Applications of Modern Computational Tools in Accelerator Physics

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Give Credit Where Credit is Due

- **CASA scientists:**
  - Alicia Hofler, Vasiliy Morozov, Yves Roblin, Fanglei Lin, Geoff Krafft

- **Old Dominion University & Center for Accelerator Science:**
  - Professors:
    - **Physics:** Alexander Godunov
    - **Computer Science:** Mohammad Zubair, Desh Ranjan
    - **Modeling, Simulation & Visualization Engineering:** Bharat Madan
  - PhD students:
    - **Computer science:** Kamesh Arumugam, Mohamed Aturban

- **REU/SULI summer undergraduate students:**
  - Matthew Kramer (UC Berkeley), Colin Jarvis (Macalester), Anton Zvezdin (Stony Brook → Northwestern), Alyssa Henderson (UVa)
Outline of the Talk

- New computational tools:
  - New methods
  - New computational hardware
- New methods: Multidimensional, non-linear optimization using a genetic algorithm (GA)
  - Brief motivation and background
  - Applications: What we have done, will do and can do
- New computational hardware:
  Parallel computation on Graphical Processing Units (GPUs)
  - Brief motivation and background
  - Applications
    - Multidimensional integration for use in CSR simulations
    - New code for simulation of long-term beam-beam dynamics
- Summary
GA Optimization: Motivation

• As the dimensionality of the problem $N$ increases, $N$-dimensional non-linear optimization becomes more challenging/impossible:
  – Traditional, gradient-based methods (Newton, conjugate-gradient, steepest descent, etc...) are not globally convergent:
    • May get stuck in a local minimum and never come out
    • Final solution depends on the initial guess
    • Generally not robust in the non-linear regime
    • Direct multi-objective optimization not possible

• This demonstrates a clear need for globally-convergent, robust, multidimensional, multi-objective, non-linear optimization methods
  – Genetic algorithm (GA):
    • Trade-off: not as efficient as traditional methods
    • Caution: still not a silver bullet
GA Optimization: Background

- GA uses principles of natural selection to solve an optimization problem

<table>
<thead>
<tr>
<th>Evolution</th>
<th>Multidimensional optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gene</td>
<td>Variable</td>
</tr>
<tr>
<td>Individual</td>
<td>Point in search space</td>
</tr>
<tr>
<td>Population</td>
<td>Set of points in search space</td>
</tr>
<tr>
<td>Mutation</td>
<td>Changing variables</td>
</tr>
<tr>
<td>Swap</td>
<td>Exchange of values of the same variable between two points in search space</td>
</tr>
<tr>
<td>Recombination (partial swap)</td>
<td>Change of values of the same variable between two points toward each other</td>
</tr>
<tr>
<td>Fitness</td>
<td>Value of the objective function</td>
</tr>
</tbody>
</table>

- Mutation
  - Asexual reproduction \( \eta_{\text{mut}} = 1 \) \( \eta_{\text{mut}} = 10 \)

- Recombination
  - Sexual reproduction \( \eta_{\text{rec}} = 1 \) \( \eta_{\text{rec}} = 10 \)

For details see: Hofler, Terzić, Kramer, Zvezdin, Morozov, Roblin, Lin & Jarvis 2013, PR STAB 16, 010101
Select independent variables = chromosomes

Initialize population

generation = 0

Evaluate model and objectives

generation ++

Assign Fitness

Select pairs

Recombination

Mutation

End condition?

Yes

Done

No

Minimize \( f_m(x) \), \( m = 1,2,\ldots,M \);

Subject to \( g_j(x) \geq 0 \), \( j = 1,2,\ldots,J \);

\( h_k(x) = 0 \), \( k = 1,2,\ldots,K \);

\( x_i^L \leq x_i \leq x_i^U \), \( i = 1,2,\ldots,n \);

← objectives

← inequality constraints

← equality constraints

← decision variable constraints

Slide courtesy of Alicia Hofler
GA Optimization: Background

- GA simulation is only as accurate as its function evaluator
- We use two paradigms depending on objective function evaluation
  - When objective function(s) are evaluated by a full simulation
    - Platform and Programming Language Independent Interface for Search Algorithms (PISA) from ETH Zürich and Alternate PISA (APISA) from Cornell:
      - Job control: spawning simulations, post-processing, communication
      - Redesigned to be more modular and script-based: easier to plug-and-play
      - Parallel execution of an entire generation: runs on JLab’s cluster and farm
  - When objective function(s) are analytically evaluated in a subroutine
    - Still can use PISA, but other options available (in various languages)
    - We choose inspyred Python package:
      - Contains various nature-inspired and other optimization methods: particle-swarm, differential evolution, simulated annealing, etc...
      - Contains efficient traditional methods: Newton, conjugate gradient, ...
      - Parallel execution possible (using pypmi package)
We applied GA optimization to many problems in accelerator physics:

- When a separate simulation is needed to evaluate objective function(s)
  - Optimizing collider working point for luminosity (BB3D)
  - Maximizing dynamic aperture in a collider ring (elegant)
  - Decoupling of the beam optics in the injector (elegant)
  - Optimizing dynamic aperture and chromaticity in a collider ring (elegant)
  - RF gun optimization for injector brightness (Astra, Superfish)
    - [Hofler 2012, PhD and elsewhere]
    - [Hofler, Terzić, Kramer, Zvezdin, Morozov, Roblin, Lin & Jarvis 2013, PR STAB 16, 010101]
  - Optimizing laser frequency modulation function in Thomson scattering
    - [Terzić, Deitrick, Hofler & Krafft 2013, PRL, submitted]

- When objective function(s) can be evaluated analytically
  - Chi-square fits to the CEBAF harp data

Future candidate for GA optimizations:

- Optimizing heat load and trip rates in the CEBAF linacs
- Injector optimization with the CEBAF as the function evaluator
• Collider luminosity is sensitive to beam-beam effect and synchro-betatron resonances of the two colliding beams
• Careful selection of a tune **working point** is essential for stable operation of a collider as well as for achieving high luminosity
• We simulate the Medium-energy Electron-Ion Collider (MEIC)

• **Optimization problem:**
  – Independent variables: betatron tunes for the two beams \((\nu_1^x, \nu_1^y, \nu_2^x, \nu_2^y)\)
    (Synchrotron tunes fixed for now; 4D problem)
  – Objective function: collider’s luminosity \(L(\nu_1^x, \nu_1^y, \nu_2^x, \nu_2^y)\)
    (Evaluated via a simulation with BeamBeam3D parallel code on the JLab cluster)
  – Subject to constraints (e.g., confine tunes to particular regions)

• GA is the only non-linear optimization method that can work in a search space so violently fraught with resonances (very sharp peaks and valleys)
Resonances occur when $m_x \nu_x + m_y \nu_y + m_s \nu_s = n$
$m_x, m_y, m_s$ and $n$ are integers ($m_s = 0$ for now)

Green lines: *difference* resonances (stable)
Black lines: *sum* resonances (unstable)

Restrict search to a group of small regions along the diagonal devoid of black resonance lines.
Restricts the search space by ~30 in 2D, ~1000 in 4D

Found an excellent working point near half-integer resonance
- e-beam: $\nu_x = 0.530, \nu_y = 0.548$
- p-beam: $\nu_x = 0.501, \nu_y = 0.527$

Luminosity about 33% above design value in only ~300 simulations (5 gen. of 64 individuals)

Systematic scan with a modest 0.01 resolution: $100^4 = 10^8$ simulations!

GA search orders of magnitude more efficient

*This is just a proof of principle* – future realistic simulations will include other important effects: magnet errors, non-linear maps, IBS, cooling ...
GA Application: Non-Linear Chi-Square Fitting

- Wire scanners (harps) are used in CEBAF to measure the beam’s position and size.
- Moved into the beamline at a constant velocity and angle (generally 45°).
- Data from these plots needs to be fit to Gaussians to obtain beam size and position.
- Non-linear fitting cast into an optimization problem: minimize chi-square ($\chi^2$)

$$\chi^2 = \sum_{i=1,N} [f(x_i) - y_i]^2$$

- Independent variables:
  Gaussian parameters: $A$, $\mu$, $\sigma$, $c$ (4D problem)
- Objective function:
  $\chi^2$ of a Gaussian $f(x) = Ae^{\frac{-(x-\mu)^2}{2\sigma^2}} - c$
- Used Python package inspyred

Slide courtesy of Alyssa Henderson

11/27
GA Application: Non-Linear Chi-Square Fitting

- A hybrid method combines strengths of the two approaches
  - Traditional, gradient-based methods converge quickly to a local optimum
  - GA and other nature-inspired methods (PSO, DE) are globally convergent – they eventually zoom in on the global minimum (generally slow)
  - First reduce the search space with a globally convergent method, then use a fast gradient-based method for improved efficiency
- Harp fitting is still mostly solvable by a traditional, globally convergent method (Newton CG)
  - GA and other nature-inspired methods used for robustness & insurance
- Switching to other models is easy (just change the objective function)
- As the dimensionality of the problem grows, gradient-based methods becomes less reliable, and the need for GA more pronounced

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCG</td>
<td>3.168</td>
</tr>
<tr>
<td>PSO</td>
<td>123.906</td>
</tr>
<tr>
<td>DE</td>
<td>41.416</td>
</tr>
<tr>
<td>GA</td>
<td>120.940</td>
</tr>
</tbody>
</table>

Average execution time for a harp scan fit with >50 points on a single CPU
GA Application: Dynamic Aperture & Chromaticity

• In a collider ring design, *dynamic aperture and momentum acceptance should both be maximized* – but they are roughly inversely proportional

• Reducing the higher-order chromaticity will lead to increased momentum acceptance
  • This, again, depends on the choice of the working point (betatron tunes)

• We simulate the MEIC ion ring

• **Optimization problem:**
  – Independent variables: betatron tunes for the beam \((v_x, v_y)\) (2D problem)
  – Objective functions:
    (Both evaluated via a simulation with *elegant* code on the JLab farm)
    • Maximize mom. acceptance \(\iff\) Minimize \(2^{\text{nd}}\)-order chromatic function \(\xi^2\)
    • Maximize area of dynamic aperture \(A\) \(\iff\) Minimize \(1/A\)
GA Application: Dynamic Aperture & Chromaticity

- Physics judgment is needed to strike a balance between the two quantities
- Pareto-optimal front: Non-dominated solutions

FIG. 17. Pareto front after 24 generations of 64 individuals. The large ×’s denote representative points A, B, and C.

[Hofler, Terzić, Kramer, Zvezdin, Morozov, Roblin, Lin & Jarvis 2013, PR STAB 16, 010101]
GA Optimization: Adapting to a New Problem

• Adapting the two paradigms to a new problem is relatively easy
  – Physics pre-screening
    • Restrict the search space as much as physically possible
      Convergence directly proportional to the volume of search space
  – Define the optimization problem:
    • **Objective function**: How is it computed? Choose paradigm
    • **Independent variables** (*parameters/“knobs to be turned”*): define ranges
    • **Constraints**: specify if present
    • Decide on simulation parameters: number of individuals/generations
  – If a separate simulation is needed for evaluating objective function(s)
    • Make sure program is installed on the platform to be used
      (Unlimited licenses are required for commercial software)
    • Modify scripts for spawning jobs and extracting results
    • Define parameter ranges for independent variables
  – If objective function(s) can be evaluated analytically
    • Write function(s)/subroutine(s)
• Second half of my talk is on parallel computation on GPUs

• Why is it important?
  • Making simulations much more efficient computationally enables studying previously inaccessible physics

• What are we doing that is new and different?
  • Interdisciplinary approach – division of labor among experts in the field:
    • Physicists: physics, algorithm development, prototyping
    • Computer scientists: algorithm development and implementation, parallel programming

• What are our goals?
  • Develop GPU-parallelized codes that will lead to accelerator physics simulations beyond the present state of the art
  • Design methods useful beyond the scope of accelerator physics
  • Develop expertise on the subject to use it on other problems
Parallel computation on GPUs
- Ideally suited for algorithms with high arithmetic operation/memory access ratio
- Same Instruction Multiple Data (SIMD)
- Several types of memories with varying access times (global, shared, registers)
- Uses extension to existing programming languages to handle new architecture
- GPUs have many smaller cores (~ 400-500) designed for parallel execution
- Avoid branching and communication between computational threads

Example: Tesla M2090 GPU has 512 cores
There are many problems in accelerator physics that can greatly benefit from a speedup from a GPU-based computation.

- Particle tracking codes (*elegant*, VORPAL, etc...)
- Collision codes (*BeamBeam3D*)
- Monte Carlo-based codes

- **Speedup**: ratio of execution times on a host CPU to that on a GPU
- Some have already been GPU-parallelized with impressive speedup
  - *elegant*: ~70 times
  - VORPAL: at least an order of magnitude

- In general, it has been shown that GPUs can deliver performance improvements of 1-3 orders of magnitude

- This kind of speedup means:
  - Simulation time: several months or a year → about a day
  - Opening the doors to studying previously inaccessible physics!
• When a charged particle beam travels along a curved trajectory (bending magnet), beam emits synchrotron radiation

Incoherent (ISR) \( \lambda < \sigma_s \)

Coherent (CSR) \( \lambda > \sigma_s \)

• CSR adversely impact beam quality:
  • Increased energy spread and emittance, longitudinal instability (microbunching)
  • CSR effects are important for FELs, light sources, ERLs, etc...
  • It is of vital importance to have a trustworthy code to simulate and mitigate the CSR effects
GPU Computation: CSR Simulations

- CSR simulations have proven to be extremely challenging
  - Computing *retarded potentials* requires integration over the retarded time $t'$:
    \[
    t' = t - \frac{|\vec{r} - \vec{r}'|}{c}
    \]
    \[
    \begin{bmatrix}
    \phi(\vec{r}, t) \\
    \vec{A}(\vec{r}, t)
    \end{bmatrix} = \int \begin{bmatrix}
    \rho(\vec{r}', t') \\
    \vec{J}(\vec{r}', t')
    \end{bmatrix} \frac{d\vec{r}'}{|\vec{r} - \vec{r}'|}
    \]
    Retarded time

- **Huge computational bottleneck!** Computations scale:
  - **Particle-particle codes:** $\sim N^2$, where $N$ is the number of particles
  - **Particle-in-cell codes:** $\sim N_{\text{res}}^2$, where $N_{\text{res}}$ is the grid resolution
    We have been developing a particle-in-cell (PIC) CSR code

- **Solution:** Develop an efficient, parallel multidimensional integrator on GPUs
  - Integration over grid is ideally suited for GPU parallelization (SIMD)
  - Used NVIDIA CUDA framework (extension to C++)
  - **Deterministic:** based on integration rules like Gauss or Newton – not Monte Carlo
  - Useful beyond this project: *outperforms Monte Carlo in medium dimensions*
Direct parallelization of the serial methods does not take advantage of GPU data parallelism and does not provide load balancing → inefficient code.

We developed a new two-phase parallel algorithm multidimensional integration on GPUs:
- Phase 1: Parallel identification of subintervals needing higher resolution
- Phase 2: Parallel evaluation of identified subregions to prescribed accuracy

GPU-based implementation outperforms the best known sequential method (CUHRE) and achieve up to 10-100 times speedup on a single GPU.

Benchmark functions in $n$ dimensions:
1. $f_1(x) = \left[ \alpha + \cos^2 \left( \sum_{i=1}^{n} x_i^2 \right) \right]^{-2}$, where $\alpha = 0.1$
2. $f_2(x) = \cos \left( \prod_{i=1}^{n} \cos \left( 2^i x_i \right) \right)$
3. $f_3(x) = \sin \left( \prod_{i=1}^{n} i \arcsin(x_i) \right)$
4. $f_4(x) = \sin \left( \prod_{i=1}^{n} \arcsin(x_i) \right)$
5. $f_5(x) = \frac{1}{2\beta} \sum_{i=1}^{n} \cos(\alpha x_i)$, where $\alpha = 10.0$ and $\beta = -0.054402111088937$

[Arumugam, Godunov, Ranjan, Terzić & Zubair 2013a]
• Next, we *optimized* our new GPU-based algorithm *for memory efficiency* and *scaled to multiple GPU devices*

• The algorithm has been implemented on a cluster of Intel® Xeon® CPU X5650 computes nodes with 4 Tesla M2090 GPU devices per node (512 cores per device)

• On a single GPU device: Reached a speedup over a serial version of up to 240 as compared to a *speedup of 70* when memory optimization was not used

• On a cluster of 6 nodes (24 GPU devices): Reached a speedup of up to 3250

1 CPU Speedup:
With Vs. Without Memory Optimization

Multi-CPU Speedup

[Arumugam, Godunov, Ranjan, Terzić & Zubair 2013a, 2013b]

Slide courtesy of Kamesh Arumugam
Monte Carlo integration on GPU (VEGAS and BASES methods) has been published previously in *The European Physical Journal C* [Kanazaki 2011, 71:1559]

- We compare Monte Carlo Vs. our method on a set of 6 functions with exact solutions
- **Preliminary results:** Even in higher dimensions our adaptive multidimensional integration method outperforms Monte Carlo method on a single GPU
- **Preliminary results:** Monte Carlo on GPU fails for large number of function evaluations
- **Possible ramifications:** Our new code can replace Monte Carlo in many physics application for improved performance

\[ f_s(x) = \frac{1}{2\beta} \sum_{i=1}^{n} \cos(\alpha x_i) \]

where \( \alpha = 10.0 \) and \( \beta = -0.0544 \)

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**Internal Relative Error**

**True Relative Error**
Studying beam dynamics in particle colliders requires specialized codes. Simulation in beam-beam codes can be divided in two parts:

- **Particle tracking** between consecutive beam collisions
  - Number of efficient tracking codes exist
- **Beam collisions** at the interaction point
  - Solving the Poisson equation: major computational bottleneck

**Long-term dynamics is of particular interest**

- Simulate operational stability and luminosity lifetime
- **Long-term**: on the order of beam lifetime (for MEIC ~ $10^9$ collisions!)
- This kind of long-term is *currently inaccessible with existing codes*
  - Too much time is spent on detailed solution of the Poisson equation

**Solution:**

- Simplify the beam-beam interaction (use Bassetti-Erskine approx.)
- Parallelize both tracking and collision on GPUs
• Bassetti-Erskine approximation:
  – When beams are *infinitesimally short* and *transversally Gaussian* the Poisson equation reduces to a *complex error function* (much faster than any other way of solving the Poisson equation)
  – Finite length of the beams simulated by stringing along thin slices
    – Collide each slice in one beam with each slice in the other beam
• *Gaussian transverse distribution a good approximation for the MEIC*

• GPU parallelization: *(preliminary results)*
  – Particle tracking:
    • Symplectic 1-turn maps of an arbitrary order from COSY-Infinity
      – Single-CPU tracking is equivalent and as fast as COSY-Infinity
      – Speedup on 1 GPU ∼ 170 times
  – Beam-beam collisions:
    • Scales better than BeamBeam3D
      – Speedup on 1 GPU ∼ 70 times

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64 CPUs: BeamBeam3D ∼ M^{7/4}

1 CPU: Our code ∼ M
Summary

• Presented two new powerful computational tools

• **GA optimization for multi-dimensional, non-linear optimization:**
  - In many physics cases, knobs have to be turned to improve performance
  - When the dimensionality of the problem is large, GA is the only hope
  - *GA simulation is only as good as its function evaluator*
  - Implemented two easily-adaptable computational paradigms
  - Applied to a number of problems in accelerator physics
  - *These serve as a template for other applications*
Summary

• Parallel computation on GPUs:
  • *Interdisciplinary approach*: physicists, computer scientists and engineers
  • Can lead to *orders-of-magnitude speedup* for some codes
  • GPU address the computational bottlenecks in two important accelerator physics problems:
    • CSR simulation
      • Numerical integration: *speedup of up to about 3 orders of magnitude*
    • Beam-beam interaction
      • Tracking and collision: *speedup of about 2 orders of magnitude*
  • We developed expertise on the subject to be used on future problems

• Overarching goal: Develop new tools to study *previously intractable* problems in accelerator physics and beyond
Details for the Work Presented

• Application of GA optimization (CASA collaboration):
  • Kramer, Jarvis & Terzić 2010, JLab Tech Note JLAB-TN-10-034
  • Terzić, Kramer & Jarvis 2011, PAC (WEP167)
  • Hofler 2012, PhD thesis
  • Hofler, Terzić, Kramer, Zvezdin, Morozov, Roblin, Lin & Jarvis 2013, PR STAB 16, 010101
    Mini-tutorial to be presented at NAPAC 2013 by Alicia Hofler
  • Terzić, Deitrick, Hofler & Krafft 2013, PRL, submitted
  • Henderson 2013, REU project

• GPU computation:
  Multidimensional integration (ODU / CAS collaboration)
  • Arumugam, Godunov, Ranjan, Terzić & Zubair 2013a,
    International Conference on Parallel Processing – 42nd Annual Conference (refereed)
  • Arumugam, Godunov, Ranjan, Terzić & Zubair 2013b,
    20th Annual International Conference on High-Performance Computing (refereed)
  • Arumugam, Godunov, Ranjan, Terzić & Zubair 2013c, in preparation

Beam-beam code (ODU / CAS / CASA collaboration)
• Roblin, Morozov, Terzić, Aturban, Ranjan & Zubair 2013, IPAC (MOPWO0080)
Backup Slides
Genetic Algorithm Basics

Crossover or Recombination

Parent-1 2-gene chromosome:

<table>
<thead>
<tr>
<th>11₀</th>
<th>27₁₀</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0 1 1 0 1 1</td>
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<tr>
<td>1 0 1 1</td>
<td>0 1 1 0 1 1</td>
</tr>
<tr>
<td>1 0 1 1</td>
<td>0 1 1 0 1 1</td>
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</table>

Parent-2 2-gene chromosome:

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<th>5₀</th>
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</thead>
<tbody>
<tr>
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<td>0 1 0 1</td>
<td>1 0 1 0 0 1</td>
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<tr>
<td>0 1 0 1</td>
<td>1 0 1 0 0 1</td>
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</tbody>
</table>

Offspring-1 2-gene chromosome:

<table>
<thead>
<tr>
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<th>0 0 1 0 0 1</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>1 0 1 1</td>
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Offspring-2 2-gene chromosome:

<table>
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<th>0 1 0 1</th>
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<tr>
<td>0 1 0 1</td>
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</table>

Mutation

2-gene chromosome:

<table>
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<tbody>
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<tr>
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</tbody>
</table>

Mutated 2-gene chromosome:

<table>
<thead>
<tr>
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<th>27₁₀</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>1 0 1 0</td>
<td>0 1 1 0 1 1</td>
</tr>
</tbody>
</table>
Real Valued Recombination

Simulated Binary Crossover

\[ \beta = \frac{X_{o_1} - X_{o_1}}{X_{p_1} - X_{p_1}} \]

\[ (X_{o_1}, X_{o_2}) = \frac{1}{2} \left[ X_{p_1} + X_{p_2} \mp \beta \left| X_{p_1} - X_{p_2} \right| \right] \]

\[ p(\beta) = \frac{1}{2} \left[ 1 + \eta_{rec} \right] \begin{cases} \beta^{\eta_{rec}} & 0 \leq \beta \leq 1 \\ \beta^{-(2+\eta_{rec})} & 1 < \beta \end{cases} \]

parent-1 3-element decision vector

parent-2 3-element decision vector

offspring-1

offspring-2

(Optionally, swap vector elements a priori)

Slide courtesy of Alicia Hofler
Real Valued Mutation

Polynomial Mutation

\[ \chi_{\text{mut}} = x + \delta \Delta_{\text{max}} \]

\[ p(\delta) = \frac{1}{2} \left[ 1 + \eta_{\text{mut}} \right] \left[ 1 - |\delta| \right]^{\eta_{\text{mut}}} \]

\[ \Delta_{\text{max}} = 1.0 \]
\[ \delta = 0.1 \]

4-element decision vector

old 1.1 2.2 3.3 4.4
new 1.1 2.2 3.4 4.4

1.1 + (0.1)(1.0)

Slide courtesy of Alicia Hofler
Optimizing Collider Working Point: GA Vs. Parameter Scan

FIG. 6. Top panel: Improvement over the design luminosity after each generation for a GA-based optimization with 20 generations of 128 individuals in each. The improved working point is about 9% better than the one found in Fig. 4. Bottom panel: Improvement over design luminosity after a systematic parameter scan with resolution \( k \) in each parameter.

Hofler, Terzić, Kramer, Zvezdin, Morozov, Roblin, Lin & Jarvis 2013, PR STAB 16, 010101