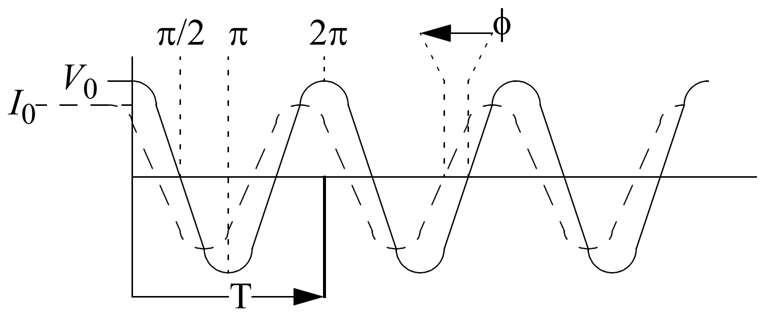
Alternating Current

- AC voltage varies in time, and a pure AC signal is sinusoidal.

$$v(t) = V_0 \cos \omega t$$

- AC current also varies in time with the same frequency but may have a different phase:

$$i(t) = I_0 \cos(\omega t + \phi)$$



- Frequency (f, Hz = cycles/s = s^{-1}), angular frequency (ω, rad/s = s^{-1}), and period (T, s) are related.

$$\omega = 2\pi f = 2\pi / T$$

- Phase measures the relative point in time within one period.

$$\phi = 2\pi t / T$$

Rms Measurement

- The time average measure of a signal:

$$\langle F \rangle = \frac{1}{T}\int_0^T F(t)dt$$

- Voltage and current: V_{rms} , I_{rms}

$$V_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\langle V_0^2 \cos^2 \omega t \rangle} = \frac{V_0}{\sqrt{2}}$$

$$I_{rms} = \sqrt{\langle i^2 \rangle} = \sqrt{\langle I_0^2 \cos^2(\omega t + \phi) \rangle} = \frac{I_0}{\sqrt{2}}$$

- Phase isn't included in rms measurement.

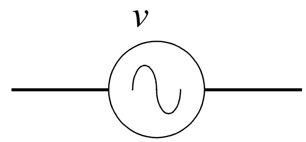
- Commercial 60 Hz AC signals are measured in rms

  120 V_{AC} = V_0 / 1.414

  V_0 = 170 V

AC Power Supply

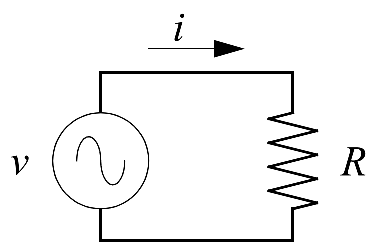
- Schematic symbol:



- Power: P

$$P_{rms} = \langle vi \rangle = \sqrt{\langle V_0^2 \cos^2 \omega t \rangle} = \frac{1}{2} V_0 I_0 \cos\phi$$

- Ohm's law applies for AC signals at each point in time.



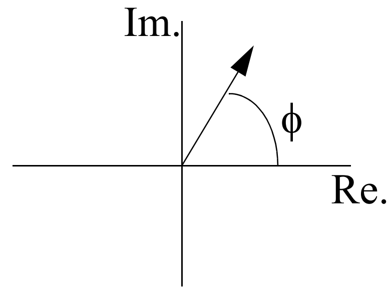
- For an AC voltage through a resistor current and voltage have the same phase.

  $\phi = 0$, $\cos\phi = 1$.

  $P = V_0 I_0 /2 = V_{\text{rms}} I_{\text{rms}}$.

Complex Numbers

- Represent an angle in the complex plane.



- The imaginary unit is j.

$$j = \sqrt{-1}$$

- A series expansion for an exponential in $j\phi$:

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$e^{j\phi} = 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \ldots$$

$$e^{j\phi} = \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \ldots\right) + j\left(\phi - \frac{\phi^3}{3!} + \ldots\right)$$

- Trigonometric formulas can be replaced by exponential ones.

$$\cos\phi = Re(e^{j\phi})$$

Gain

- Gain is the ratio of voltage (or current) out of a circuit compared to the voltage (or current) in

$$A = \frac{v_{out}}{v_{in}} \qquad A = \frac{i_{out}}{i_{in}}$$

- Unit of gain: decibel (dB)
- Decibels are a logarithmic measure,

  Voltage and current, $A_{dB} = 20 \log_{10} A$

  Power = voltage * current, $A_{dB} = 10 \log_{10} A$
- Useful rules:

  A factor of 10 is a 20 dB measure

  A factor of 2 is about a 6 dB measure

  Negative dB is a reduction in magnitude

- Gain vs. frequency

  Use log-log graph: power law relations become straight lines - *Bode plot*

  eg. $A = c\omega^{-2}$ becomes $A_{dB} = -2\omega + \log(c)$

Complex Gain

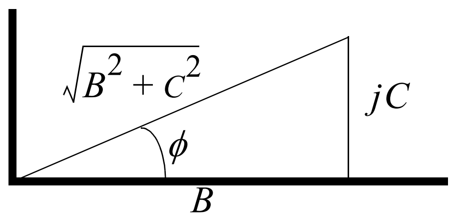
- Includes magnitude of gain and phase shift
- The absolute magnitude (a real) is the magnitude of the gain

$$|A| = |B + jC| = \sqrt{(B + jC)(B - jC)} = \sqrt{B^2 + C^2}$$

- The angle in the complex plane is the phase shift

$$\tan\phi = C/B$$

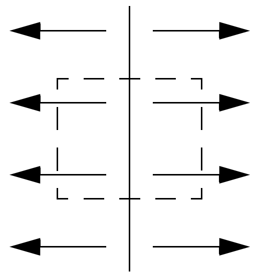
- Graphically:

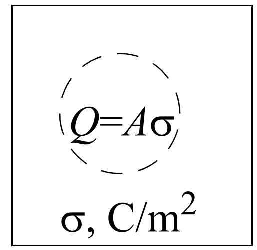


$$\phi = \operatorname{atan}\frac{C}{B}$$

Capacitance

- For an infinite plane with a charge density σ, $Q = A\sigma$:

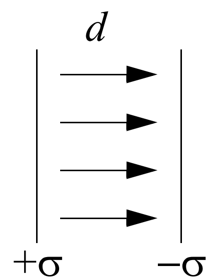




- The electric field and potential are related to the charge, and the voltage

$$\int_S \vec{E} \cdot d\vec{A} = 2EA = \frac{Q}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0} \qquad V = Ed = \frac{\sigma d}{2\varepsilon_0}$$

- For two planes of opposite charge, the field outside is 0 and the field inside is the sum of the two separate fields, $E = \sigma/\varepsilon_0$, and the voltage is $\sigma d/\varepsilon_0$.

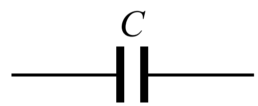


- Capacitance is the charge stored unit of potential, $C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\varepsilon_0} = \frac{\varepsilon_0 A}{d}$

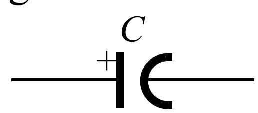
- Unit of capacitance: farad (F) = coulomb (C) / volt (V)

Capacitors

- Schematic symbol:



- Capacitors are measured in farads (F) and can range from 10^{-12} to 1 F.
- Capacitors come with maximum voltage ratings from 10 to 10^4 V.
- Some capacitors are polarized, and voltage must be maintained in a particular direction at all times.



- Capacitors are usually marked in either μF (10^{-6}) or pF (10^{-12}), rarely nF.

  μF values are usually less than 1

  pF values are usually greater than 1

  Like resistors the number is often either a direct measure or uses the three digit code.

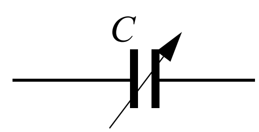
  With three digits, the first two are a number and the last is an exponent for a power of 10, usually based on pF. (eg. $103 = 10 * 10^3 = 10{,}000$ pF = 0.01 μF)
- Other codes also exist for capacitors (EIA codes).

Capacitor Types

- Ceramic disk capacitors are cheap, cover capacitances from 1 pF up to 1 μF, and come in wide range of performance specifications.
- Polyester film (Mylar) capacitors are cheap, cover capacitances from 0.001 μF up to 100 μF and have reasonable accuracy (5-10%).
- Polypropylene film (PP) capacitors can have 1% accuracy from 100 pF to 100 μF.
- Mica capacitors are bulky and expensive, but have very high quality (1 pF to 3,300 pF).

- Electrolytic capacitors have very high capacitance (0.1 μF to 1 F), but poor accuracy (-20% to +80%) and are polarized.
- Tantalum capacitors have high capacitance in a small package (0.1 μF to 1000 μF), with poor accuracy and are polarized and very sensitive to voltage ripples and cannot be used in all circuits.

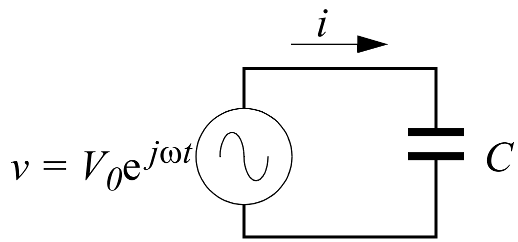
- Variable capacitors



C refers to maximum capacitance

Capacitors in AC Circuits

- Capacitive impedance



- The relation between current and voltage is:

$$v = \frac{q}{C}$$

$$\frac{dv}{dt} = \frac{1}{C}\frac{dq}{dt} = \frac{1}{C}i$$

- Using the AC voltage and current:

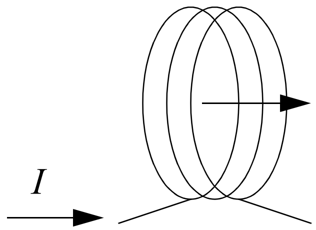
$$\frac{dv}{dt} = j\omega v = \frac{1}{C}i$$

$$v = \frac{1}{j\omega C}i$$

- This looks like Ohm's law, but with R replaced by $1/j\omega C$. This is called the *impedance* Z.

Inductance

- Ampere' law: the magnetic field in a solenoid with N loops in l meters is related to the current.



$$B = \frac{\mu N I}{l} \qquad \Phi = \int_S \vec{B} \cdot \vec{dA} = \frac{\mu N I A}{l}$$

- Units from magnetism: field -- tesla (T) = N/A-m; flux -- weber = T-m^2

- Faraday's law: a change in flux induces a electric potential.

$$v = -N\frac{d\phi}{dt} = -\left(\frac{\mu N^2 A}{l}\right)\frac{di}{dt} = -L\frac{di}{dt}$$

  The minus sign indicates the potential is opposite the change in the current.

- Inductance L defined for a coil:

$$L = \frac{N\Phi}{I} = \frac{\mu N^2 A}{l}$$

- Unit of inductance: henry (H) = weber / amp (A)

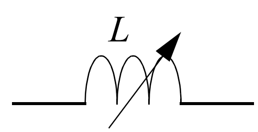
Inductors

- Schematic symbol:

- Inductors are measured in henrys (H) in the range 10^{-6} to 1 H.
- Inductors are usually marked in either μH (10^{-6}) or mH (10^{-3}).
  A resistor color code is used on some inductors.

- Inductors are wound wires, the simplest are in a coil around air.
- Inductors can be wound around iron or ferrite to increase the permeability and thus the inductance.

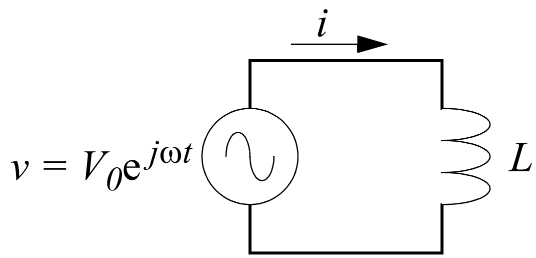
- Variable inductors



L refers to the maximum inductance for a variable inductor

Inductors in AC Circuits

- Inductive impedance



- Kirchhoff's voltage law gives $v + \left(-L\frac{di}{dt}\right) = 0$.

- The relationship between current and voltage is:

$$v = L\frac{di}{dt}$$

$$\int v dt = Li$$

- Using the AC voltage and current:

$$\int v dt = \frac{v}{j\omega} = Li$$

$$v = j\omega Li$$

- This looks like Ohm's law, but with R replaced by $j\omega L$. This is the *impedance* Z for an inductor.