

Assignment: HW2 [40 points]

Assigned: 2006/09/27

Due: 2006/10/04

P2.1 [1 + 3 + 3 + 3 + 2 = 12 points]

Let the motion of a point mass be governed by the law

$$\ddot{\mathbf{r}} = \dot{\mathbf{r}} \times \mathbf{a}, \quad \mathbf{a} = \text{const.} \quad (1)$$

- Show that $\dot{\mathbf{r}} \cdot \mathbf{a}$ is constant in time.
- Reduce Eq. 1 to an inhomogeneous differential equation of the form $\ddot{\mathbf{r}} + \omega^2 \mathbf{r} = f(t)$.
- Solve the above equation by using a particular function of the form $\mathbf{r}_p = \mathbf{c}t + \mathbf{d}$.
- Express the constants of integration in terms of the initial values $\mathbf{r}(0)$ and $\dot{\mathbf{r}}(0)$.
- Describe the curve $\mathbf{r}(t) = \mathbf{r}_g(t) + \mathbf{r}_p(t)$, where $\mathbf{r}_g(t)$ is the solution of the homogeneous equation $\ddot{\mathbf{r}} + \omega^2 \mathbf{r} = 0$.

P2.2 [2 + 5 = 7 points]

Use variational calculus to find

- The shortest connection between two points (x_1, y_1) and (x_2, y_2) on the 2-dimensional Euclidean plane.
- The shape of a line of uniform mass density supported between (x_1, y_1) and (x_2, y_2) in a uniform gravitational field $\mathbf{g} = g\hat{\mathbf{e}}_y$.

P2.3 [2 + 3 = 5 points]

If a conservative force field in 3-dimensional space is axially symmetric, and we choose the z axis of a cylindrical coordinate system $\{r, \phi, z\}$ along the axis of symmetry, then show that

- The corresponding potential has the form $U = U(r, z)$,
- The force always lies in a plane containing the z axis.

P2.4 [3 + 2 = 5 points]

In Problem P1.4 (see Homework assignment 1), is

$$H \equiv \theta \frac{\partial L}{\partial \theta} - L \quad (2)$$

conserved? Evaluate H and compare it to the energy $E \equiv T + V$.

P2.5 [3 + 6 + 2 = 11 points]

The Lagrangian for a particle of mass m and electric charge e moving in the xy plane is given by

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + eEy - \frac{eB}{c}y\dot{x}.$$

This describes the motion of the particle in a uniform electric field E in the y direction and a uniform magnetic field B in the z direction. c is the speed of light.

- (a) Write down the Euler-Lagrange equations.
- (b) Find the Hamiltonian. Simplify the expression (eliminating \dot{x} by making use of a first integral of motion). What can you say about the general allowed motions in $y(t)$?
- (c) At $t = 0$, $x = y = 0$. What critical value v_c must $\dot{x}(0)$ take in order for the particle to go in a uniform motion? What is the corresponding value of \dot{y} ? (You can either make use of the results in part (a) or that in part (b).)