20 Lecture 20: Cosmic Microwave Background Radiation  
— continued

“Innocent light-minded men, who think that astronomy can be learnt by looking at the stars without knowledge of mathematics will, in next life, be birds.”

Plato

The Big Picture: Today we are finishing the discussion of the CMB radiation, including the analysis of the acoustic peaks and effects leading to anisotropies.

Scales in the Angular Power Spectrum

The angular power spectrum quantifies the correlation of different parts of the sky we observe separated by an angle $\theta$. This angle is related to a multipole $l$ of the expansion as $\theta = 180^\circ/l$. The size of the observable Universe (horizon) at the time of decoupling corresponds to about $1^\circ$ on the sky today ($l \approx 200$). The part of the angular spectrum which correlates portions on the sky separated by angles appreciably larger than the size of the horizon at decoupling (corresponding to $l \lesssim 20$) represent initial conditions: these parts of the Universe have not been in causal contact since (before) inflation (Fig. 34). The other part of the angular spectrum — at high $l$ values — feature peaks corresponding to acoustic oscillations (Fig. 35). The positions and magnitudes of the peaks of acoustic oscillations contain fundamental properties about the geometry and structure of the Universe.

CMB Observations

![CMB Observations Diagram](image)

Figure 34: CMB horizon (Courtesy of W. Hu)
Acoustic Oscillations

In the early Universe before decoupling, rapid scattering couples photons and baryons into a plasma which behaves as perfect fluid. Initial quantum overdensities create potential (gravitational) wells — inflationary seeds of the Universe’s structure. Infall of the fluid into the potential wells is resisted by its pressure, thus forming acoustic oscillations: periodic compression (overdensities in the fluid; hot spots) and rarefactions (underdensities; cold spots). These acoustic oscillations of the early Universe are frozen at recombination and give the CMB spectrum a unique signature.

The CMB data reveals that the initial inhomogeneities in the Universe were small. An overdense regions would grow by gravitationally attracting more mass, but only after the entire region is in causal contact. This means that only regions which are smaller than the horizon at decoupling had time to compress before then. Regions which are sufficiently smaller than the horizon had enough time to compress gravitationally until the outward-acting pressure halted the compression via Thomson scattering, and possibly even go through a number of such acoustic oscillations. Therefore, perturbations of particular sizes may have gone through: (i) one compression (fundamental wave); (ii) one compression and one rarefaction (first overtone); (iii) one compression, one ramification and one compression again (second overtone); etc... (Fig. 36).

The most pronounced temperature variation in the CMB radiation will be due to the fundamental sound wave. This is because the portions of the sky separated by the scale equal to the horizon at decoupling — corresponding to the fundamental sound wave — will be completely out of phase.

Consider a standing wave \( A_k(x, t) \propto \sin(kx) \cos(\omega t) \), going through space at the speed of sound (in plasma \( v_s \approx c/\sqrt{3} \)), with the frequency \( \omega \) and wave number \( k \), related by \( \omega = kv_s \). The displacement — and hence the correlation in temperature — will be maximal at the decoupling time \( t_{\text{dec}} \) for \( \omega t_{\text{dec}} = kv_s t_{\text{dec}} = \pi, 2\pi, 3\pi... \) The subsequent peaks in the power spectrum represent...
Figure 36: Sound waves in a pipe (top) and acoustic waves in the early Universe (Hu & White, *Scientific American*, February 2004).
the temperature variations caused by overtones. The series of peaks strongly supports the theory that inflation halted all of the sound waves at the same time. If the perturbations had been continuously generated over time, the power spectrum would not be so harmoniously ordered.

**Dampening of the overtones.** Both ordinary matter and dark matter supply mass to the primordial plasma and enhance the gravitational pull, but only ordinary matter undergoes the sonic compressions and rarefactions (dark matter has decoupled from the plasma at a much earlier time). At recombination, the fundamental wave is frozen in a phase where gravity enhances its compression of the denser regions of plasma (Fig. 37). The first overtone, which corresponds to scales half of the fundamental wavelength, is caught in the opposite phase (Fig. 37, bottom panel) — gravity is attempting to compress the plasma while the plasma pressure is trying to expand it. As a consequence, the temperature variations caused by this overtone (and all subsequent ones) will be less pronounced than those caused by the fundamental wave (fundamental peak).

This dampening of the magnitudes of the overtones allows for quantification of the relative strength of gravity and radiation pressure in the early Universe.

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**Figure 37:** Gravitational modulation: gravity and acoustic oscillation work in phase in the first peak (top); gravity and acoustic oscillations attenuate each other’s effects (Hu & White, *Scientific American*, February 2004).
Dampening of the small-scale acoustic waves. The theory of inflation also predicts that the sound waves should have nearly the same amplitude on all scales. The power spectrum, however, shows a sharp drop-off in magnitude of temperature variations after the third peak. This is due to the dissipation of the sound waves with short wavelengths: sound is carried by oscillation of particles in gas or plasma, a wave cannot propagate if its wavelength is shorter than the typical distance traveled by particles between collisions.

Polarization of the CMB

Researchers have recently detected that the CMB radiation is polarized. Careful and precise study of this area is believed to be the most promising avenue toward discovering new fundamental physics.

The polarization, unlike the temperature anisotropies is only generated by scattering. When we observe the polarization we are looking directly at the surface of the last scattering of photons. It is therefore our most direct probe of the Universe at the epoch of recombination as well as the later reionization of the Universe by the first stars. The latter can really only be probed by the CMB through its polarization.

Figure 38: Generation of polarization: unpolarized but anisotropic radiation incident on an electron produces radiation. Intensity is produced by line thickness. To an observer looking along the direction of the scattered photons ($z$), the incoming quadrupole pattern produces linear polarization along the $y$-direction.

The polarization, which carries directional information on the sky (as a tensor field), contains more information than the temperature field. Measurements of the polarization power spectrum can greatly enhance the precision with which one can extract the physical parameters associated with acoustic oscillations.

Furthermore, the polarization through its directional information provides a means of isolating the gravitational waves predicted by models of inflation. As such polarization provides our most direct window onto the very early Universe and the origin of all structure in the Universe.

Origin of polarization. Quadrupole anisotropy polarizes the anisotropic (but unpolarized) radiation (Fig. 38). The CMB radiation is polarized by Thomson scattering in the following manner. Consider incoming radiation from the left being Thomson-scattered by $90^\circ$ out of the screen. Since
light cannot be polarized along its direction of motion, only one linear polarization gets Thomson-scattered. However, there is nothing special about light coming in from the left: if the light also comes from the top, the resulting scattered radiation will have both polarization states. The degree of polarization will depend on intensity of the incoming radiation, so the 90° anisotropies in the radiation will result in linear polarization (Fig. 38).

**Shift to high l.** Because the polarization arises from scattering, which in turn dilutes the quadrupole, the anisotropies in polarization are much weaker than anisotropies in temperature. With each scatter that the photon experiences on its way to equilibrium, the polarization is reduced. The remaining polarization is a direct result of the stoppage of scattering. The local quadrupole on the scales which are much larger than the mean-free path of photons (for instance, the scale of the horizon) will be diluted by multiple scattering, and therefore not dominant in the spectrum. The peak of the spectrum is shifted toward smaller scales (large \( l \) values), where the local quadrupole is close to the mean-free path of photons.

**Physical Effects Affecting the CMB Radiation**

**The Sunyaev–Zel’dovich Effect.** The Sunyaev-Zel’dovich (SZ) effect refers to the Compton scattering of CMB photons by hot, ionized gas in clusters of galaxies. It was first predicted in 1969 by Sunyaev and Zel’dovich. The effect is a foreground anisotropy to the CMB. The SZ effect causes a “hotspot” in the CMB due to the kinetic SZ effect (due to the bulk motion of the cluster with respect to the CMB) and a noticeable change in the shape of the CMB spectrum due to the thermal SZ effect.

The SZ effect is important to the study of cosmology and the CMB for two main reasons:

1. the observed “hotspots” created by the kinetic effect will distort the power spectrum of CMB anisotropies. These need to be separated from the primary anisotropies in order to probe properties of inflation.

2. The thermal SZ effect can be measured and combined with X-ray observations in order to determine values of cosmological parameters, in particular the present value of the Hubble rate \( H_0 \).

Interaction between photons of the CMB and charged particles they encounter as they pass through the hot, ionized gas in clusters of galaxies causes them to scatter, thus polarizing the CMB radiation across wide swaths of the sky. Observations of this large-angle polarization by the WMAP spacecraft imply that about 17 percent of the CMB photons were scattered by a thin fog of ionized gas a few hundred million years after the Big Bang.

This relatively large fraction is perhaps the biggest surprise from the WMAP data. Cosmologists had previously theorized that most of the Universes hydrogen and helium would have been ionized by the radiation from the first stars, which were extremely massive and bright. (This process is called reionization because it returned the gases to the plasma state that existed before the emission of the CMB.) But the theorists estimated that this event occurred nearly a billion years after the Big Bang, and therefore only about 5 percent of the CMB photons would have been scattered. WMAPs evidence of a higher fraction indicates a much earlier reionization and presents a challenge for the modeling of the first rounds of star formation. The discovery may even challenge the theory of inflations prediction that the initial density fluctuations in the primordial Universe were nearly the same at all scales. The first stars might have formed sooner if the small-scale fluctuations had higher amplitudes. The WMAP data also contain another hint of deviation from scale invariance that was first observed by the COBE satellite. On the biggest scales, corresponding to regions
stretching more than 60 degrees across the sky, both WMAP and COBE found a curious lack of temperature variations in the CMB. This deficit may well be a statistical fluke: because the sky is only 360 degrees around, it may not contain enough large-scale regions to make an adequate sample for measuring temperature variations. But some theorists have speculated that the deviation may indicate inadequacies in the models of inflation, dark energy or the topology of the Universe.

**Sachs-Wolfe Effect.** At last scattering the baryons and photons decouple and the photons suddenly find themselves free to travel in straight paths through the Universe. However, the baryons are clustered together in gravitational potential wells prior to last scattering. Since the photons are tightly coupled to the baryons before last scattering, they are confined to potential wells too. Thus the photons have to climb out of potential wells when they are suddenly freed at last scattering. This climb requires some energy and the photons are therefore redshifted. The subsequent rise at low \( l \) in the CMB power spectrum is known as the Sachs-Wolfe (SW) effect, and since it is imprinted on the CMB power spectrum at the time of last scattering, it is considered a primary anisotropy.

This effect is the predominant source of fluctuations in the CMB for angular scales above about ten degrees — the regions in the early Universe which were too big to undergo acoustic oscillations.

**Integrated Sachs-Wolfe Effect.** The Integrated Sachs-Wolfe (ISW) effect is also caused by gravitational redshift, however here it occurs between the surface of last scattering and the Earth, so it is not a fundamental part of the CMB.

The ISW effect can arise after last scattering as the photons free stream through the Universe. Although the photons are no longer tightly coupled to the baryons, they can still slip into potential wells and have to climb back out. When they fall in, the photons gain some energy (are blueshifted) and when they climb back out, they are redshifted. Assuming that the depth of the potential well remains constant while the photon traverses it, the redshift exactly cancels the blueshift. No trace of the photon’s passage through the potential well remains, assuming that both sides of the dip are the same height and no energy is dissipated. Suppose, however, that the potential well through which the photon passes either decays or deepens while the photon is inside. Then its redshift and blueshift will not exactly cancel; instead the photon gains or loses some energy (respectively) from its passage through the potential well.

There are two main contributions to the integrated effect. The first occurs shortly after photons leave the last scattering surface, and is due to the evolution of the potential wells as the Universe changes from being dominated by radiation to being dominated by matter. The second, sometimes called the ‘late-time integrated Sachs-Wolfe effect’, arises much later as the evolution starts to feel the effect of the cosmological constant (or, more generally, dark energy), or curvature of the Universe if it is not flat. The latter effect has an observational signature in the amplitude of the large scale perturbations of the CMB and their correlation with the large scale structure.

The primary anisotropies (SW) on the CMB power spectrum tell us about the initial conditions of the photons, and any passage through a potential well that results in a net energy loss or gain changes these conditions and leaves a mark on the spectrum — the secondary anisotropy (ISW).

**Determining the Cosmic Parameters from CMB Radiation**

**Baryonic matter content** \((\Omega_b)\). Relative magnitudes of the first overtone to the fundamental peak in the power spectrum of the CMB radiation enables precise quantification of relative strengths of gravity and radiation in the early Universe. It has been determined that the energy in baryons was about the same as the energy in CMB photons at the time of decoupling, which — through scaling which we have done in previous classes (recall \( \rho_r \propto a^4 \)) — puts the baryonic content of the Universe at about 5 percent. This is in excellent agreement with the predictions of the BBN.
Dark energy ($\Omega_\Lambda$). Because dark energy accelerates the expansion of the Universe, it weakens the gravitational-potential wells associated with galaxy clustering (ISW effect). These effects can be detected and quantified at the large-scale variations of the CMB radiation (low $l$ values).

Hubble rate ($H_0$). SZ effect is used to measure the present-day value of the Hubble rate ($H_0$).