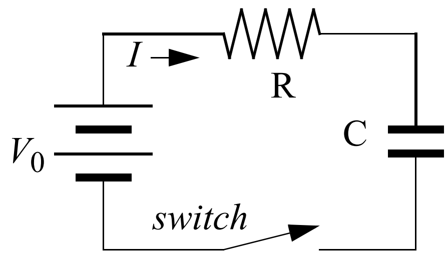
Transients

Transient Response of a Capacitor



What happens when the switch is closed?

- Kirchoff's law for a single loop: $V_0 = V_R + V_C$
- The voltage drop for the resistor is $V_R = IR$, and for the capacitor is $V_C = Q/C$.
- Using the relationship between charge and voltage

$$I = \frac{dQ}{dt}$$

$$V_0 = R\frac{dQ}{dt} + \frac{1}{C}Q$$

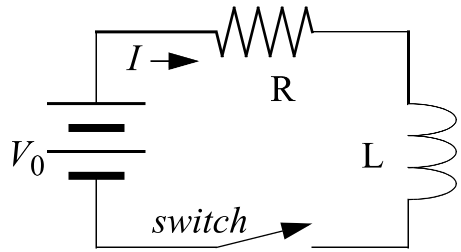
- The solution of the differential equation for Q, then I and V_C:

$$Q = Q_0 e^{-t/RC} + V_0 C(1 - e^{-t/RC})$$

$$I = -\frac{Q_0}{RC}e^{-t/RC} + \frac{V_0}{R}e^{-t/RC} = \left(\frac{V_0}{R} - \frac{Q_0}{RC}\right)e^{-t/RC}$$

$$V_C = \frac{Q_0}{C}e^{-t/RC} + V_0(1 - e^{-t/RC}) = V_0 + \left(\frac{Q_0}{C} - V_0\right)e^{-t/RC}$$

Transient Response of an Inductor



What happens when the switch is closed?

- Kirchoff's law for a single loop:

$$V_0 + V_L = V_R$$

- The voltage induced by the inductor is $V_L = -L\, dI/dt$,

$$V_0 = RI + L\frac{dI}{dt}$$

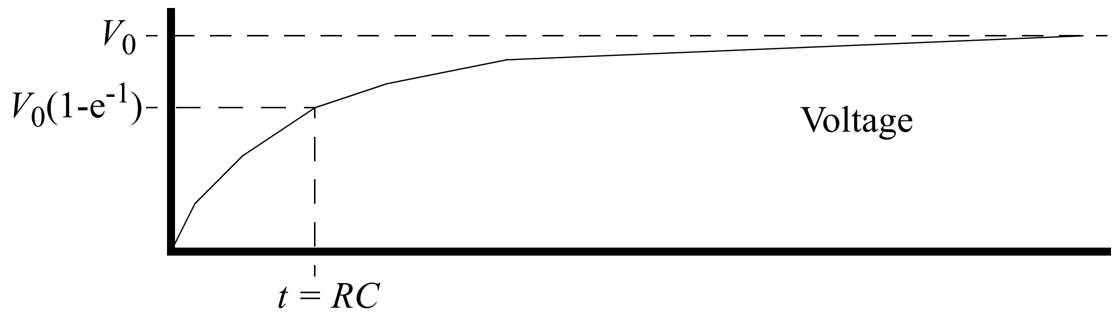
- The solution of the differential equation for I and V_L:

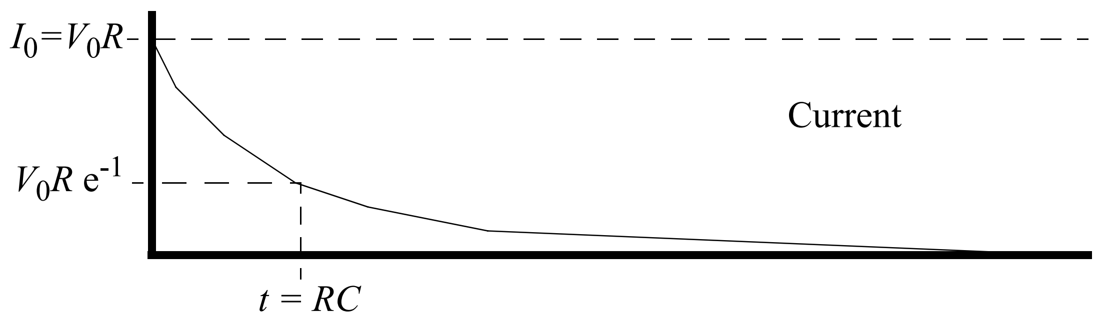
$$I = I_0 e^{-tR/L} + \frac{V_0}{R}(1 - e^{-tR/L})$$

$$V_L = -I_0 R e^{-tR/L} + V_0 e^{-tR/L} = (V_0 - I_0 R)e^{-tR/L}$$

RC Time Constant

- *RC* has units of time (1 Ω-F = 1 C/A = 1 s)
- For a closing switch, $Q_0 = 0$

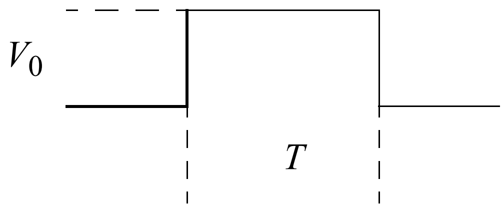




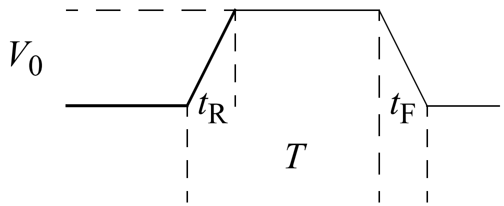
- For an inductor: L/R has units of time (1 H/Ω = 1 s).
- This is the time constant for inductive circuits.

Pulses

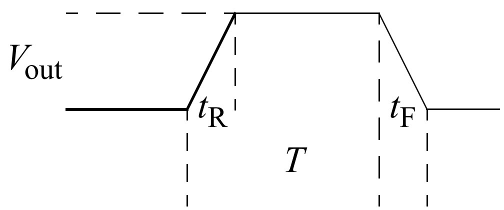
- A perfect square pulse has an amplitude V_0 and duration T like a perfect switch on and off.



- Real pulses have a finite rise time t_R and fall time t_F.

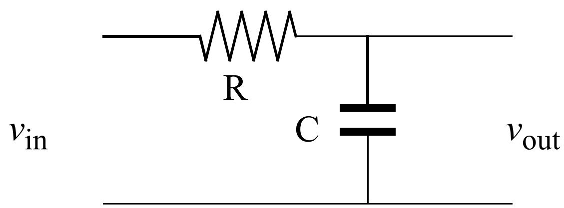


- A perfect voltage divider reduces the voltage of the amplitude but leaves the time unchanged.



Integrator

- Low-pass filter at short times



- The relationship between charge and voltage

$$v_{in} = R\frac{dQ}{dt} + \frac{1}{C}Q$$

- The solution of the differential equation for Q and v_{out} with $Q_0 = 0$:

$$Q = v_{in}C(1 - e^{-t/RC})$$

$$v_{out} = v_{in}(1-e^{-t/RC}) \cong \frac{v_{in}t}{RC} \qquad t \ll RC$$

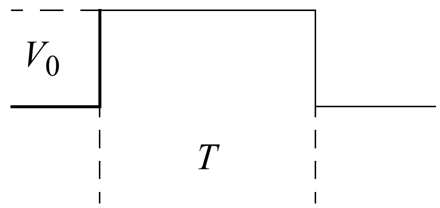
- For v_{in} slowly varying in time compared to RC

$$v_{out} \cong \frac{v_{in}t}{RC} \cong \frac{1}{RC}\int_0^t v_{in}dt$$

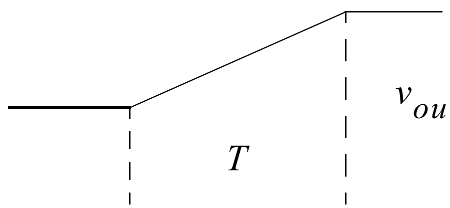
- The gain of the integrated signal is proportional to 1/RC

Integrated Pulses

- Square pulse



- Integrated square pulse



$$v_{out} = \frac{V_0 T}{RC}$$

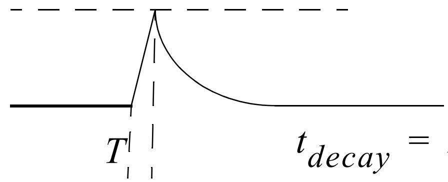
- The peak of the integrated pulse is proportional to the area of the initial pulse.

Noise Suppression

- Noise pulse

- Integrated noise pulse

$$v_{max} = \frac{V_0 T}{2RC}$$

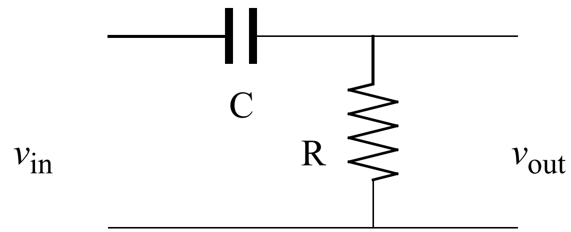


$$t_{decay} = RC$$

- The magnitude of the noise is reduced by $T/2RC$.
- The pulse is spread by a time comparable to RC.

Differentiator

- High-pass filter at long times



- The relationship between charge, current and voltage

$$v_{in} = Ri + \frac{1}{C}Q$$

$$\frac{dv_{in}}{dt} = R\frac{di}{dt} + \frac{1}{C}i$$

- The solution of the differential equation for i and v_{out} with $I_0 = 0$:

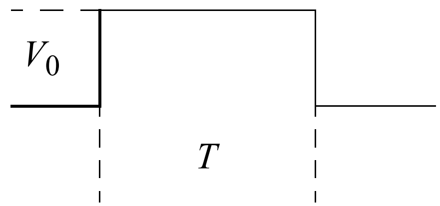
$$i = \frac{dv_{in}}{dt}C(1 - e^{-t/RC})$$

$$v_{out} = \frac{dv_{in}}{dt}RC(1-e^{-t/RC}) = RC\frac{dv_{in}}{dt} \qquad t \gg RC$$

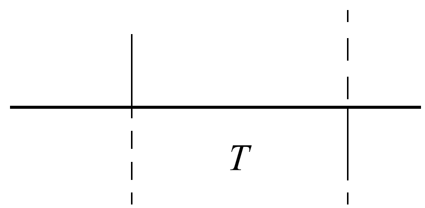
- The gain of the differentiated signal is proportional to RC.

Differentiated Pulses

- Square pulse



- Differentiated square pulse



The peak of the differentiated pulse is proportional to the rise (or fall) time.