Assignment: HW3 [25 points]

Assigned: 2012/02/20
Due: 2012/02/27

P3.1 [4 points]
Find the inverse of each element of the permutation group of 3 labelled objects using the 3×3 matrix representation.

P3.2 [3 points]
Show that topologically speaking, the group $SU(2)$ is a 3-sphere of unit radius embedded in the real euclidian 4-dimensional space.

P3.3 [$3 + 3 = 6$ points]
A bounded operator P on a Hilbert space $\mathcal{H}$ is a *projection operator* if

$$P^\dagger = P \qquad \text{and} \qquad P = P^2. \tag{1}$$

(a) Show that $(\mathbf{1} - P)$ is also a projection operator (where $\mathbf{1}$ is the identity operator).

(b) Find the eigenvalues of P.

Note: the first condition implies that P is hermitian.

P3.4 [6 points]
Give a pictorial representation of the $SU(3)$ raising and lowering operators $I_\pm$, $U_\pm$, and $V_\pm$ on the t_3, t_8 plane.

P3.5 [6 points]
If a theory is to be invariant under a unitary local gauge (or phase) transformation $\psi' = U\psi$, where ψ represents a fermion wave function, then the general expression for the transformation of the gauge field is given by

$$A^{\mu\prime} = -\frac{i}{g}(\partial^\mu U)U^{-1} + UA^\mu U^{-1}. \tag{2}$$

Show that this gives the expected answer for the electromagnetic field (i.e. the electromagnetic field tensor $F^{\mu\nu}$ is unaffected by such a transformation), where $U = e^{-ig\theta(x)}$ and $g = -e$.