# Science in the News


Saturn as seen by
the Cassini Spacecraft

# Status: Unit 4 - Circular Motion and Gravity

- ✓ Uniform circular motion, angular variables, and the equations of motion for angular motion (3-9, 10-1, 10-2)
- ✓ Applications of Newton's Laws to circular motion (5-2, 5-3, 5-4)
- ✓ Harmonic Motion (14-1, 14-2)
- ✓ The Universal Law of Gravitation, Satellites (6-1, 6-2, 6-3, 6-4)
- Kepler's Law's, Fields and Force (6-4,6-5)
- Fields, Forces and What Not (6-6, 6-7, 6-8, 6-9)

# Quiz Number Two

- Wednesday, February 7$^{th}$.
- Same format as first quiz.
- Total of 50 points
  - 4 multiple choice @ 5 pts each
  - 3 problems @ 5, 10, 15 points.
- Bring a calculator.

**Newton's 1st Law of Motion**: Every body continues in its state of rest or of uniform speed in a straight line as long as no net force acts on it.

**Newton's 2nd Law of Motion**: The acceleration of an object is directly proportional to the net force on it and inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.

$$\boxed{\sum \vec{F} = m\vec{a}}$$

**Newton's 3nd Law of Motion**: Whenever one object exerts a force on a second object, the second exerts an equal and opposite force on the first.

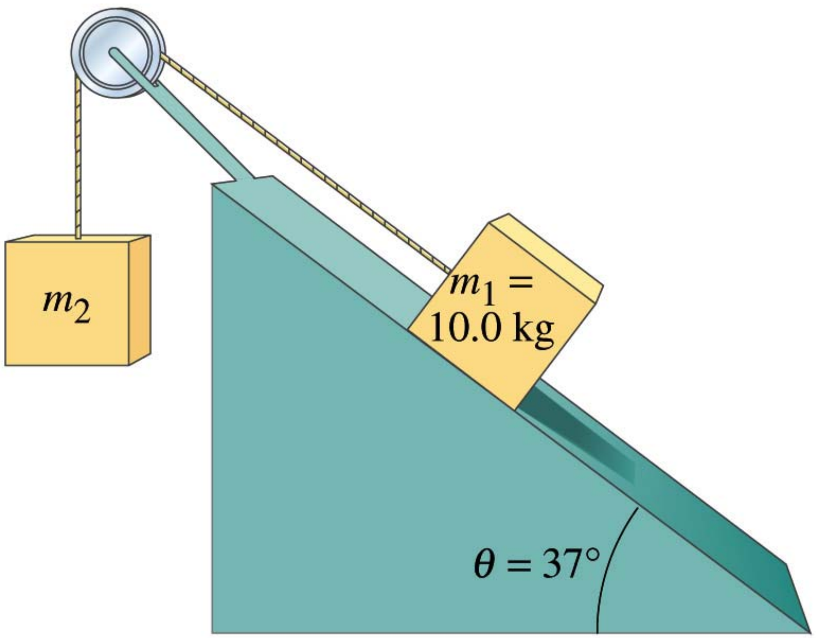
# You'll Be Asked to Solve Two Problems Based on Newton's Laws

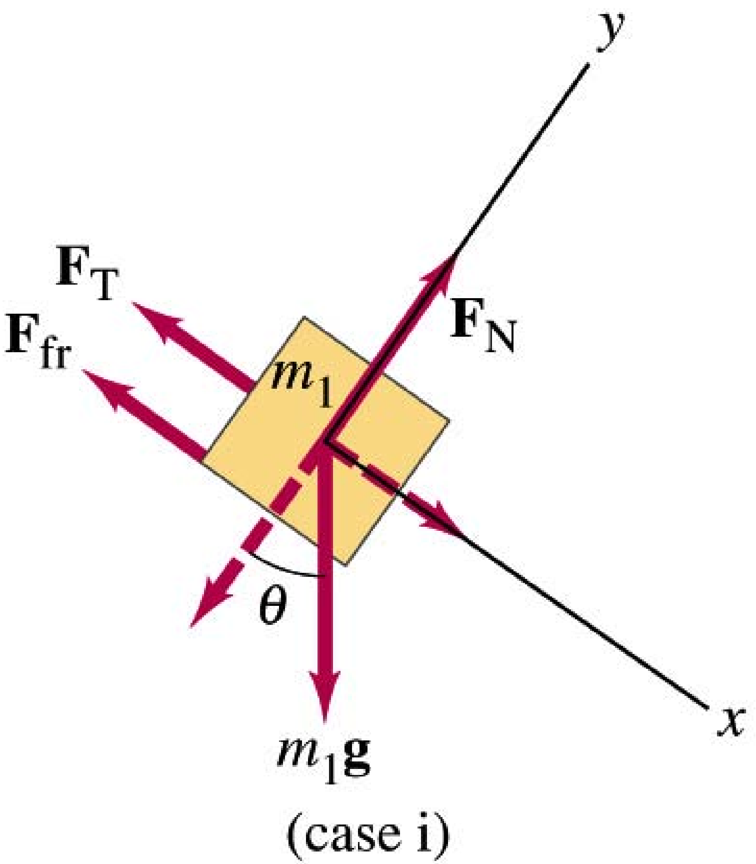
1. Read Carefully
2. Draw the Free-Body Diagram for each object.
3. Choose convenient xy coordinate system. Resolve vectors into components. Apply 2nd Law to each direction independently
4. List knowns and unknowns and choose equations relating them.
5. Solve approximately or at least qualitatively
6. Solve algebraically and numerically.
   1. You need one equation for each unknown.
   2. Retain algebraic formulation until the very end. This increases insight.
7. Check units
8. Check if reasonable, the "smell" test.

# You'll Be Asked for a Free Body Diagram

- A key element in understanding motion
- A free-body diagram has:
  1. A convenient coordinate system,
  2. Representative vectors
     - I. For <u>all</u> forces acting <u>on</u> a body
     - II. Including those that are unknown.
     - III. If translational only, at the center of the body.
  3. Descriptive labels for each force vector.
- Doesn't show forces the body exerts on other objects.

# Our Most Complicated Free-Body Diagram



# Derivation and Applications of the Normal Force and the Forces of Friction

- Know how to calculate the normal force of an object on a surface- it's usually present to counteract the weight of an object.
- Know the relationship between the normal force and static and kinetic friction
- You'll have two problems which require calculation of the normal force and associated frictional forces.

$$F_{fr} = \mu_k F_N$$

$$F_{fr}(\max) = \mu_s F_N$$

# Understand and Know How to Use Angular Variables

$$v = \omega R$$

$$a_R = \frac{v^2}{R} = \omega^2 R$$

$$a_{\tan} = R\alpha$$

$$\omega = 2\pi f$$

$$T = \frac{1}{f}$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\overline{\omega} = \frac{\omega_0 + \omega}{2}$$

**You’re as likely to need the linear eqs. of motion as well as the angular ones!**

# Be Familiar with the Universal Law of Gravitational, its Applications, and Artificial Gravity

$$F = \frac{Gm_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2} = G\frac{m_E m}{r_E^{\,2}}$$

$$mg = G\frac{m_E m}{r_E^{\,2}} \rightarrow$$

$$g = G\frac{m_E}{r_E^{\,2}}$$

$$F = \frac{Gmm_E}{r^2} = m\frac{v^2}{r}$$

Note the mass of the satellite cancels:

$\frac{Gm_E}{r^2} = \frac{v^2}{r}$ as does one power of r:

$\frac{Gm_E}{r} = v^2$ solving for v we see:

$$v = \sqrt{\frac{Gm_E}{r}}$$

# Simple Harmonic Motion

- Understand the derivation of the equations of motion.
- Know the characteristics of the equations describing position, velocity, and acceleration.

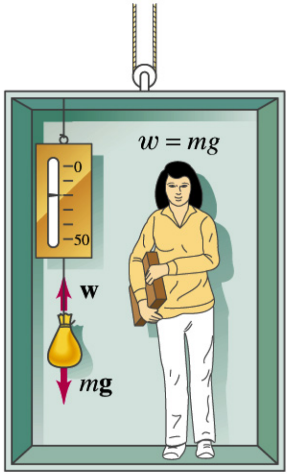
$$x(t) = A\cos(\omega t + \phi)$$
$$x_{\max} = A$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$
$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A$$

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$
$$a_{\max} = \omega^2 A = \frac{k}{m}A$$

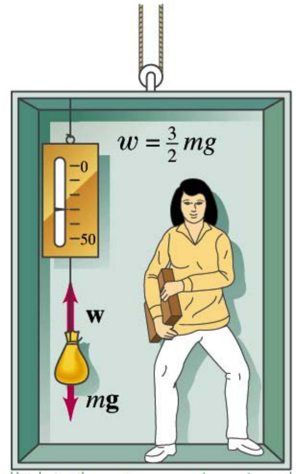
# Apparent Weightlessness

- We can get a good handle on the ideas behind weightlessness by considering what happens to our apparent weight in an elevator going up or down.
- The limiting cases really tell it all!
  - If the elevator was going up with large acceleration
    - We would press hard against the floor
    - If we were standing on a scale, our effective weight would increase.
  - If the elevator were falling with an acceleration of g
    - We and everything in the elevator would be floating
    - The scale would read zero.
  - So our effective weight would go up when the elevator was accelerating up and would go down when the elevator was accelerating down.
- To be quantitative requires free body analysis.



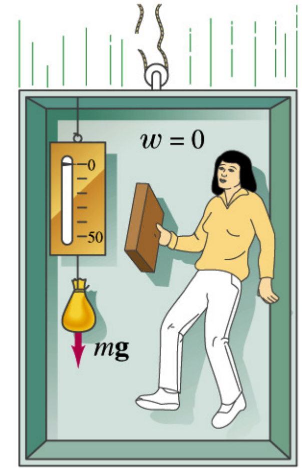

$\boldsymbol{a} = 0$

(a)




$\boldsymbol{a} = -\frac{1}{2}\,\mathbf{g}$(up)

(b)




$\boldsymbol{a} = \mathbf{g}$(down)

(c)

- Elevator at rest, a=0
- For the bag:
  - $\Sigma F = w - mg = 0 \rightarrow$
  - $w = mg$
- From the 3rd Law the scale reads w.
- Nothing new.

- Elevator with an upward acceleration of g/2
  - $\Sigma F = w - mg = mg/2$
  - $w = 3mg/2$
- That is, an increased effective weight
- The women's weight has increased and so would the weight of an astronaut propelled upward.

- Elevator with a downward acceleration of g
  - $\Sigma F = w - mg = -mg \rightarrow$
  - $w = 0$
- The scale would read zero.
- It's the same with a satellite and the objects inside, they all are in free fall accelerating inward at $v^2/r$.

- More generally
  - $\Sigma F = w - mg = ma$
  - $w = mg + ma = m(g+a)$
- If the direction is up, the right side will increase and the effective weight will be larger.
- If the direction is down, the left side will decrease and the weight will be smaller.

# Back to Orbital Motion

- So as you can see an astronaut is not strictly "weightless" a better phrase is "apparent weightlessness‟.
- If you were standing on a scale in a freely falling elevator you’d register no weight at all.
- If you were to stand on a scale in an orbiting satellite, you’d also register no weight because both you and the satellite are falling toward earth with the same acceleration. The scale won’t push back.
- The only difference between the satellite and the elevator is that the satellite moves in a circle. The acceleration associated with "falling" just "turns" the “elevators” velocity vector into the circular orbit. In both cases although the apparent weight is zero, the true weight is given by the universal law of gravitation.
- The point here is that there is apparent weightlessness relative to the elevator or satellite.
- On the other hand the true weight is nonzero, small but non-zero.

# Effects of Micro-gravity or near-Weightlessness

- There have been many interesting articles on the effects of prolonged apparent weightlessness.
- A past issue of “Scientific American” gives a really interesting summary. These have to do with perception and physiology.
- Here’s and interesting current www site: http://www.spacefuture.com/archive/artificial_gravity_and_the_architecture_of_orbital_habitats.shtml
- Oddly many people become congested and get "fat heads" as the blood migrates from the lower extremities to an even distribution throughout the body.
- Another interesting effect has to do with frames of reference: If an astronaut pushes off a wall it seems that the wall retreats rather than the person!
- Perhaps bone and muscle loss are the most serious long-term issues and may in the end by the real reason for the provision of artificial gravity.

**Large Scale Solution**

# The Principle Behind Artificial Gravity

- As you've seen many times in the movies, artificial gravity in a satellite can be provided by spinning the vessel. Although the movies often mess up the physics of space, they at least get this right! (For, instance weapons wouldn't go "zap" in an airless environment).
- We can ask: At what speed must the surface of a space station of radius 1700 m move so that astronauts on the inner surface experience a push that equals earth weight?

Remember that the centripetal force goes as $\frac{mv^2}{r}$ which we would like to equal the force of gravity on earth. That is, $mg = \frac{mv^2}{r}$. Solving for velocity we see that the mass cancels and : $v = \sqrt{rg} = \sqrt{1700m \times 9.8m/s^2} = 130m/s$ or 300mph!

- Note that the foot and head actually feel different forces since these are at different radii. Something to be considered when considering really fast rotations!
- Space stations could provide different "gravity" at different radii.

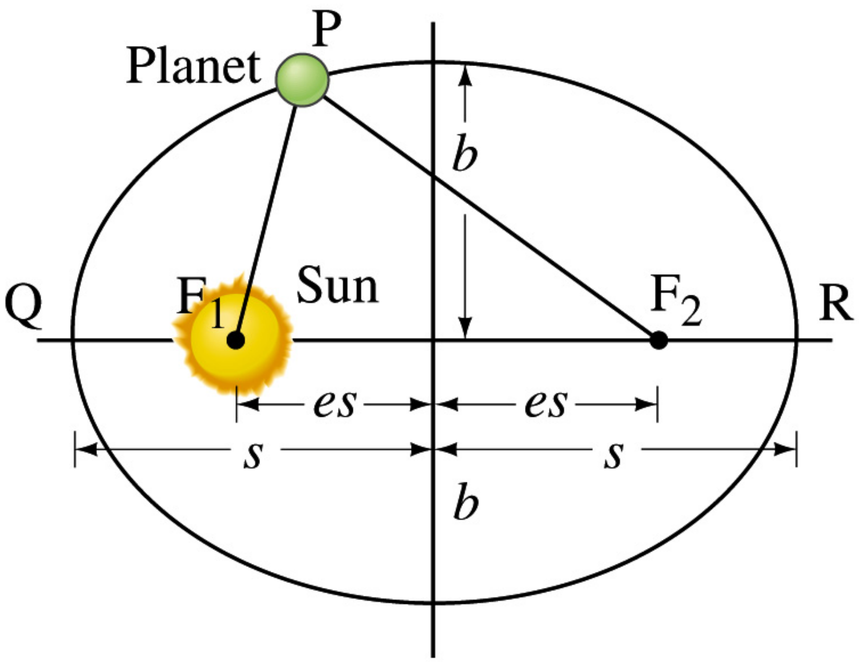
**Small Scale Solution**

# Kepler's Laws of Planetary Motion

- Prior to the publication of the three laws of motion there were extensive astronomical data available on the motion of astronomical objects.
- The German Astronomer Johannes Kepler(1571-1630) using data collected by Tycho Brahe(1546-1601) had thoroughly described the motion of the heavenly bodies about the Sun.
- He had also formulated a set of observational laws.

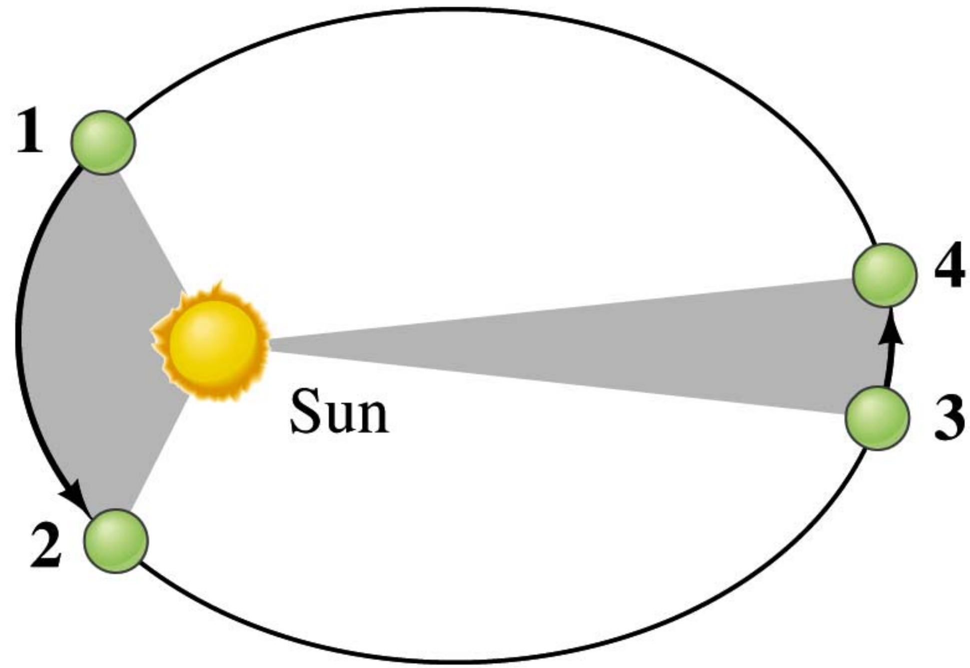
# Kepler's First Law

- The path of each planet about the Sun is an ellipse with Sun at one focus.
- Ellipse:
  - $C= F_1P + PF_2$
  - Semimajor axis =s
  - Semiminor axis =b
  - Eccentricity related to distance from foci to center.



# Kepler's Second Law

- Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.
- If an orbit is elliptical this means that planets move fastest when they are closest to the sun.



# Kepler's Third Law

- The ratio of the squares of the period of two planets revolving around the Sun is equal to the ratio of the cubes of their semi-major axis.
- The semi-major axis is the average of the closest and furthest distance from the Sun.
- For a circular orbit the semi-major axis is just the radius.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3 \rightarrow \frac{s_1^3}{T_1^2} = \frac{s_2^3}{T_2^2}$$

- This happens to be experimentally verified for the eight plants:
  - Mercury 3.34
  - Venus 3.35
  - Earth 3.35
  - Mars 3.35
  - Jupiter 3.34
  - Saturn 3.34
  - Uranus 3.35
  - Neptune 3.34

- Newton derived Kepler's Laws using the Laws of Motion and the Law of Universal Gravitation.
- This must have had a profound effect and certainly helped cement the correctness of Newton's Principia.
- Newton even went further and showed that the only an inverse square law described the observational data.
- With the tools at hand and the simple and fairly accurate assumption that the orbits of the planet are circular we can derive Kepler's 3rd Law ourselves!

We began with Newton's 2nd Law

$$\Sigma F = ma$$

and consider a mass subject to the gravitation of the Sun and in uniform circular motion

$$G\frac{m_1 M_S}{r_1^2} = m_1\frac{v_1^2}{r_1},$$

where $m_1, v_1, r_1$ are the mass, velocity, and radius of the planet and $M_S$ is the mass of the Sun. As we have done in the past we can relate the velocity of the planet to the radius and period of orbit: $v_1 = 2\pi r_1 / T_1$.

Substituting we find:

$$G\frac{m_1 M_S}{r_1^2} = m_1\frac{(2\pi r_1 / T_1)^2}{r_1} = m_1\frac{4\pi^2 r_1}{T_1^2}$$

$$G\frac{m_1 M_S}{r_1^2} = m_1 \frac{4\pi^2 r_1}{T_1^2}$$

Notice the mass cancels!

$$G\frac{M_S}{r_1^2} = \frac{4\pi^2 r_1}{T_1^2},$$ dividing through by $r_1$,

$$G\frac{M_S}{r_1^3} = \frac{4\pi^2}{T_1^2},$$ multiplying through by $T_1^2$,

$$GM_S\frac{T_1^2}{r_1^3} = 4\pi^2,$$ and dividing through by $GM_S$,

$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM_S},$$ on the right we have variables of the orbit and on the left a constant. The analysis would be identical for any planet and always result in the ratio of the period sqaured to the radius cubed equal to a constant. So for any planet 1 and planet 2:

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

- The second to last equation gives us another method to measure the mass of distance planets, by looking at the orbital characteristics of their moons!
- As measurements improved departures from predicted orbits were observed.
- But Newton expected these due to the influence of the planets on one another.
- These perturbations were required for the Law of Gravitation to be universal
- They also led to the discovery of the outermost planets.
  - Neptune perturbed the orbit of Uranus
  - Likewise Pluto that of Neptune

# Applications of Kepler's 3rd Law

- Given that Mars required 687 days to orbit the Sun how many earth-sun distances is Mars from the Sun?

We start with $\frac{T_E^2}{r_{ES}^3} = \frac{T_M^2}{r_{MS}^3}$

and solve for the ratio of radi:

$$\frac{r_{MS}^3}{r_{ES}^3} = \frac{T_M^2}{T_E^2} \rightarrow \frac{r_{MS}}{r_{ES}} = \left(\frac{T_M}{T_E}\right)^{2/3} =$$

$\left(\frac{687d}{365d}\right)^{2/3} = 1.52.$ That is, Mars is about 52% farther from the sun or $2.28\times10^{11}m$.

- Given that earth is 1.5x10m from the Sun was is the mass of the Sun?

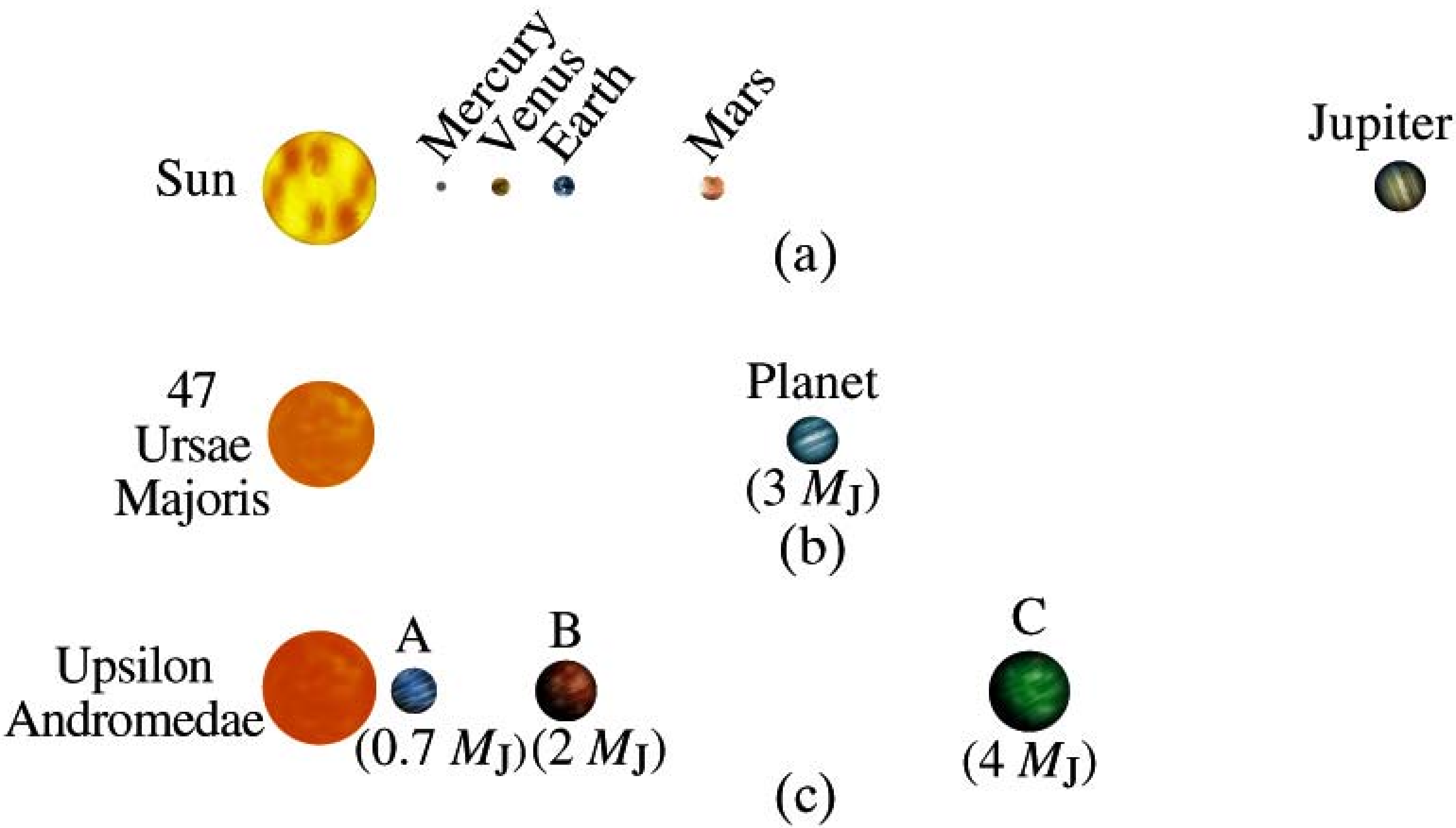
We start with $\frac{T_E^2}{r_E^3} = \frac{4\pi^2}{GM_S}$

and solve for the solar mass:

$$M_S = \frac{4\pi^2 r_E^3}{GT_E^2} =$$

$$\frac{4\pi^2(1.5\times10^{11}m)^3}{(6.67\times10^{-11}Nm^2/kg^2)(3.16\times10^7 s)^2}$$

$$= 2.0\times10^{30}kg$$



Sun
Mercury
Venus
Earth
Mars
Jupiter
(a)
47
Ursae
Majoris
Planet
(3 $M_J$)
(b)
Upsilon
Andromedae
A
B
C
(0.7 $M_J$)
(2 $M_J$)
(4 $M_J$)
(c)

# Halley's Comet

- Halley's comet orbits the earth every 76 years in an elliptical orbit and nearly grazes the surface of the sun on it's closest approach. Estimate it's farthest distance from the sun.
- We can use Kepler' s 3rd Law and compare the earth to the comet.

$$\left(\frac{T_H}{T_E}\right)^2 = \left(\frac{s_H}{s_E}\right)^3 \to s_H = s_E\left(\frac{T_H}{T_E}\right)^{2/3} =$$

$$1.5\times10^{11}m\left(\frac{76yr}{1yr}\right)^{2/3} = 2.7\times10^{11}m$$

$5.4\times10^{12}m$ which is near Pluto