Assignment: HW7 [40 points]

Assigned: 2016/11/27
Due: 2016/12/05

P7.1 [6 + 4 = 10 points]

(a) Show that the combination of two successive Lorentz transformations with parallel velocities β_1 and β_2 is equivalent to a single Lorentz transformation with velocity $\beta = \dfrac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$.

(b) Show that the set of all Lorentz transformations with this composition rule forms a group.

P7.2 [8 + 2 + 2 = 12 points]

(a) A rocket propels itself along a straight line in empty space by burning its fuel in such a way that the velocity of the exhaust gases ejected from the nozzle is v_0 in the instantaneous (inertial) rest frame of the rocket. If the rocket starts from rest with a total mass M_i (icluding fuel), find the relationship between its velocity and mass during its journey.

(b) Typical chemical fuels yield exhaust speeds of the order of 10^3 m/s. Let us imagine we had a fuel that gives $v_0 = 3 \times 10^5$ m/s. What initial mass of fuel would the rocket need in order to attain a final velocity of $0.1c$ for a final mass of 1 ton?

(c) Matter-antimatter fuel yields $v_0 = c \equiv 1$ (the exhaust consists of photons). What initial mass of fuel would be required in this case (in order to attain a final velocity of $0.1c$ for a final mass of 1 ton)?

P7.3 [8 points]

Determine the maximum energy that can be carried off by any one of the decay particles, when a particle of mass m_0 at rest decays into three particles with masses m_1, m_2, and m_3.

P7.4 [5 + 5 = 10 points]

(a) A particle of mass m is produced with energy E and decays after traveling a distance ℓ. How long did the particle live in its own rest frame?

(b) Use the result of part (a) to determine the spatial separation between the production and decay vertices (a.k.a. "decay length") of a τ lepton carrying an energy of 89 GeV if it lived 0.3 ps in its own rest frame. The mass of the τ lepton is 1.78 GeV.