Assignment: HW3 [40 points]

Assigned: 2016/10/04
Due: 2016/10/11

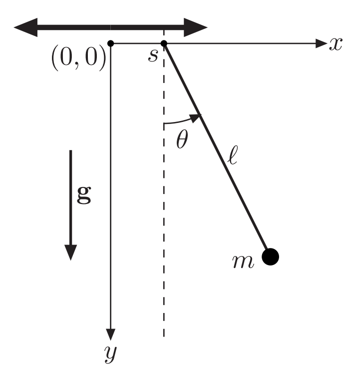
P3.1 [6 points]

A smooth rod of length ℓ rotates in a plane with a constant angular velocity ω about an axis fixed at one end of the rod and perpendicular to the plane of motion. A bead of mass m, free to move along the rod, is initially positioned at the fixed end of the rod and given a slight push such that its initial speed directed towards the other end of the rod is $\omega\ell$. Using Lagrange's method, find the time it takes the bead to reach the other end of the rod.

P3.2 [7 points]

Using Lagrange's method, find the two-dimensional equation of motion of a pendulum of mass m suspended at the end of a massless rod of length ℓ in a gravitational field of uniform acceleration $\mathbf{g}$, whose point of support is executing a simple harmonic motion in the direction perpendicular to gravity, as shown in the figure below, i.e., the coordinates of the point of support are given as functions of time by

$$x_s(t) = x_0 \cos(\omega t); \qquad y_s(t) = 0.$$



Use θ, the angle between the pendulum and the direction of gravity, as the generalized coordinate, and express your answer in terms of θ (and its time derivatives). Assume θ to be small and use the corresponding approximations to simplify your answer. Compare your result to the equation of motion of a forced harmonic oscillator.

P3.3 [$5 + 2 = 7$ points]

(a) Obtain the Hamiltonian and the canonical equations for a particle in a central force field (in 3 dimensions).

(b) Take two of the initial conditions to be $p_\phi(0) = 0$ and $\phi(0) = 0$ (this is essentially the choice of a particular spherical coordinate system). Discuss the resulting simplification of the canonical equations.

P3.4 [$5 + 2 = 7$ points]

A particle of mass m is constrained to move under gravity without friction on the inside of a paraboloid of revolution whose axis is vertical.

(a) Find the one-dimensional problem equivalent to its motion.

(b) What is the condition on the particle's initial velocity to produce circular motion?

P3.5 [6 points]

A particle moves in a central force field given by the potential

$$V = -k\frac{k^{-\alpha r}}{r},$$

where k and a are non-negative constants. When are circular orbits possible? Comment on the condition of circular orbits in this problem to that in Problem 3.4.

P3.6 [$2 + 5 = 7$ points]

(a) For circular and parabolic orbits in an attractive $\frac{1}{r}$ potential having the same angular momentum, show that the perihilion distance of the parabola is one half the radius of the circle.

[Hint: Refer to formulae in Goldstein.]

(b) Prove that in the same central force as in Part (a), the speed of a particle at any point of a parabolic orbit is $\sqrt{2}$ times the speed on a circular orbit passing through the same point.