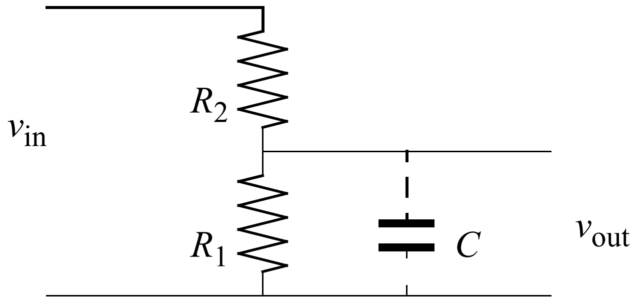
$$v_{out} = \frac{R_2}{R_2 + R_1 / (j\omega R_1 C + 1)} v_{in} = \frac{j\omega R_1 R_2 C + R_2}{j\omega R_1 R_2 C + R_2 + R_1} v_{in}$$

$$A = \sqrt{\frac{(\omega R_1 R_2 C)^2 + (R_2)^2}{(\omega R_1 R_2 C)^2 + (R_2 + R_1)^2}}$$

- For $\omega \ll 1/R_1C$, $A = R_2/(R_1 + R_2)$; $\omega \gg 1/R_1C$, $A = 1$.
- High frequencies are enhanced, so a pulse edge becomes sharper.

High Frequency Effects

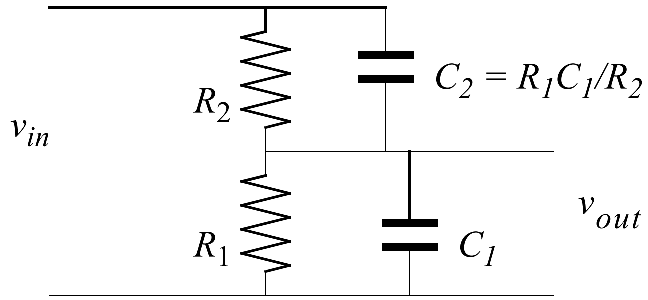
- Actual resistive voltage dividers have some small capacitance between wires.



- This acts as a low-pass filter and attenuates high frequencies.

Compensated Voltage Divider

- Precisely selected capacitors can compensate for this effect.



$$v_{out} = \frac{(R_1 \parallel C_1)}{(R_1 \parallel C_1) + (R_2 \parallel C_2)} v_{in}$$

$$v_{out} = \frac{\dfrac{R_1 / j\omega C_1}{R_1 + 1/j\omega C_1}}{\dfrac{R_1 / j\omega C_1}{R_1 + 1/j\omega C_1} + \dfrac{R_2 / j\omega C_2}{R_2 + 1/j\omega C_2}} v_{in} = \frac{R_1}{R_1 + R_2} v_{in}$$