Kirchhoff's Laws

1. **Kirchhoff Current Law**

   The sum of currents entering a junction is equal to the sum of currents leaving the junction.

2. **Kirchhoff Voltage Law**

   The sum of voltage increases around a closed loop of a circuit is equal to the sum of voltage drops around the loop.

- **Branch Method**

  Label a current through each branch of the circuit.

  Set up a set of independent equations for the junctions (current law) and loops (voltage law) in the circuit.

- **Loop Current Method**

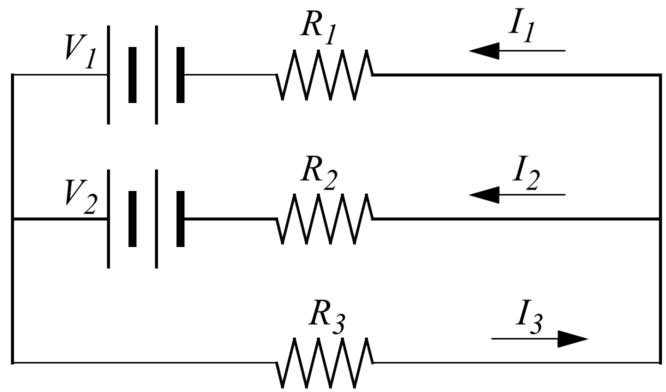
  Label a current around each independent loop of the circuit.

  Set up an equation for each current loop (voltage law), counting all the currents in any given element.

- **Nodal Method**

  Like loop current, but an unknown voltage is assigned to each junction. Currents found from Ohm's Law

Branch Example



$V_1 = 16$ V

$V_2 = 6$ V

$R_1 = 2$ kΩ

$R_2 = 4$ kΩ

$R_3 = 5$ kΩ

- Current law: $I_3 = I_1 + I_2$
- Voltage law: $V_1 - V_2 = R_1 I_1 - R_2 I_2$ and $V_2 = R_2 I_2 + R_3 I_3$
- Solve for I_1, substituting for I_3 and I_2:

$$V_2 = R_2 I_2 + R_3 I_1 + R_3 I_2$$

$$V_2 = R_1 I_1 + V_2 - V_1 + R_3 I_1 + (R_3 / R_2)(R_1 I_1 + V_2 - V_1)$$

$$R_1 R_2 I_1 + R_2 R_3 I_1 + R_1 R_3 I_1 = R_2 V_1 + R_3 V_1 - R_3 V_2$$

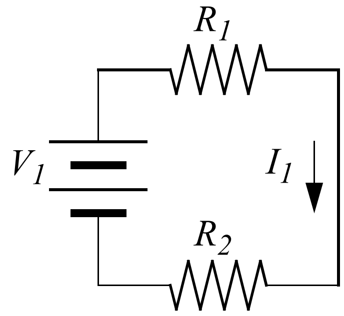
$$I_1 = (R_2 V_1 + R_3 V_1 - R_3 V_2) / (R_1 R_2 + R_2 R_3 + R_1 R_3)$$

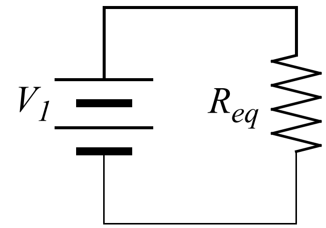
- Finally,

  $I_1 = 3$ mA, $I_2 = -1$ mA, $I_3 = 2$ mA.

Series Resistors

In *series* the same current flows through all elements





- Use a one loop application of Kirchhoff's voltage law:

$$V_1 = R_1 I_1 + R_2 I_1$$

$$V_1 = (R_1 + R_2) I_1 = R_{eq} I_1$$

$$R_{eq} = R_1 + R_2$$

- In general, the series resistance is the sum of the individual resistances.

Voltage Divider

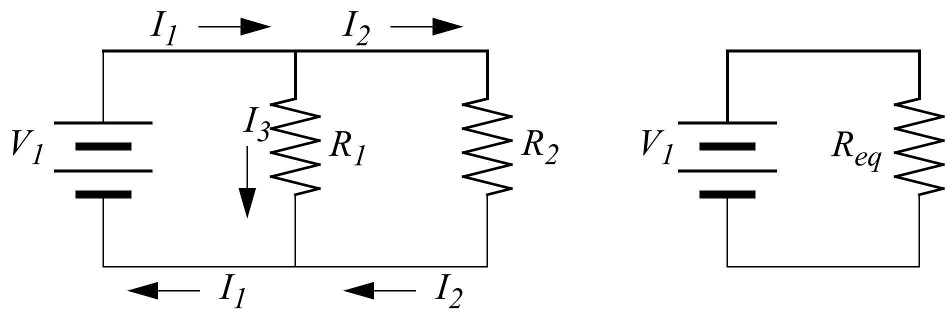
- The voltage at the point between R_1 and R_2 is given by:

$$V_{out} = I_1 R_2 = \frac{R_2}{R_1 + R_2} V_1$$

- The *voltage divider* is perhaps the most common circuit combination!

Parallel Resistors

In *parallel* the same voltage is present across all elements



- Kirchhoff’s current law gives: $I_1 = I_2 + I_3$
- There are two loops for Kirchhoff’s voltage law:

$$V_1 = R_1 I_3 \qquad 0 = -R_1 I_3 + R_2 I_2$$

- Eliminate I_3 from the equations, and solve for I_1:

$$V_1 = R_2 I_2 \qquad I_1 = (V_1/R_2) + (V_1/R_1)$$

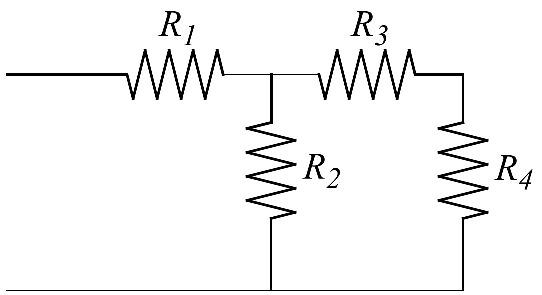
$$V_1 = \left[\frac{R_1 R_2}{R_1 + R_2}\right] I_1 = R_{eq} I_1$$

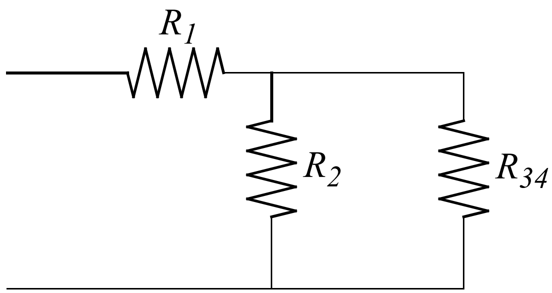
- In general, the inverse of the parallel resistance is the sum of the inverse of the individual resistances.

$$R_{eq} = R_1 R_2/(R_1 + R_2) \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

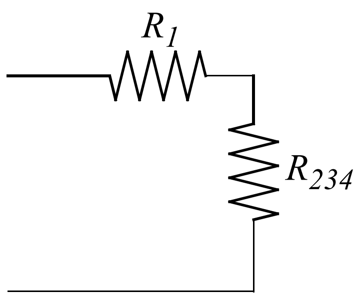
Resistor Networks

- Repeat applications of series or parallel rules as needed:





$$R_{34} = R_3 + R_4$$



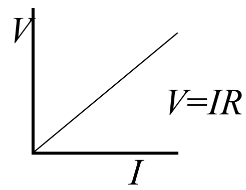
$$R_{234} = \frac{R_2 R_{34}}{R_2 + R_{34}} = \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

The equivalent network is $R_{eq} = R_1 + \dfrac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$

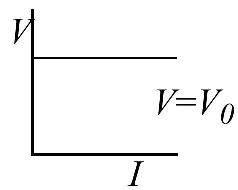
V-I Curves

Devices have a characteristic relationship between current and voltage:

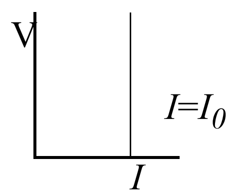
- Ohmic device (resistor):



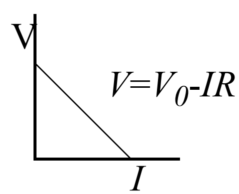
- Voltage source (power supply):



- Current source:



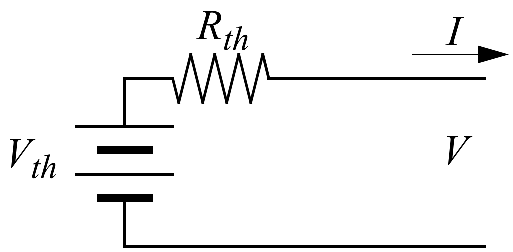
- Battery and resistor in series:



Thevenin's Theorem

Any circuit of batteries and resistors can be reduced to one battery and one resistor in series.

- Theorem is based on the linear form of Ohm's law and Kirchoff's laws.
- Any set of circuit equations is reduced to $V(I) = V_{th} - IR_{th}$.

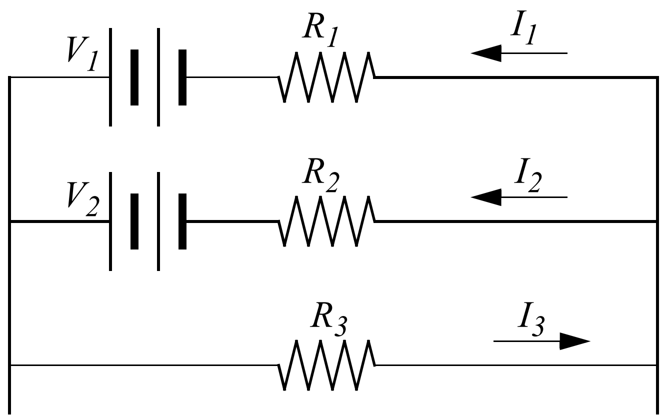


- **Procedure**

1. Find the voltage with no external circuit as V_{th}.
2. Find the current that would flow through an external short circuit.
3. Find the equivalent resistance as $R_{th} = V_{th}/I_{sc}$.

Thevenin Example

- Consider the branch circuit from before, but connect it to an unknown circuit.



$V_1 = 16$ V

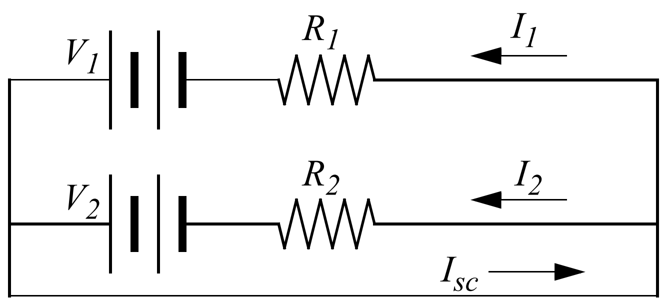
$V_2 = 6$ V

$R_1 = 2$ kΩ

$R_2 = 4$ kΩ

$R_3 = 5$ kΩ

- From the Kirchoff example $I_3 = 2$ mA. $V_{th} = R_3 I_3 = 10V$
- If the output is short circuited:



$V_1 = 16$ V

$V_2 = 6$ V

$R_1 = 2$ kΩ

$R_2 = 4$ kΩ

$$I_3 = V_1/R_1 + V_2/R_2 = 9.5mA$$

- The R_{th} is the same if all batteries are shorted and resistance measured. $R_{th} = V_{th}/I_{sc} = 1.05k\Omega$

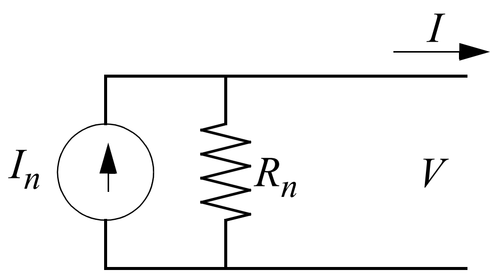
Norton's Theorem

Any circuit of batteries and resistors can be reduced to one current source and one resistor in parallel.

- Electrically equivalent to a Thevenin circuit.

  $R_n = R_{th}$.

  $I_n = I_{sc} = V_{th} / R_{th}$.



- **Procedure**

1. Find the voltage with no external circuit as V_{th}.
2. Find the current that would flow through an external short circuit I_{sc}.
3. Find the equivalent resistance as $R_n = V_{th}/I_{sc}$.