Science News for St. Valentines Day: A 6000 Year Old Neolithic Embrace

http://www.cnn.com/2007/TECH/science/02/07/prehistoric.love.ap/index.html

Review of Test

Answers Test 1: Multiple Choice

- Problem 1(5 points): What is the percent uncertainty in the measurement 3.26 +/- 0.25m?
- Answer:
  - Circle D
  - 0.25m/3.26M x 100% = 7.7%

- Problem 2 (5 points): Suppose an astronaut has a vertical leap of 0.5m on Earth what would be his vertical leap on Pluto?
- Answer: Circle D

$$v^2 = v_0{}^2 + 2a(y - y_0)$$

at the top of the trajectory

$$0 = v_0{}^2 - 2gy$$

$$y = v_0{}^2 / g$$

so the ratio of leaps:

$$\frac{y_M}{y_E} = \frac{v_0{}^2 / g_M}{v_0{}^2 / g_E} = \frac{g_E}{g_M} \rightarrow$$

$$y_M = \frac{g_E}{g_M}(y_E) = \frac{9.8m/s^2}{0.8m/s^2}(0.5m)$$

$$y_M = 6.1m$$

- Problem 3 (5 points): How many kilometers are there in a light year?
- Answer: Circle B

$$(3.00x10^{8}\frac{m}{s})(\frac{1km}{1000m})(\frac{365d}{yr})(\frac{24hr}{d})(\frac{3600s}{hr}) =$$

$$(3.00x10^{8})(\frac{1}{1000})(365)(24)(3600)x$$

$$(\frac{m}{s})(\frac{km}{m})(\frac{d}{yr})(\frac{hr}{d})(\frac{s}{hr}) =$$

$$9.46x10^{12}km/yr$$

So in one year

$$9.46x10^{12}km$$

- Problem 4 (5 points): A pendulum has mass m and length L, using dimensional analysis determine which of the following quantities has the right dimensions for the oscillation time of the pendulum.
- Answer: Circle D

We want a dimension of T. Ignore the numerical constants as they don't matter for dimensional analysis.

$$\text{a)}\sqrt{\frac{\mathrm{g}}{\mathrm{L}}} \rightarrow \sqrt{\frac{L/T^2}{L}} = \sqrt{\frac{1}{T^2}} = \frac{1}{T} \text{ doesn't give time}$$

$$\text{b)}\sqrt{\frac{\mathrm{mg}}{\mathrm{L}}} \rightarrow \sqrt{\frac{ML/T^2}{L}} = \sqrt{\frac{M}{T^2}} = \frac{\sqrt{M}}{T} \text{ nope}$$

$$\text{c)}\frac{\sqrt{\mathrm{Lg}}}{\mathrm{m}} \rightarrow \frac{\sqrt{L^2/T^2}}{M} = \frac{L/T}{M} \text{ also no}$$

$$\text{d)}\sqrt{\frac{L}{g}} \rightarrow \sqrt{\frac{L}{L/T^2}} = \sqrt{\frac{1}{1/T^2}} = \sqrt{T^2} = T$$

Test 1: Problems

- Problem 5 (5 points): A ball is dropped from a height, h, give the time of fall in terms of h & g.

Start with $y = y_0 + v_{y0}t - (1/2)gt^2$

Put the origin at the surface of the earth

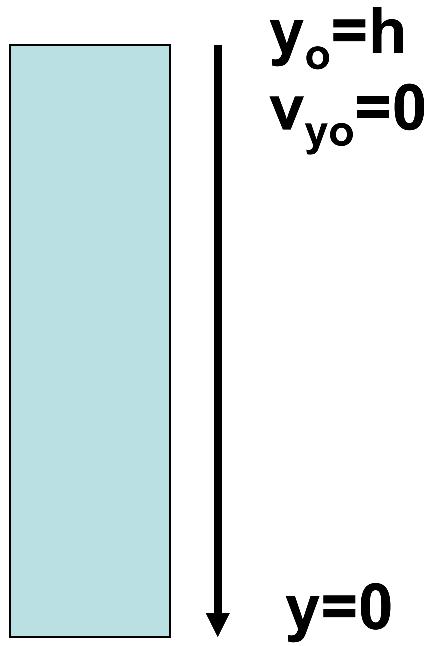
then $y = 0$ and $v_{yo} = 0$ so

$$0 = h + 0 - (1/2)gt^2 \rightarrow$$

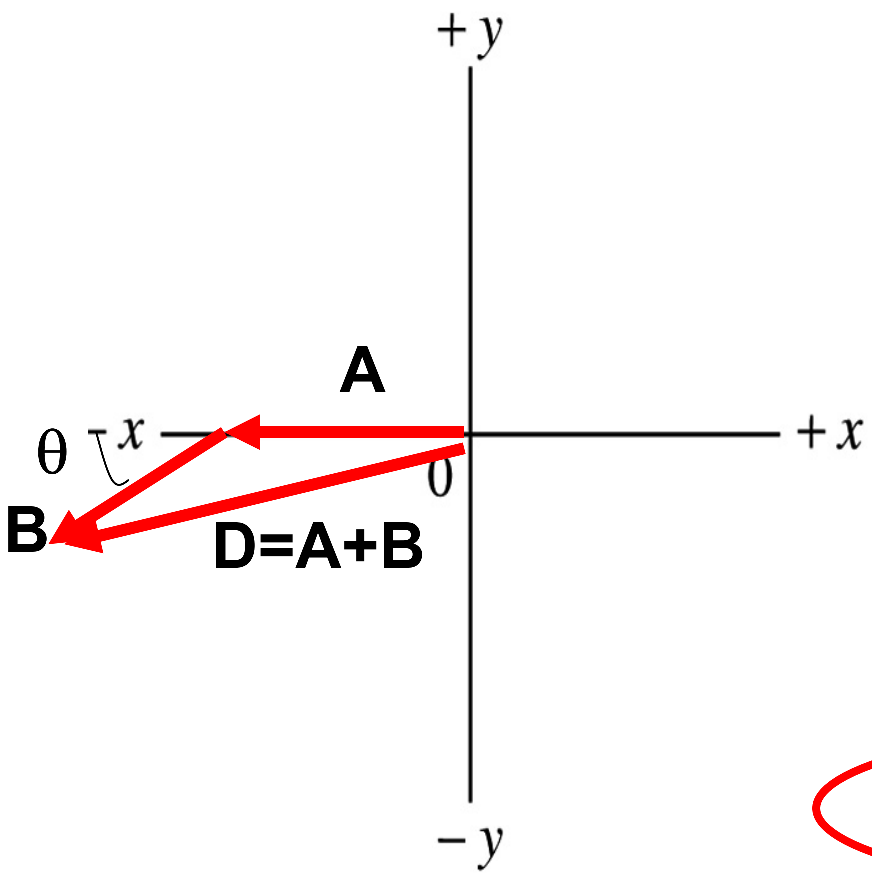
$$(1/2)gt^2 = h \rightarrow$$

$$t^2 = 2h/g \rightarrow$$

$$t = \sqrt{\frac{2h}{g}}$$



- Problem 6 (10 points): A grasshopper makes two jumps: The displacement vectors are (1) **A**=27.0cm due west and (2) **B**= 23.0 cm, 45^o south of west. Draw a picture of the movement and express the final displacement vector **D**=**A**+**B** in terms of the unit vectors **i**,**j**, and **k**.



A=-27.0cm**i** + 0**j**

Bx=-23.0cm(cos45^o)=-16.3cm

By=-23.0cm(sin45^o)=-16.3cm

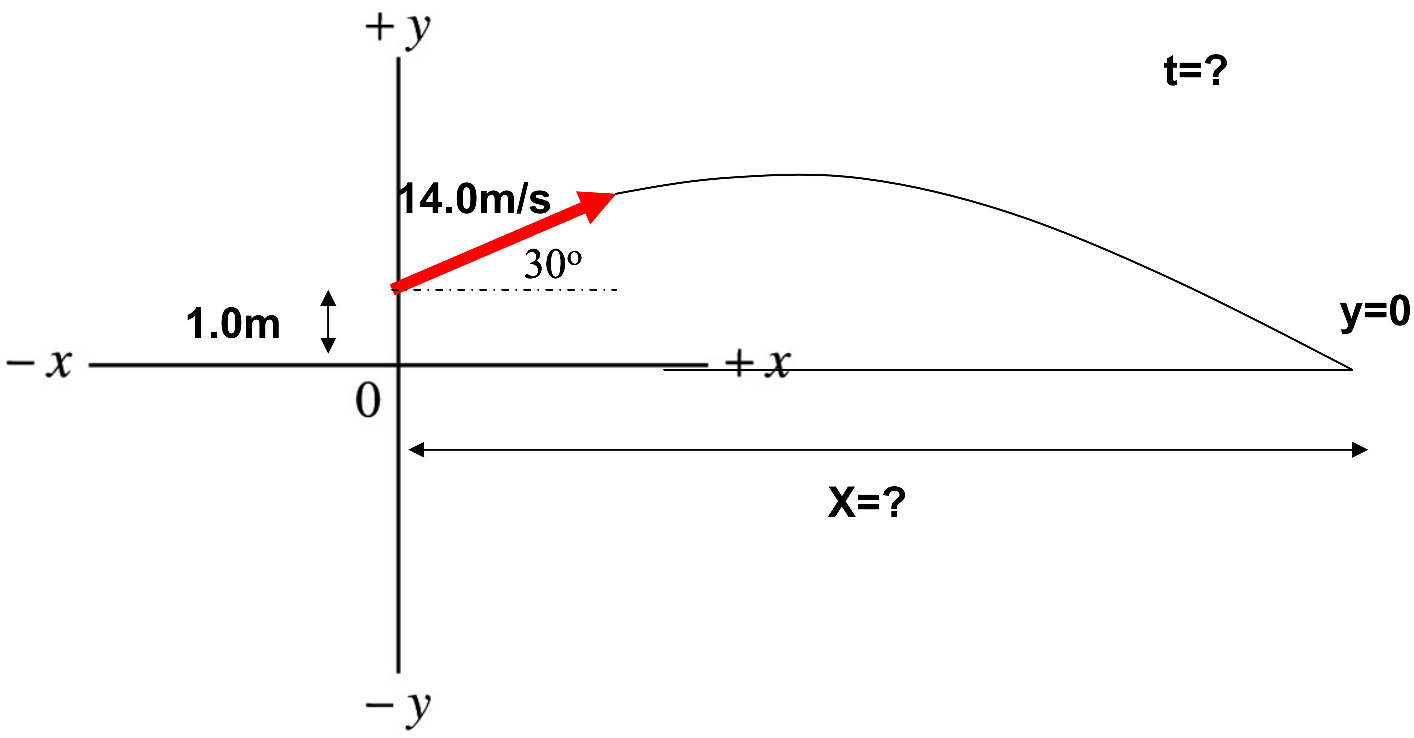
B=-16.3cm**i** – 16.3cm**j**

Dx= -43.3cm

Dy= -16.3cm

D=-43.3cm**i** – 16.3cm**j** + 0**k**

- Problem 7 (15 points): A shot-putter throws at an initial speed of 14.0m/s at a 30^o angle to the horizontal. The shot leaves the athlete's hand at a height of 1.0 m above the ground. Draw a picture of the situation and calculate a) the time of flight and b) the horizontal distance traveled.



Let's decompose the vector to get the initial velocities in x and y.

$$v_{xo} = \cos(30^o)(14.0m/s) = 12.1m/s$$

$$v_{yo} = \sin(30^o)(14.0m/s) = 7.0m/s$$

Use $y = y_0 + v_{y0}t - (1/2)gt^2$ to find the time

$$0 = 1.0m + (7.0m/s)t - (1/2)(9.8m/s^2)t^2$$

or

$$(4.9m/s^2)t^2 - (7.0m/s)t - 1.0m = 0$$

Applying the quadratic formula

$$t = \frac{7.0m/s \pm \sqrt{(-7.0m/s)^2 - 4(4.9m/s^2)(-1.0m)}}{2(4.9)}$$

$$t = \frac{7.0m/s \pm \sqrt{49.0m^2/s^2 + 19.6m^2/s^2}}{9.8m/s^2}$$

$$t = \frac{7.0m/s \pm \sqrt{68.6m^2/s^2}}{9.8m/s^2}$$

$$t = \frac{7.0m/s \pm 8.3m/s}{9.8m/s^2}$$

$$t = 15.3m/s / 9.8m/s^2$$

$$t = 1.6s$$

$$x = x_o + v_{xo}t$$

$$x = 0 + (12.1m/s)(1.6s)$$

$$x = 18.9m$$

Status: Unit 3

- ✓ Force, Mass, & Newton's 1st, 2nd, 3rd Laws of Motion (4-1 through 4-5)
- Weight – the Force of Gravity: and the Normal Force, Problem solving (4-6, 4-8, 4-7)
- Solving Problems w/ Newton's Laws, Problems w/ Friction (4-7, 5-1)
- Problems w/ Friction and Terminal Velocity (5-1, 5-5)

Newton's 1st Law of Motion: Every body continues in its state of rest or of uniform speed in a straight line as long as no net force acts on it.

Newton's 2nd Law of Motion: The acceleration of an object is directly proportional to the net force on it and inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.

$$\boxed{\sum \vec{F} = m\vec{a}}$$

Newton's 3nd Law of Motion: Whenever one object exerts a force on a second object, the second exerts an equal and opposite force on the first.

Weight - The Force of Gravity

- We now connect the acceleration of gravity to the force of gravity simply by using Newton's 2nd Law:

$$\sum \vec{F} = m\vec{a} \rightarrow$$

$$\vec{F}_G = m\vec{g}$$

- The magnitude of the force of gravity is usually called the weight of an object.
- Note how weight is a force, NOT mass!
- If we assume up is +y:

$$\vec{F}_G = -(mg)\vec{j}$$

The Normal Force: A Key element of Mechanics

- When an object is falling the consequences of the force of gravity are easy to see. But what about when an object is at rest, say a book on a table?
- The force hasn't disappeared. Just stand on a scale, there's a force present!
- So why doesn't the object accelerate? Well the key is the 2nd Law, if the acceleration is zero then the net force must be zero.

$$\boxed{\sum \vec{F} = m\vec{a}}$$

- There must be another force completely canceling the force of gravity or

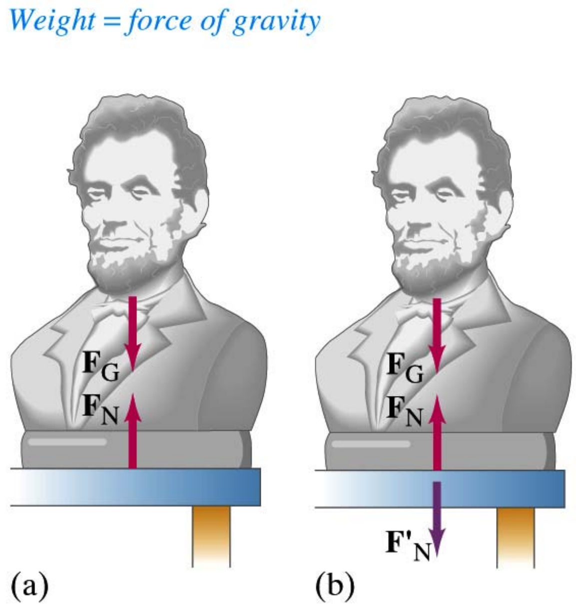
$$\boxed{\vec{F} = +(mg)\vec{j}}$$

- It's from the surface upon which an object rests. In the case of the book, it's the table.
- Note the properties of this other force:
  - Magnitude = Weight
  - Direction = +y

The Normal Force

- The force exerted by the surface is called the contact force
- Since it's usually perpendicular to the surface of contact it's also called the normal force and so labeled.
- As shown in the drawing, these are not the opposite and equal forces of the 3rd Law.

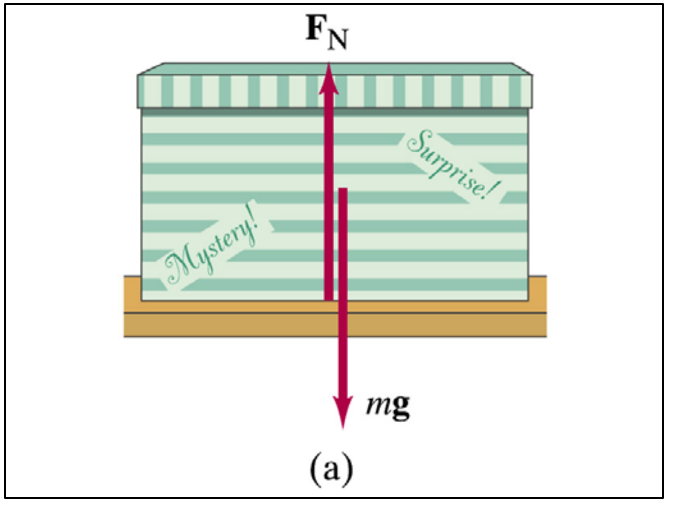
$$\boxed{\vec{F}_N}$$



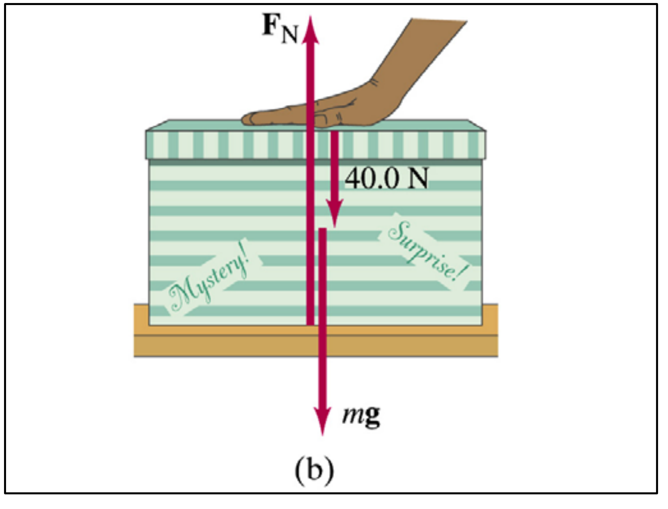
- Plus the bust imposes an equal and opposite force on the earth!

Example 1: Weight & Normal Force

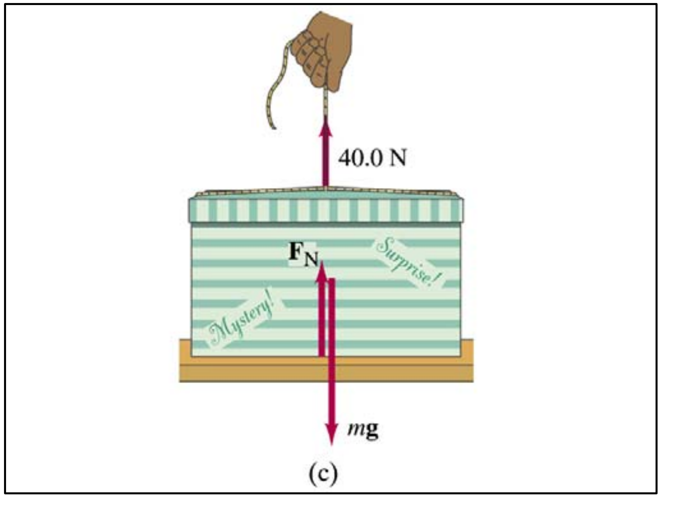
- For the pictured 10.0 kg box:
  - What are the weight and normal force?
  - If someone pushes down with 40.0 N force, what is the normal force?
  - If someone pulls up with 40.0 N force, what is the normal force?
  - If someone pulls up with 100.0N?



(a)



(b)

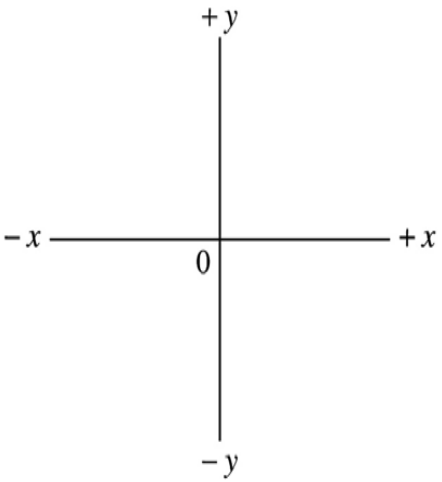


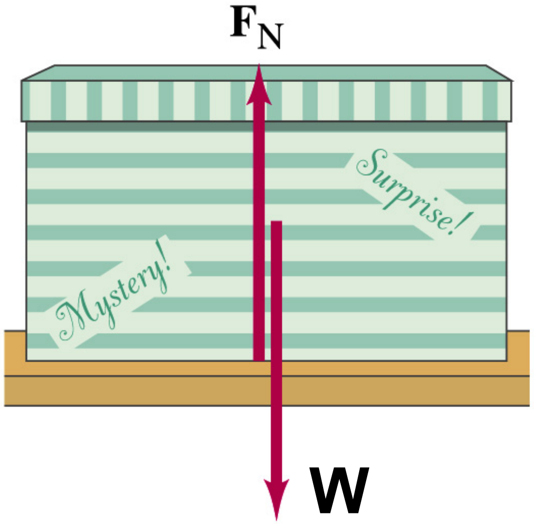
(c)

- Weight and Normal Force?
- Assign +y up.
- Two forces:

$$\vec{W} = -mg\vec{j} =$$
$$-(10.0kg)(9.80m/s^2)\vec{j}$$
$$= -98.0N\vec{j}$$

Since at rest:
$$\Sigma F_y = F_N + W = 0 \rightarrow$$
$$F_N = -W$$
$$\vec{F}_N = +98.0N\vec{j}$$





(a)

- Pushing down with 40.0N?
- Assign +y up.
- Three forces:

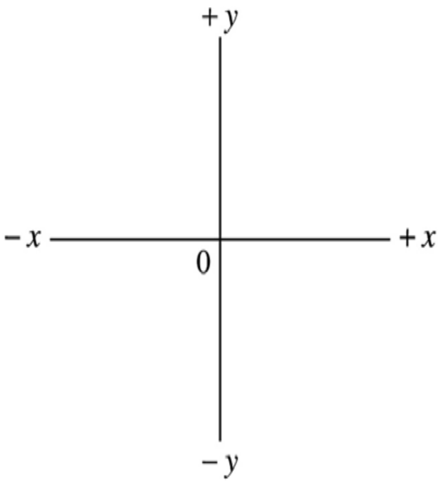
Since at rest :

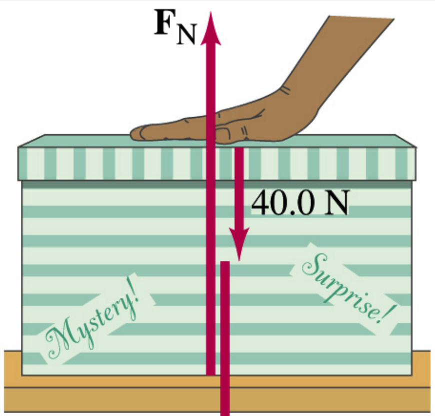
$$\Sigma F_y = F_N + W - 40.0N = 0 \rightarrow$$

$$F_N = -W + 40.0N$$

$$F_N = +98.0N + 40.0N = 138.0N$$

$$\vec{F}_N = 138.0N\vec{j}$$

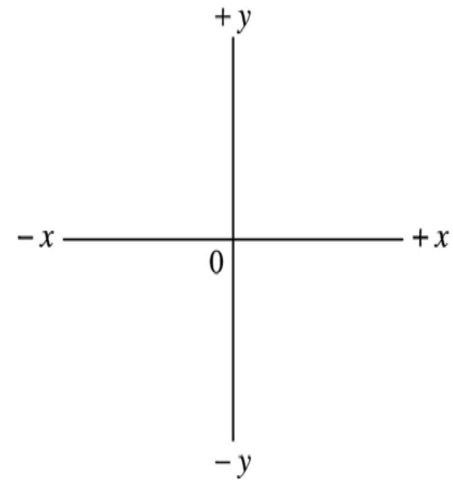




The table is "pushing" harder

(b)

- Pulling up with 40.0N?
- Assign +y up.
- Three forces:



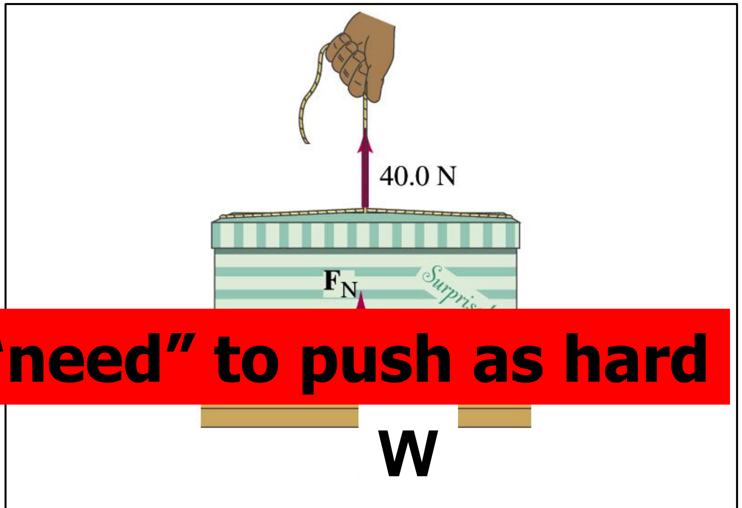
Since STILL at rest:

$$\Sigma F_y = F_N + W + 40.0N = 0 \rightarrow$$

$$F_N = -W - 40.0N$$

$$F_N = +98.0N - 40.0N = 58.0N$$

$$\vec{F}_N = 58.0N\vec{j}$$



(c)

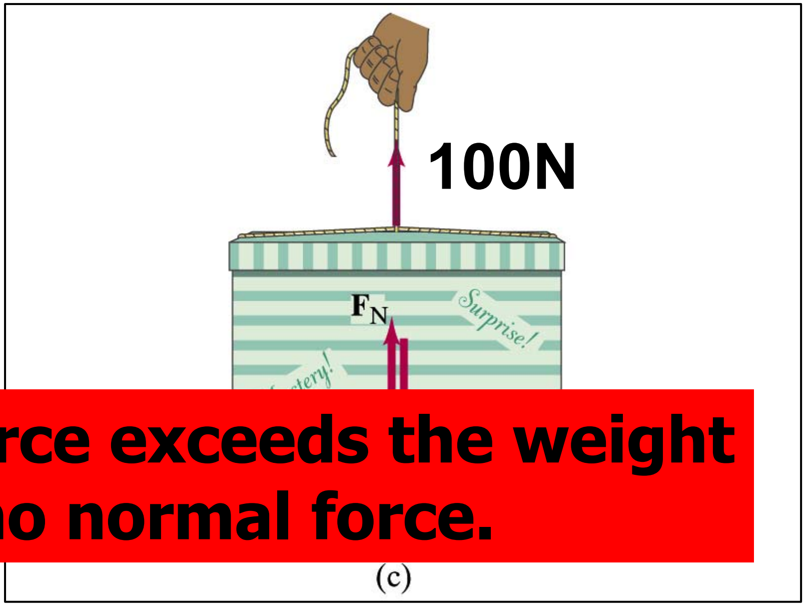
The table doesn't "need" to push as hard

- Pulling up with 100.0N?
- Assign +y up.
- Three forces:

$$\Sigma F_y = F_N + W + 100.0N$$
$$\Sigma F_y = F_N - 98.0N + 100.0N$$
$$\Sigma F_y = F_N + 2.0N$$

- But if this is set to zero the normal force is -2N which is unphysical! The table can't "pull" the b force must

$$\Sigma F_y = W + 100.0N$$
$$\Sigma F_y = -98N + 100.0N$$
$$\Sigma F_y = 2.0N$$
$$a_y = 2.0N / 10.0kg = 0.20 m/s^2$$

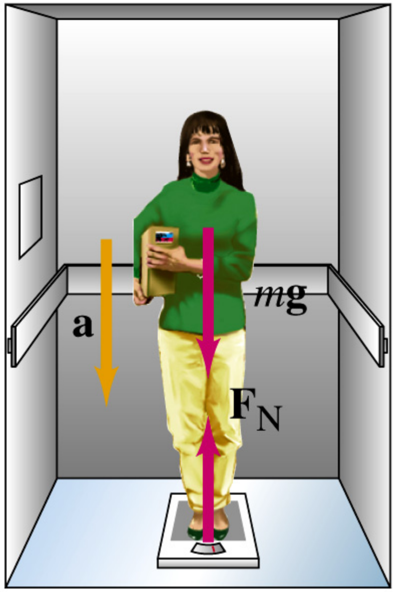


(c)

If the upward force exceeds the weight there is no normal force.

An Illustrative Example: Descending in an Elevator

+y



- A 65-kg woman descends in an elevator accelerating at 0.20g.
- What does the scale read?
- Note:
  - She is accelerating at 0.20g
  - But there are only two forces involved.

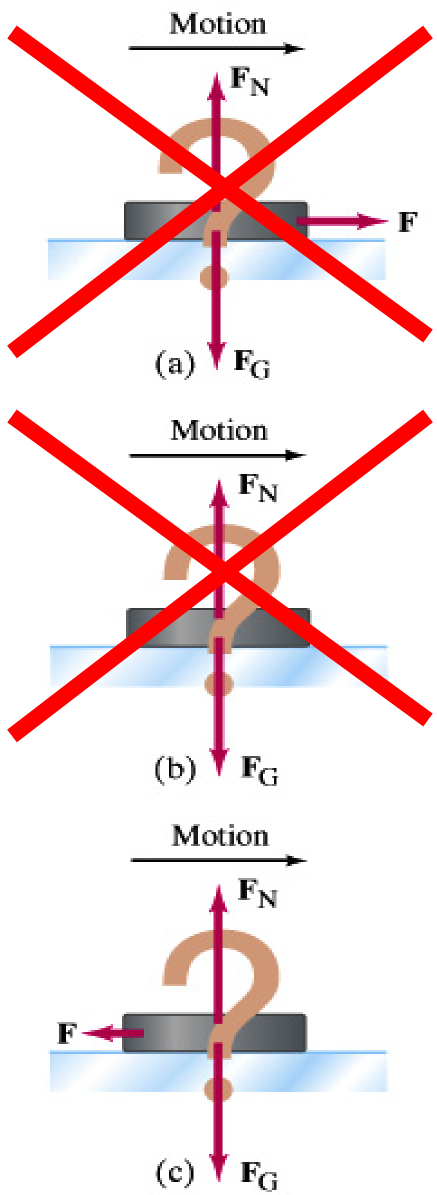
$$\Sigma F = ma$$
$$mg - F_N = ma$$
$$F_N = mg - ma$$
$$F_N = m(g - a)$$
$$F_N = m(g - 0.2g)$$
$$F_N = 0.80mg$$

- So the scale pushes up with a force of 0.80mg and will show a mass of 0.80m or 52kg. A dandy way to loose weight.

Free Body Diagrams and Problem Solving

- Unknowingly you just learned about free-body diagrams. A crucial tool for understanding Newton's Laws and motions for a body.
- A free-body diagram has:
  1. A convenient coordinate system,
  2. Representative vectors
     - I. For all forces acting on a body
     - II. Including those that are unknown.
     - III. If translational only, at the center of the body
  3. Descriptive labels for each force vector.
- Don't show forces the body exerts on other objects.

Free Body Diagram Example: A Hockey Puck

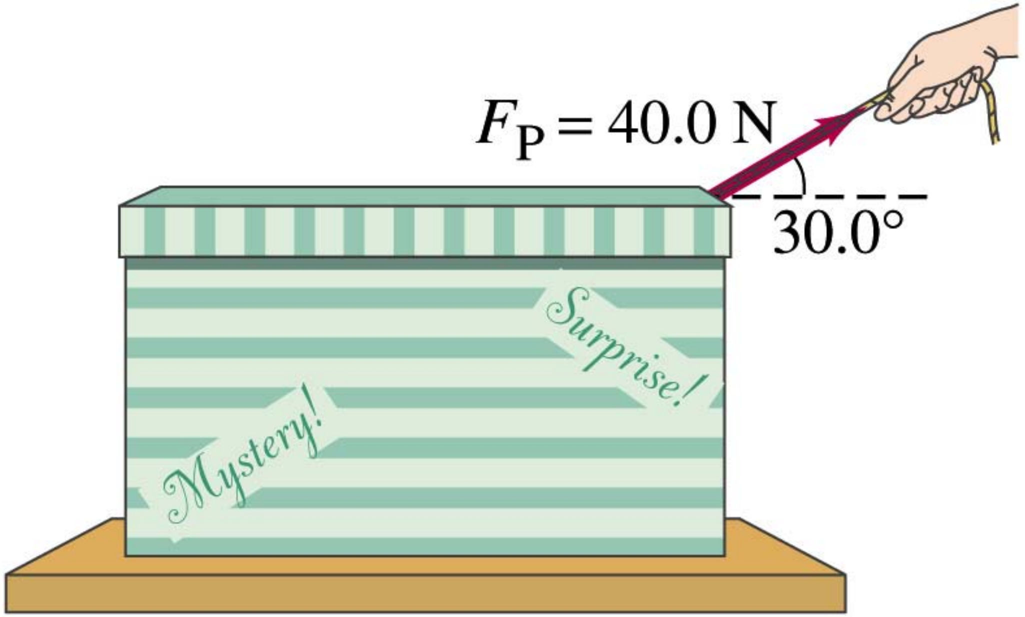


Modification of the "8-Step" Way

1. Read Carefully
2. Draw the Free-Body Diagram for each object.
3. Choose convenient xy coordinate system. Resolve vectors into components. Apply 2nd Law to each direction independently
4. List knowns and unknowns and choose equations relating them.
5. Solve approximately or at least qualitatively
6. Solve algebraically and numerically.
    1. You need one equation for each unknown.
    2. Retain algebraic formulation until the very end. This increases insight.
7. Check units
8. Check if reasonable, the "smell" test.

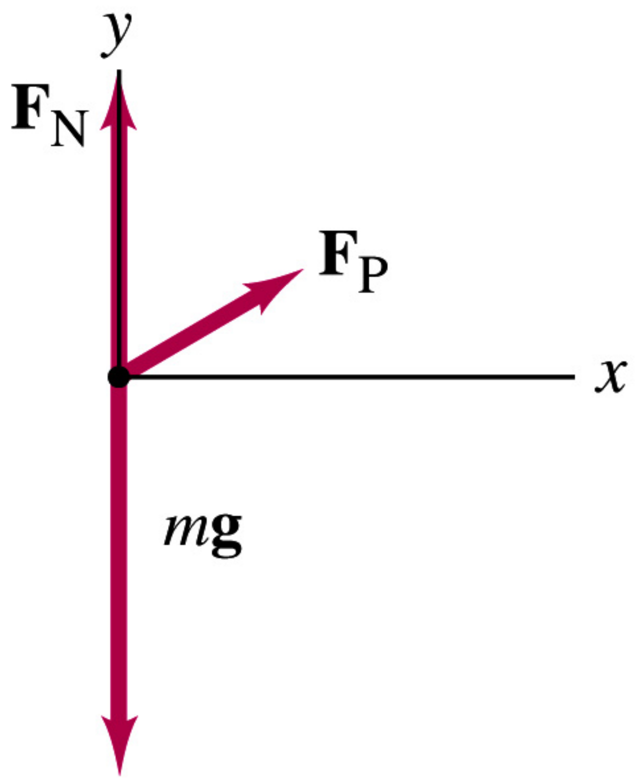
An Example: Pulling the Box

Mass=10.0kg
F_P=40.0N, 30°



What is the acceleration?
What is F_N?

The Free-Body Diagram



Resolving Vectors/Apply 2nd Law in 2D

- $\mathbf{F}_P$

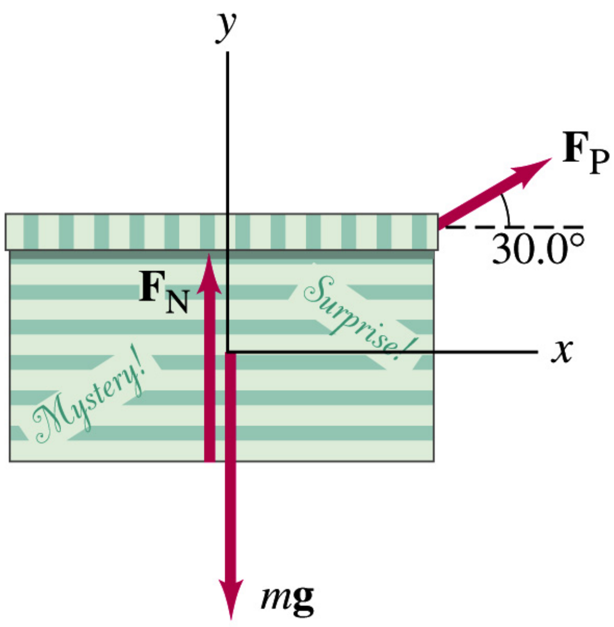
  $FP_x = (40.0N)(\cos 30.0^o) = +34.6N$

  $FP_y = (40.0N)(\sin 30.0^o) = +20.0N$

  $\mathbf{F}_P = 34.6N\mathbf{i} + 20.0N\mathbf{j}$
- $M\mathbf{g}$

  $(10.0kg)(-9.80m/s^2)\mathbf{j} = -98.0N\mathbf{j}$
- $\mathbf{F}_N$

  $+98.0N\mathbf{j}$



- **F**=m**a** in x dimension:

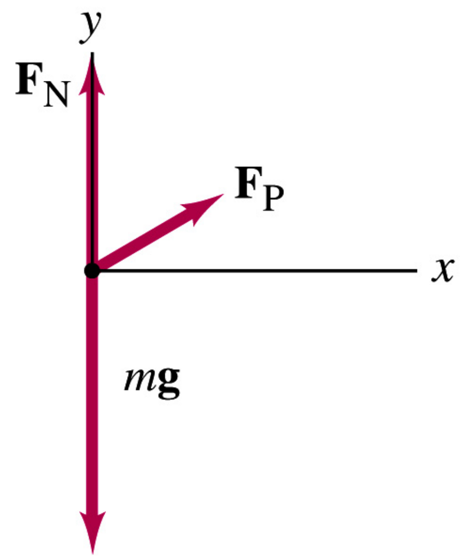
  $\Sigma F_x = ma_x \rightarrow$

  $34.6\text{N} = (10.0\text{kg})(a_x) \rightarrow$

  $a_x = 3.46\ \text{m/s}^2$
- Now y dimension :

  $\Sigma F_y = ma_y \rightarrow$

  $F_N - mg + F_{Py} = ma_y \rightarrow$
- Note mg = 98.0N and F_{Py}=20.0N, thus the pull does not lift the box off the table and the box does not move vertically.



As a result, $a_y = 0 \rightarrow$

$F_N - 98.0\text{N} + 20.0\text{N} = 0 \rightarrow$

$F_N = 78.0\text{N}$

Our Valentine's Day Lesson in Retrospect

- The three laws present a formalism to treat the motion of objects.
- We've looked at one dimensional examples in some detail and introduced the idea of free-body diagrams.
- We've also considered our first two dimensional problem.
- Next lesson we'll look at increasingly more realistic, interesting, and useful situations.