Collective Modes and Colored Noise as Beam-Halo Amplifiers

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Abstract. As illustrated herein, collective modes and colored noise conspire to produce beam halo with much larger amplitude than could be generated by either phenomenon separately. Collective modes are inherent to nonequilibrium beams with space charge. Colored noise arises from unavoidable machine transitions and/or errors that influence the internal space-charge force. Lowest-order radial eigenmodes calculated self-consistently for a direct-current, cylindrically symmetric, warm-fluid Kapchinskij-Vladimirskij equilibrium serve to model the collective modes. Even with weak space charge, small-amplitude collective modes, and weak noise strength, a pronounced halo is seen to develop if these phenomena act on the beam over a sufficiently long time, such as in a synchrotron or storage ring.

INTRODUCTION

A recent investigation revealed that collective modes and colored noise can synergistically conspire to produce large halo in beams wherein self-interaction, e.g., space charge, is important [1]. The investigation involved three distinctly different potentials: one mimicking a global collective mode in a cylindrical beam for which the envelope remains stationary (i.e., an "envelope-matched" beam), another mimicking a global collective mode in a spherical beam that is otherwise in thermal equilibrium, and the third doing likewise in a self-gravitating Plummer sphere (an example of interest in the context of galactic dynamics). In each case, copious halo developed. Colored noise, an unavoidable phenomenon arising from the presence of accelerator-hardware irregularities that lead to space-charge fluctuations as the beam self-consistently responds, keeps a small fraction of particles more in phase with the collective mode. Not only does the halo become much larger than that produced by either phenomenon (global modes or noise) acting alone, but also the synergism removes the popular notion of a hard upper bound to the halo extent [2].

Subsequently the original investigation was extended both to improve the inherent statistics and to explore large portions of the parameter space [3]. The new study concerned a direct-current, cylindrically symmetric beam modeled as a warm-fluid Kapchinskij-Vladimirskij (KV) equilibrium configuration possessing a self-consistent spectrum of collective, stable axisymmetric flute modes [4]. The associated time-dependent space-charge force combines with the external focusing force to determine the equation of motion of test particles. Quantifying the evolving halo entailed populating the full configuration space with very many (typically 10^6) test particles,

assigning each test particle its own random manifestation of colored noise, and then tracking all of their orbits. The resulting halo depends on the beam parameters (the space-charge tune depression), the collective-mode parameters (amplitudes and frequencies), and the noise parameters (strength and autocorrelation time).

The context of the work outlined above was linear accelerators of intense hadron beams as might drive heavy-ion fusion or a source of spallation neutrons. As such, the orbital-integration time was correspondingly short and the space charge was likewise strong. A natural question is: Can weak space charge, weak modes, and weak noise, by acting over a long time, lead to copious halo? The answer would be of central importance in the context of, e.g., synchrotrons and storage rings wherein the beam resides for relatively very long times. This paper presents a preliminary study, done in the context of the warm-fluid KV model, which suggests the answer is "yes". It commences with a synopsis, for the reader's convenience, of the underlying models of the beam and colored noise (details appear in Ref. [3]). It then presents the results and concludes by summarizing future plans as motivated by the results.

EQUATION OF TEST-PARTICLE MOTION

Following Ref. [5], we consider an intense, direct-current charged-particle beam propagating in the z-direction at constant speed through a transport channel that imposes a constant, cylindrically symmetric, linear transverse focusing force. The beam's equilibrium configuration is modeled as a warm-fluid Kapchinskij-Vladimirskij equilibrium, and collective modes are superposed on this equilibrium configuration. These modes correspond to stable, axisymmetric flute perturbations derived from linearizing the respective Vlasov-Maxwell-Poisson equations [4]. The dimensionless self-field perveance K incorporates the beam parameters:

$$K = \frac{2\rho q^2}{\beta^2 \gamma^3 m c^2}, \tag{1}$$

wherein ρ is the line density (number of particles per unit length), q and m are the particle charge and mass, respectively, β and γ are the usual relativistic factors, and c is the speed of light. The space-charge tune depression η is defined in terms of K as

$$\eta \equiv \left[1 - \left(\frac{\beta c}{\omega_f R_0} \right)^2 K \right]^{1/2}, \tag{2}$$

wherein R_0 is the radius of the equilibrium beam and ω_f is the angular frequency associated with the bare (linear) external focusing force. The frequency of the nth axisymmetric flute mode is

$$\omega_n(\eta^2) = \omega_f \sqrt{2[1 + \eta^2 (2n^2 - 1)]}. \tag{3}$$

For the analysis in this paper, only the n=1 mode is excited. The radial coordinate r is normalized in terms of the envelope radius R_0, and the time t is normalized in terms of the angular frequency ω_f , i.e., $t \rightarrow \omega_f t$; effectively this entails setting R_0=1 and ω_f =1.

The axisymmetric flute modes differ from breathing modes: the beam boundary is static ("envelope-matched," meaning R_0 is constant) in the case of flute modes, but oscillates in the case of breathing modes. In both cases the beam is root-mean-square (rms) mismatched. For the warm-fluid KV beam, the equilibrium density profile exhibits a step-function discontinuity at the boundary. In turn, the flute modes likewise involve a discontinuity in the density profile at the boundary. For example, consider the KV beam excited by the n=1 flute mode. The density profile inside the beam is always uniform, but its magnitude oscillates. To conserve particle number, this mode includes an oscillating surface charge, i.e., the density profile exhibits a Dirac delta function at the (stationary) envelope radius such that the integral over the beam cross section is independent of time. By contrast, the lowest-order breathing mode entails a self-similar oscillation; the envelope radius itself oscillates, and the number density likewise oscillates but remains everywhere uniform.

To explore the dynamics of halo formation, we compute the motion of test particles constrained to lie on radial orbits (the choice of orbital angular momentum has little impact on the overall halo dynamics [3]). By definition, test particles contribute nothing to the total potential and do not interact with each other. The equation of test-particle motion decomposes into two regimes, one internal to the beam for which the normalized radial coordinate $r < 1$, and the other external to the beam for which $r \geq 1$. With only the n=1 flute mode excited, it is [5]

$$\begin{aligned} &\ddot{r} + [\eta^2 - (1-\eta^2)\sqrt{\Gamma_1}\cos(\omega_1 t)]r = 0 \qquad \text{for} \quad r < 1; \\ &\ddot{r} + r - \frac{1-\eta^2}{r} = 0 \qquad\qquad\qquad\qquad\quad \text{for} \quad r \geq 1; \end{aligned} \tag{4}$$

in which Γ_1 is the ratio of the rms electrostatic energy contained in the n=1 collective mode to that contained in the equilibrium beam, and ω_1 is given by Eq. (3) upon setting ω_f =1 and n=1. Because Eq. (4) devolves from linear perturbation theory, Γ_1 must be small compared to unity for it to be valid.

Our goal is to assess the extent to which colored noise, in combination with the collective mode, influences the particle dynamics. Noise is inherent to accelerators because they are imperfect. Machine errors, irregularities, and transitions will feed space-charge fluctuations as the beam evolves self-consistently in response to these external influences. Examples include forces from image charges due to irregularities in the accelerator hardware as well as errors in radiofrequency and magnetic fields. In the lab frame the errors may themselves be time-independent, or they may fluctuate due to, e.g., jitter. From the perspective of a beam particle, the effect of all of these machine imperfections is to impart time-dependent noise on the particle orbit; thus, the next step is to include this noise in the equation of test-particle motion.

The noise is modeled as gaussian colored noise sampling an Ornstein-Uhlenbeck process, and it is quantified in terms of a frequency fluctuation $\delta\omega(t)$. The first two moments of $\delta\omega(t)$ determine the statistical properties of the noise:

$$\langle \delta\omega(t) \rangle = 0; \quad \langle \delta\omega(t)\delta\omega(t_1) \rangle = A^2 \exp(-|t-t_1|/t_c) \; ; \qquad (5)$$

in which t_c denotes the autocorrelation time, i.e., the time scale over which the noise signal changes appreciably. The special case of white noise corresponds to the limit $t_c \rightarrow 0$. After generating a colored-noise signal using, e.g., the technique developed in Ref. [6], the next step is to compute $|A| \leftrightarrow \langle |\delta\omega| \rangle$ which then constitutes the measure of noise strength. In a beam each particle will have its own distinct initial conditions and thus experience a manifestation of the noise differing from that seen by each of the other particles. Hence, while integrating an orbit, at each successive time step a randomly generated frequency fluctuation is computed in keeping with the statistical properties specified in Eq. (5). This frequency fluctuation leads to a fluctuation in the space-charge tune depression; calculated in a manner consistent with Eq. (3), it is:

$$\omega_1 \rightarrow \omega_1 + \delta\omega(t); \; \eta^2 \rightarrow \eta^2 + \delta\eta^2(t); \; \delta\eta^2(t) \equiv \omega_1 \delta\omega(t) = \sqrt{2(1+\eta^2)}\,\delta\omega(t). \; (6)$$

Where they occur in the equation of motion, Eq. (4), the quantity ω_1 is replaced by $\omega_1 + \delta\omega(t)$, and the quantity η^2 is replaced by $\eta^2 + \delta\eta^2(t)$, with $\delta\eta^2(t)$ given by the last expression in Eq. (6). The fluctuation thereby influences the propagation of the orbit to the next time step.

The methodology briefly summarized above is elaborated in Ref. [3]. This reference also provides a detailed study of how the various parameters influence the halo dynamics in linear accelerators of high-average-current ion beams. What follows presents results pertaining to long-lived beams such as those in circular machines.

HALO PRODUCTION IN LONG-LIVED BEAMS

An example of a "long-lived" beam would be one that transits the Fermilab Booster synchrotron. This machine accelerates long proton bunches from 0.4 GeV at injection to 8 GeV at output. Accordingly the beam becomes more relativistic and space charge becomes correspondingly weaker; a representative choice for the space-charge tune depression is η=0.95. We thus choose this value for η and then consider modest mode excitations, specifically, Γ_1=0.01 and 0.05. A rough measure of percent rms mismatch associated with this n=1 mode is $50\Gamma_1^{1/2}$ [3], so the choices of Γ_1 correspond roughly to 5% and 10% rms mismatches, respectively. Test-particle orbits are then integrated over a 'long' time, a time corresponding to roughly 3,000 turns around the Booster, which is about 16% of the total time the beam resides in the Booster.

Although our model of the beam and transport channel is, respecting the Booster, irrelevant at worst and rudimentary at best, it nevertheless serves as a platform to explore whether modest collective modes and weak noise might, if acting over a long period of time in a beam with weak space charge, lead to substantial emittance degradation and halo formation. The autocorrelation time is chosen to be t_c=80; this corresponds to about eight orbital periods of a typical beam particle. Then, for each value of Γ_1, noise strengths $\langle |\delta\omega| \rangle$=0, 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1} are considered. These choices correspond to frequency fluctuations spanning from 0 to about 10% of the collective-mode frequency ω_1. The strength of the frequency fluctuation is *not*

quantitatively identical to the magnitude of machine errors; in a recent paper Gerigk [7] shows, for example, that a 1% rms focusing error translates into a few percent (i.e., significantly greater than 1%) rms fluctuation in the oscillation frequency.

For each set of input parameters, 1,000 test-particle orbits are then integrated. This is a relatively small sample (the studies in Ref. [3] typically incorporate 10^6 test particles), but is one that permits a quick survey of the parameter space. At time t=0, these particles are distributed throughout the interior of the beam in keeping with a uniform density profile over the range $0 \leq r < 1$; their initial velocities are all zero.

The results of this exploratory study appear in Fig. 1 below, wherein the halo amplitude R_H, defined to be the radius of the outermost particle at the specific time step under consideration, is plotted versus time. With zero noise, the collective mode periodically enlarges the halo, but only by a small amount. However, the addition of colored noise, even noise as weak as 0.01% of the oscillation frequency, results in a substantial growth of the halo. For $\langle|\delta\omega|\rangle$ =1-10% of ω_1, the halo is *strongly* enhanced, and the time scale for this enhancement is rapid. Using a larger number of test particles would yield a larger halo; a crude estimate based on inspection of Fig. 3 of Ref. [3] is that 10^6 test particles would, compared to the results shown in Fig. 1, yield a roughly 25% larger halo amplitude at later integration times.

Faced with these results, one might be concerned that, since they are unrealistic, density discontinuities inherent to collective modes in the warm-fluid KV beam could somehow lead to unrealistic dynamics, thereby vitiating the findings. This seems not to be the case because, for example, adding noise to a perturbed thermal-equilibrium beam, one completely devoid of discontinuities and wherein the perturbation mimics the presence of a global collective mode, likewise yields enhanced halo [1].

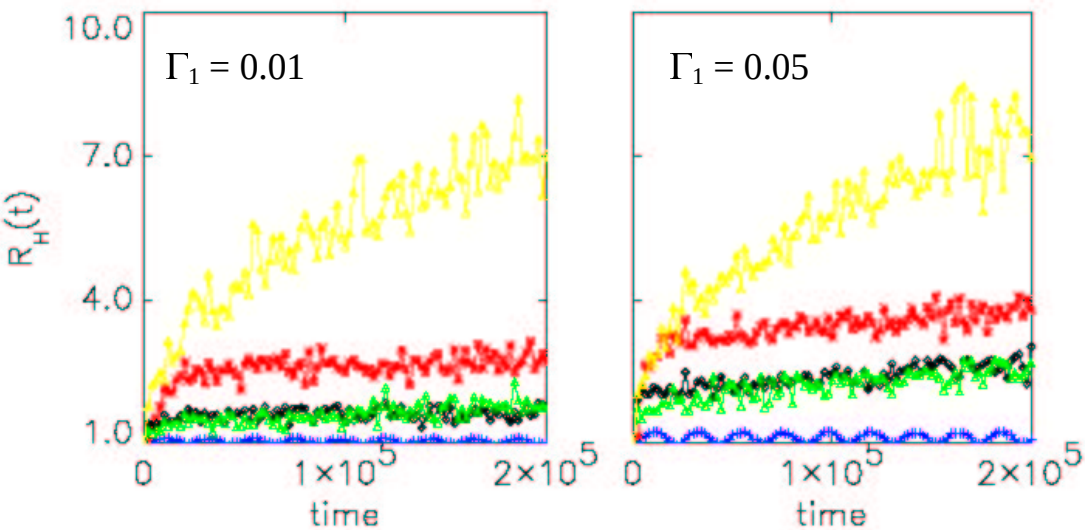

FIGURE 1. Evolution of halo amplitude [$R_H(t)$ vs. time t] for exploratory long-run-time integrations involving 1,000 test particles with Γ_1=0.01 (left) and 0.05 (right). The run time (2×10^5 differential-equation time units) corresponds roughly to 3,000 transits of Fermilab's Booster synchrotron. Progressing from bottom to top in each panel, the curves correspond to noise strengths $\langle|\delta\omega|\rangle$ = 0, 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1}.

IMPLICATIONS AND FUTURE WORK

Despite the limitations inherent to our idealized model, the qualitative features of Fig. 1 may provide new insight into emittance dilution observed in Fermilab's Booster synchrotron. Specifically, the beam emittance is observed to grow continually after injection. Synergism between noise from machine errors (such as hardware alignment errors and jitter in the magnet power supplies) and collective modes arising from injecting an imperfectly matched beam stands as a candidate explanation. More work is required before one can conclude that this actually *is* the explanation. Thus, as they presently stand, the findings herein have motivated further study in collaboration with Fermilab's Booster Space Charge Study Group. Fermilab staff are now in the process of including realistic values for hardware alignment errors and jitter in the ORBIT code [8] used to simulate the Booster, and will then use this code to compute how these errors influence a beam injected with an imperfect match as represented by, e.g., an asymmetric transverse emittance [9]. In parallel, we will be doing production runs with our model involving much larger numbers of test particles in an effort to aid in interpreting the results of the ORBIT simulations.

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