Assignment: HW4 [40 points]

Assigned: 2006/10/25
Due: 2006/11/01

P4.1 [$4 + 4 = 8$ points]

(a) Find the moment of inertia tensor I of a uniform cube of side s and mass M whose pivot is at a corner and whose sides are lined up along the axes of an orthonormal coordinate system.

(b) Find the principal axis system and the moments of inertia.

P4.2 [4 points]
The cube in Problem 1 rotates instantaneously about the edge that is lined up along the x_1 axis. Find the angle between the angular momentum $\mathbf{L}$ and the angular velocity $\vec{\omega}$.

P4.3 [4 points]
Consider the symmetric dumbbell rotating in a "double cone" about its CM as shown in Fig. 4.3: two equal point masses m connected by a massless inextensible link of length 2ℓ. Find the angular momentum of the system and the torque required to maintain the motion.

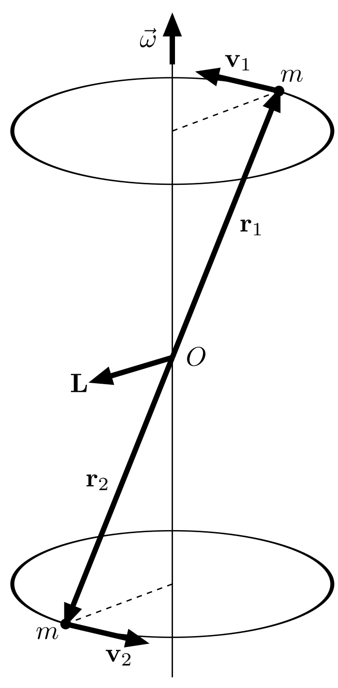

Figure 4.3

P4.4 [8 points]
Find the characteristic frequencies of the coupled circuits in Fig. 4.4.

Comment on the two modes of oscillation (*Hint: only one mode is damped*). Examine how the damped mode depends on the relation between R^2 and $\frac{L}{C}$.

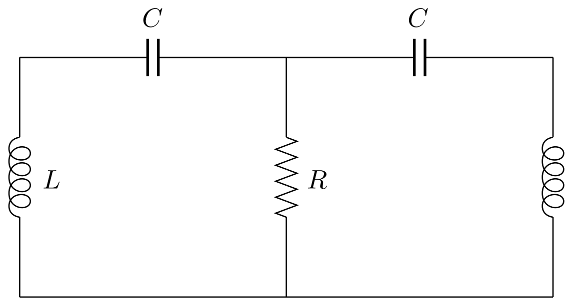

Figure 4.4

P4.5 [10 points]

A mass M moves horizontally along a smooth rail. A pendulum of mass m hangs from M by a massless rod of length ℓ in a uniform vertical gravitational field $\mathbf{g}$ as shown in Fig. 4.5. Ignore all terms of order θ^3 and higher in expansions of trigonometric functions, as well as terms of order $\theta^2\dot{\theta}$ and higher in the Lagrangian. Find the eigenfrequencies and describe the normal modes.

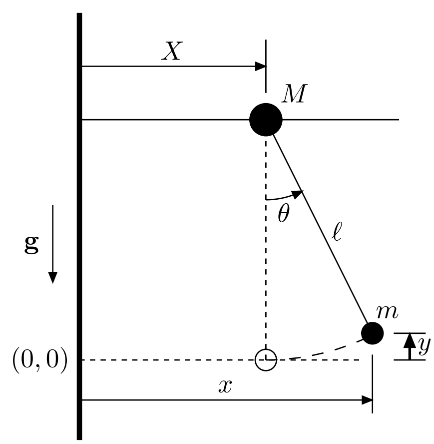

Figure 4.5

P4.6 [6 points]

Three oscillators of equal mass m moving in one dimension are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1(x_1^2 + x_3^2) + \kappa_2 x_2^2 + \kappa_3(x_1 x_2 + x_2 x_3)\right] \tag{1}$$

where

$$\kappa_3 = \sqrt{2\kappa_1\kappa_2}. \tag{2}$$

Find the eigenfrequencies by solving the secular equation. What is the physical interpretation fo the zero-frequency mode?