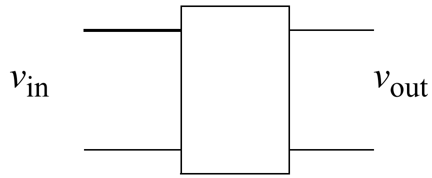
# *Filter Circuits*

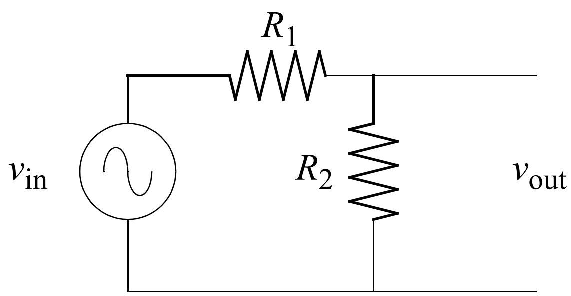
- A filter is a 4-terminal circuit element - input and output voltage, gain $A$.




$$v_{out} = A v_{in}$$

- Input and output may both have one terminal at ground.

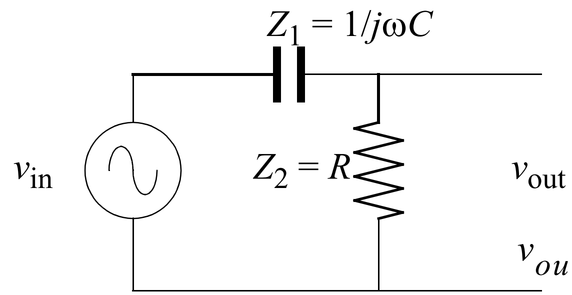
- A voltage divider with resistors is a filter.




$$v_{out} = \frac{R_2}{R_1 + R_2} v_{in}$$

# High Pass Filter

- Impedances act like resistances in circuits and obey Kirchoff's laws.
- If $R_1$ is replaced by a capacitor the divider can be calculated with impedances.




$$v_{out} = \frac{Z_2}{Z_1 + Z_2} v_{in} = \frac{R}{1/j\omega C + R} v_{in}$$

- This can be rewritten in terms of the complex gain $A$:

$$A = \frac{v_{out}}{v_{in}} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega RC + \omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2}$$

- The terms in the denominator are comparable when $\omega_{\mathrm{b}} = 1/RC$, the *breakpoint frequency.*

# Bode Plot

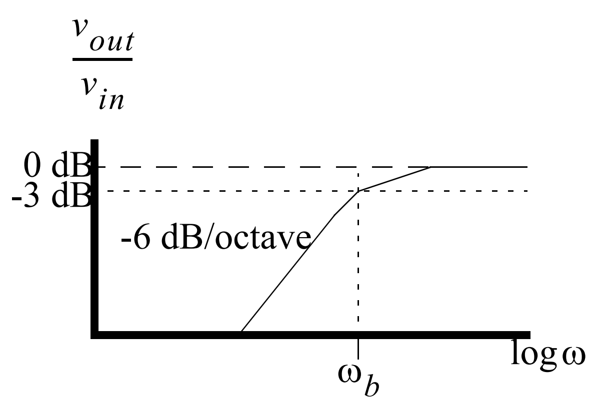
- The magnitude of a complex number is $|A| = |B + jC| = \sqrt{B^2 + C^2}$

  The magnitude of the gain is

$$|A| = \sqrt{\frac{\omega^2 R^2 C^2 + (\omega^2 R^2 C^2)^2}{(1 + \omega^2 R^2 C^2)^2}} = \sqrt{\frac{\omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2}}$$

- For $\omega$ much less than $1/RC$, $A$ is very small. $|A| \cong \omega RC$
- For $\omega$ much greater than $1/RC$, $A$ is nearly 1.
- The high-pass filter has a breakpoint frequency $\omega_b = 1/RC$.

- A Bode plot compares log gain (dB) to log frequency.

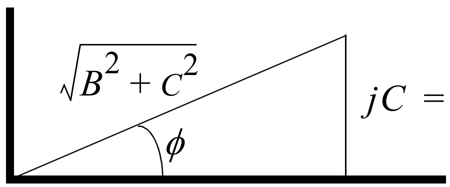


# Phase Changes

- The phase of the current compared to the voltage can be determined from the complex impedances: the angle in the complex plane is the phase shift

$$\tan\phi = C/B$$

- For the high pass filter:




$$jC = \frac{j\omega RC}{1 + \omega^2 R^2 C^2}$$

$$B = \frac{\omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2}$$

$$\phi = \operatorname{atan}\frac{\omega RC}{\omega^2 R^2 C^2} = \operatorname{atan}\frac{1}{\omega RC}$$

- The phase depends on the frequency

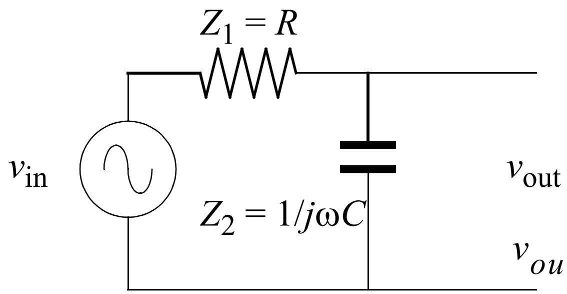
  At low frequency, $\phi$ -> 90

  At $\omega = 1/RC$, $\phi = 45$

  At high frequency, $\phi$ -> 0

# Low Pass Filter

- If the capacitor replaces $R_2$ in a voltage divider:

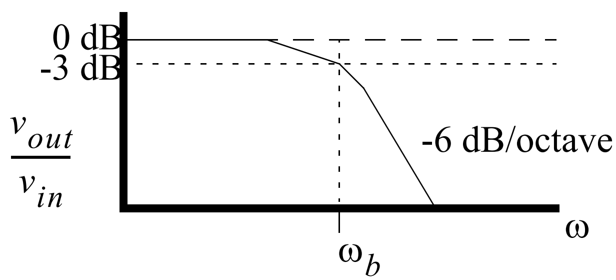



$$v_{out} = \frac{Z_2}{Z_1 + Z_2} v_{in} = \frac{1/j\omega C}{1/j\omega C + R} v_{in}$$

- Again this can be rewritten in terms of the complex gain $A$:

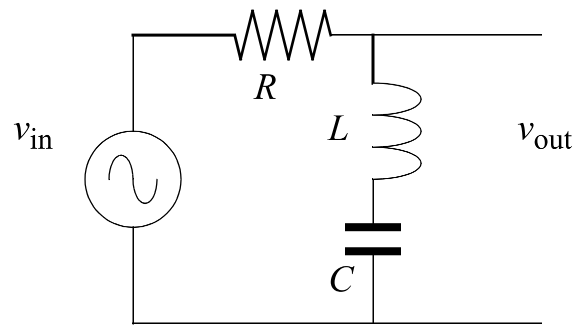
$$A = \frac{v_{out}}{v_{in}} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2} \qquad |A| = \sqrt{\frac{1}{1 + \omega^2 R^2 C^2}}$$

- For $\omega$ much less than $1/RC$, $A$ is nearly 1.
- For $\omega$ much greater than $1/RC$, $A$ is very small.
- This is called a low-pass filter with a break frequency $\omega_b = 1/RC$.
- Low pass Bode plot



# Series RLC Circuit

- A voltage divider with one impedance due to an inductor and capacitor in series



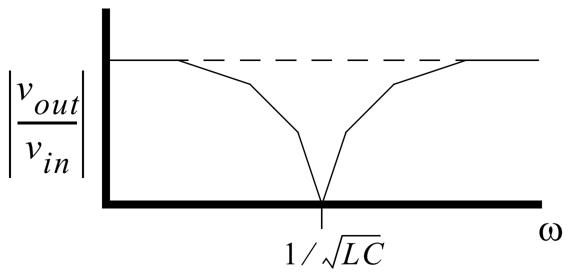

$$v_{out} = \frac{Z_{LCser}}{R + Z_{LCser}} v_{in}$$

- The series impedance can be calculated and inserted to find the gain:

$$Z_{LCser} = 1/j\omega C + j\omega L = \frac{1 - \omega^2 LC}{j\omega C}$$

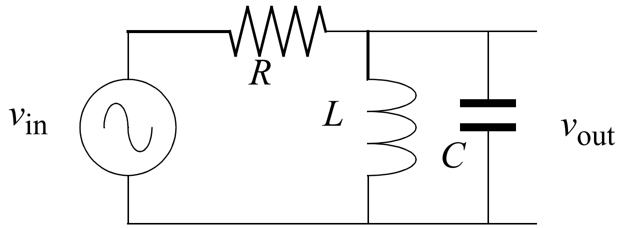
$$\frac{v_{out}}{v_{in}} = \frac{1 - \omega^2 LC}{j\omega RC + 1 - \omega^2 LC} \qquad \left|\frac{v_{out}}{v_{in}}\right| = \sqrt{\frac{(1 - \omega^2 LC)^2}{R^2(1 - \omega^2 LC)^2 + \omega^2 L^2}}$$

- Graphically



# *Parallel RLC Circuit*

- A voltage divider with one impedance due to an inductor and capacitor in parallel



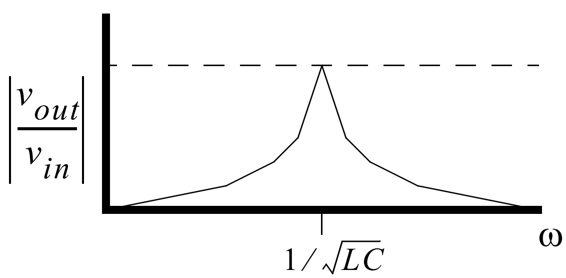

$$v_{out} = \frac{Z_{LCpar}}{R + Z_{LCpar}} v_{in}$$

- The impedance can be calculated and inserted to find the gain:

$$Z_{LCpar} = \frac{j\omega L / j\omega C}{1/j\omega C + j\omega L} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$\frac{v_{out}}{v_{in}} = \frac{j\omega L}{R(1 - \omega^2 LC) + j\omega L} \qquad \left|\frac{v_{out}}{v_{in}}\right| = \sqrt{\frac{\omega^2 L^2}{R^2(1 - \omega^2 LC)^2 + \omega^2 L^2}}$$

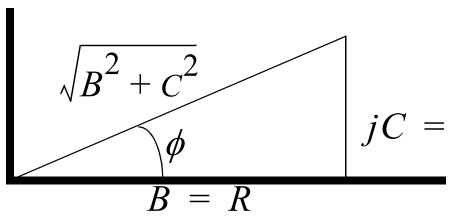
- Graphically



# *Phase Changes at Resonance*

- The frequency $\omega = 1/\sqrt{LC}$ is the *resonant frequency.*
- Use $\tan\phi = C/B$
- For the series RLC circuit:

$$Z_{LC} = 1/j\omega C + j\omega L = \frac{1-\omega^2 LC}{j\omega C}$$




$$jC = j\frac{\omega^2 LC - 1}{\omega C}$$

$$\phi = \operatorname{atan}\frac{\omega^2 LC - 1}{\omega RC}$$

- The phase depends on the frequency

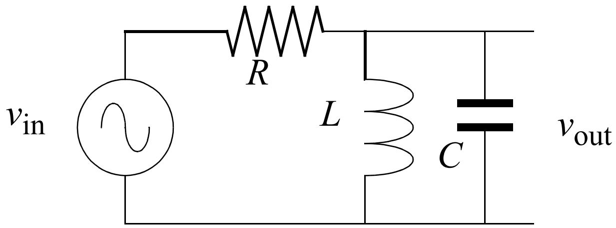
  At low frequency, $\phi$ -> -90

  At $\omega^2 = 1/LC$, $\phi = 0$

  At high frequency, $\phi$ -> +90

# *Quality Factor*

- Consider the power gain in a parallel RLC circuit.




$$\left|\frac{v_{out}}{v_{in}}\right|^2 = \frac{\omega^2 L^2}{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}$$

- At resonance the gain is 1.
- The frequency where the power is halved:

$$\frac{1}{2} = \frac{\omega^2 L^2}{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}$$

$$\frac{1}{2}R^2(1-\omega^2 LC)^2 = \frac{1}{2}\omega^2 L^2$$

$$1-\omega^2 LC = \frac{\omega L}{R}$$

- This has solutions at roughly:

$$\omega = \frac{1}{\sqrt{LC}} \pm \frac{R}{2L} = \omega_0 \pm \omega_0 \frac{R}{2\omega_0 L} = \omega_0 \pm \frac{\omega_0}{2Q}$$

- $Q = L/R$ is the *quality factor* for the circuit.