We report on a successful implementation of a wavelet-based Poisson solver for use in 3D particle-in-cell (PIC) simulations. One new aspect of our algorithm is its ability to treat the general inhomogeneous Dirichlet boundary conditions. The solver harnesses advantages afforded by the wavelet formulation, such as sparsity of operators and data sets, existence of effective preconditioners, and the ability usually to remove numerical noise and further compress relevant data sets. Having tested our method as a stand-alone solver on two model problems, we merged it into IMPACT-T to obtain a fully functional serial PIC code. We present and discuss preliminary results of application of the new code to the modelling of the Fermilab NICADD and the AES/JLab photoinjectors.

**Applications**

| Objective | to develop wavelet-based Poisson solver which can easily be integrated into existing particle-in-cell codes. |

**Cone Strain Equation**

\[ u(x, y, z) = \frac{1}{\rho} \nabla \cdot \sigma \]

**Wavelet Transform Analysis**

The continuous wavelet transform of a function \( f(x) \) is

\[ \psi_\alpha(x) = \frac{1}{\sqrt{\alpha}} \int f(t) \psi^{*}(\frac{x-t}{\alpha}) \, dt \]

\( \psi \) is the mother wavelet, \( \alpha \) and \( \tau \) are scale and translation factors.

- Simultaneous time and frequency localization (FDT only frequency)
- Orthogonal basis of functions
- Discrete wavelet transform: a family of perfect reconstruction filters with compact support (non-zero over a finite domain)
- Thresholding wavelet coefficients: if \( |\psi_\alpha| < \varepsilon \), set \( \psi_\alpha = 0 \)
- Compact representation of data: wavelet data compression
- Removes numerical noise from the signal: wavelet denoising
- Wavelet denoising: removing numerical noise due to granularity and discreteness from N-body simulations has the same effects as running these simulations with orders of magnitude more many particles

**CONCLUSIONS**

- Presented an iterative wavelet-based Poisson solver (CPGC)
- Wavelet compression and denoising achieves computational speedup
- Computational load additionally reduced by
  - Preconditioning the Laplacian operator
  - Exploiting sparsity of the Laplacian operator in wavelet space
- Integrated CPGC into a particle-in-cell code (IMPACT-T) for beam simulations
- Results of simulations with the new wavelet-based Poisson solver are in excellent agreement with results of standard 3D Poisson solvers
- Original contributions
  - First wavelet-based solver for 3D Poisson equation with general Dirichlet boundary conditions
  - First application of wavelet-based multiscale methodology to 3D simulation of beams
  - Still to be done
    - Denoising by thresholding the wavelet coefficients
    - Parallelization, optimization and benchmarking
  - Use CPGC in N-body simulations of self-gravitating systems (galaxies, star clusters...)

**PARTICLE-IN-CELL METHOD**

Solve the Poisson equation with general Dirichlet boundary conditions

\[ \Delta u = f \]

- Discretized on \( N_x \times N_y \times N_z \) grid

Potential on the boundaries of the computational grid \( u_{n_{\text{bc}}} \) is computed via Green's function corresponding to the physical boundary and the geometry of the system

**WAVELET-BASED POISSON SOLVER**

- Iterative solver with convergence rate \( \tau \) - condition number of the operator \( L \)
- Initial guess \( u \) at previous time step
- Diagonal efficient preconditioner in wavelet space
  - Removing numerical noise subspace compressing
  - Convergence greatly improved by preconditioning
- Weaknesses:
  - Need potential on the surface of the grid (Compute gradient EPG)

**APPLICATIONS**

- Without constraining effective dimensionality of the problem by thresholding in wavelet space, IMPACT-T with CPGC is already as fast as the standard IMPACT-T
- Significant computational speedup will follow the implementation of wavelet thresholding
- Other areas of computational optimization
  - Finding a more effective preconditioner
  - Further optimizing computation of boundary conditions

**IMPACT-T vs. IMPACT-T with CPGC: computational speedup**

- Comparison between IMPACT-T (top panel) and IMPACT-T with CPGC (lower panel) for transverse distributions at different \( z \)-locations along the AES/JLab low-energy photoinjector

**IMPACT-T vs. IMPACT-T with CPGC: level of detail**

- Comparison of IMPACT-T, IMPACT-T with CPGC and PARAMELA for the AES/JLab low-energy photoinjector (left 4 plots) and high-energy photoinjector (right 4 plots)