Physics 375, Laboratory 3
Time Constants

Overview

The purpose of these experiments is to investigate the transient properties of RC circuits. review the properties of capacitors and inductors and to use them to investigate frequency dependence in AC circuits using an oscilloscope.

Background

The equations for RLC circuits can be expressed as differential equations, and solved for current or voltage for an input signal. The equation for an RC circuit has the form:

$$V_0 = R\frac{dQ}{dt} + \frac{1}{C}Q$$

The output voltage for an input square wave measured across the resistor or capacitor is:

$$V_R = -\frac{Q_0}{C}e^{-t/RC} + V_0 e^{-t/RC} = \left(V_0 - \frac{Q_0}{C}\right)e^{-t/RC}$$

$$V_C = \frac{Q_0}{C}e^{-t/RC} + V_0(1-e^{-t/RC}) = V_0 + \left(\frac{Q_0}{C} - V_0\right)e^{-t/RC}$$

where RC is the characteristic time constant for the circuit. In the limit of small t, the measurement across the capacitor looks like the integral of the input voltage. The measurement across the resistor behaves like the derivative of the input voltage.

For an RLC circuit the equation is more complicated and takes the form of a second order differential equation.

$$V_0 = L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q$$

The general solution is:

$$V = V_0\left(1-e^{-Rt/2L}e^{-j\omega_0 t}\right)$$

where $\omega_0^2 = 1/LC$ and is a characteristic frequency of oscillation and $2L/R$ is a damping time.

1. RC Phase Shift

Connect the function generator (50 Ω output), oscilloscope, a resistor and capacitor in a circuit:

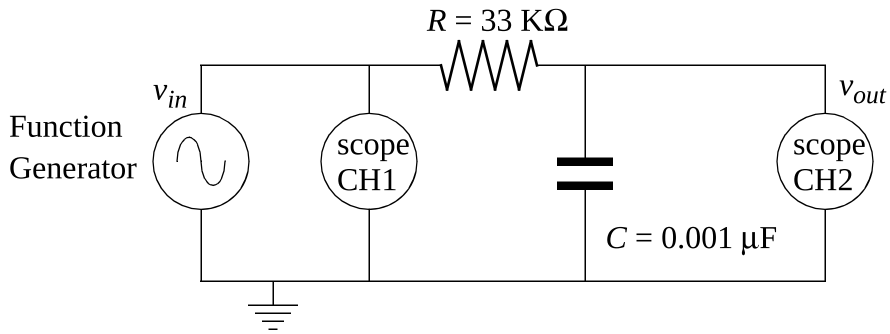

Figure 1: RC Integrator Circuit

Set the function generator to produce sine waves at a frequency of 1 KHz. Attach v_{in} to CH1 of the scope, and v_{out} to CH2 of the scope, with both channels on a 2 V vertical scale and DC coupling. Adjust the level of the scope to start a trace at V=0. Measure the time displacement of the zero of the period seen on CH2 compared to the period of CH1, and express it in radians. Repeat the measurement this for frequencies of 2.0 KHz, 4.0 KHz, 5.0 KHz, 6.0 KHz, 8.0 KHz and 10.0 KHz, and plot it. Find the breakpoint frequency, f_b, from the values of the components in the circuit. Set the frequency on the function generator to that value and measure the phase at the breakpoint frequency.

2. RC Integrator

Use the circuit from part 1 and set the function generator for a square wave output with a frequency equal to f_b/10. Make sure the oscilloscope is set to DC coupling, positive trigger and observe the output. The expected behavior is $V_C = V_0 + \left(\frac{Q_0}{C} - V_0\right)e^{-t/RC}$ with $V_0 = -v_{in}$, and $Q_0/C = +2v_{in}$, where v_{in} is the amplitude of the square wave. Measure the time it takes for the signal to rise from $-v_{in}$ to a value equal to $-v_{in}+2v_{in}(1-e^{-1})$. How does this compare to the expected time constant $t=RC$? Set the signal generator for a square wave output with a frequency equal to f_b*10. Observe the output signal and measure the slope of the output signal. How does this compare with the expected constant of integration (1/RC) for the circuit?

3. RC Differentiator

Swap the resistor and capacitor in the circuit in part 1 to build the following circuit:

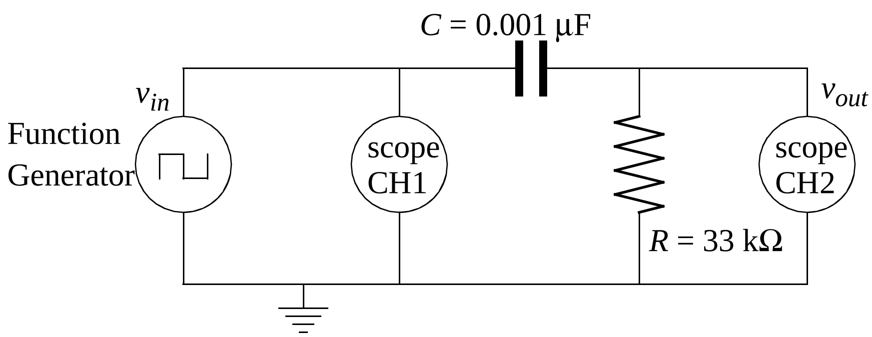

Figure 2: RC Differentiator Circuit

Calculate the break frequency, f_b. Set the function generator for a square wave output with a frequency equal to $f_b/10$. Set the oscilloscope to DC coupling and observe the output. Measure the time it takes for the signal to change from v_{in} to a value equal to $v_{in}e^{-1}$. How does this compare to the expected time constant $t=RC$? Change the signal generator to a triangle wave at the same frequency. Observe the output signal and measure the amplitude of the signal. How does this compare with the expected constant of differentiation for the circuit?

4. Ringing

Build the following parallel RLC circuit:

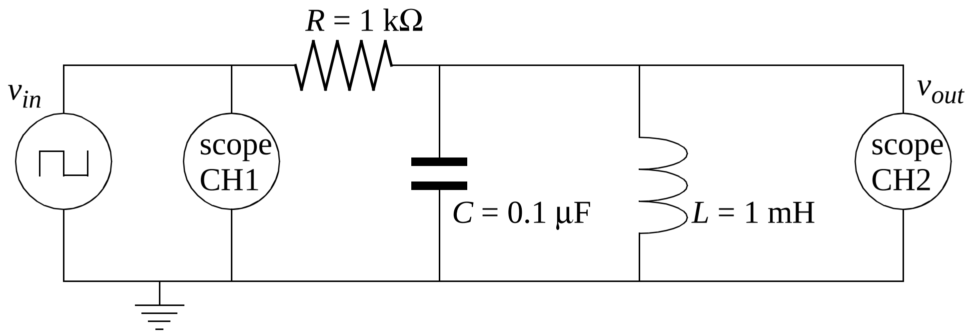

Figure 3: Parallel LC Circuit

Calculate the break frequency, f_b. Set the function generator for a square wave output with a frequency equal to $f_b/10$. Observe the output. Measure the peak amplitude of the ringing, the time it takes the ringing amplitude to drop by a factor of e and the period of the ringing oscillation. How does the period of the ringing compare to the characteristic oscillation time, ω_0? How does the damping time compare to $2L/R$?