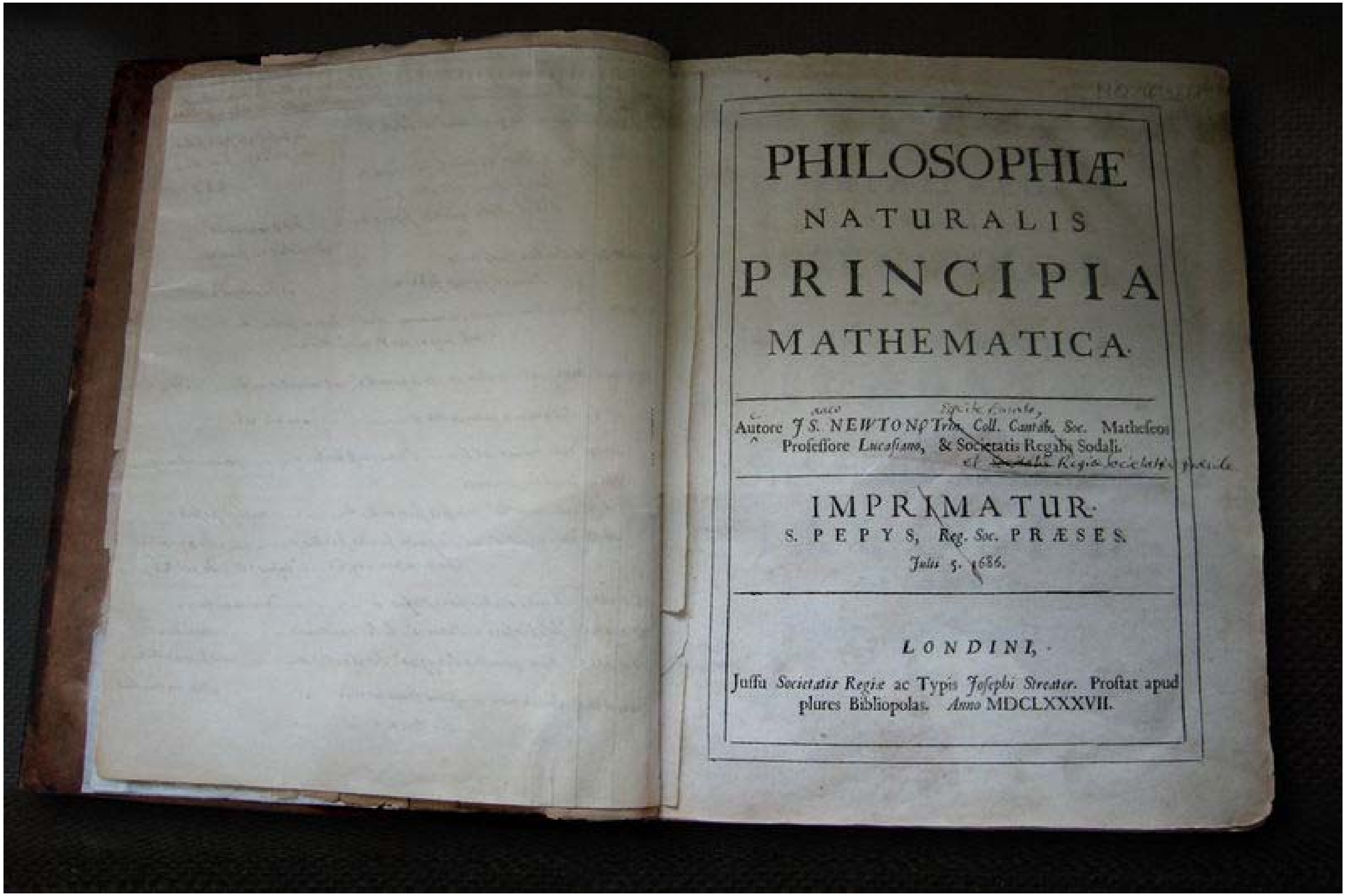
Unit 3: Force and Laws of Motion

- We've done a good job discussing the *kinematics* under constant acceleration including the practical applications to free-fall and projectile motion.
- Now we turn to the *dynamics* of motion or the connection between the forces that act on an object and impel it into motion.
- In this lesson we'll cover Newton's three Laws of Motion.

Sir Isaac Newton
4/1/1643-3/31/1727
Calculus/Gravitation/Optics



Philosophiae Naturalis Principia Mathematica
(Mathematical Principles of Natural Philosophy)
July 5, 1687

Force and Mass

- Before we begin a few concepts need development, at least in words:
- Force: Push or pull on an object
  - If something is a rest a force is required to accelerate it to a non-zero velocity
  - If something is already moving a force is require to change direction or speed.
- Already two ideas become apparent
  - A relationship exists between force and acceleration
  - Force is a vector with magnitude and direction
- Mass: "the quantity of matter"
  - Something like a boulder is obviously very massive since it is extremely hard to move.
  - As opposed to something like a marble that can be easily put into motion.
- A pair of definitions seem reasonable
  - Mass is the response of an object to a force.
  - Or a measure of the inertia of a body
- We can compare the mass of objects and so need a standard: In SI units the kilogram cylinder of platinum-iridium stored in France.

A Subtlety

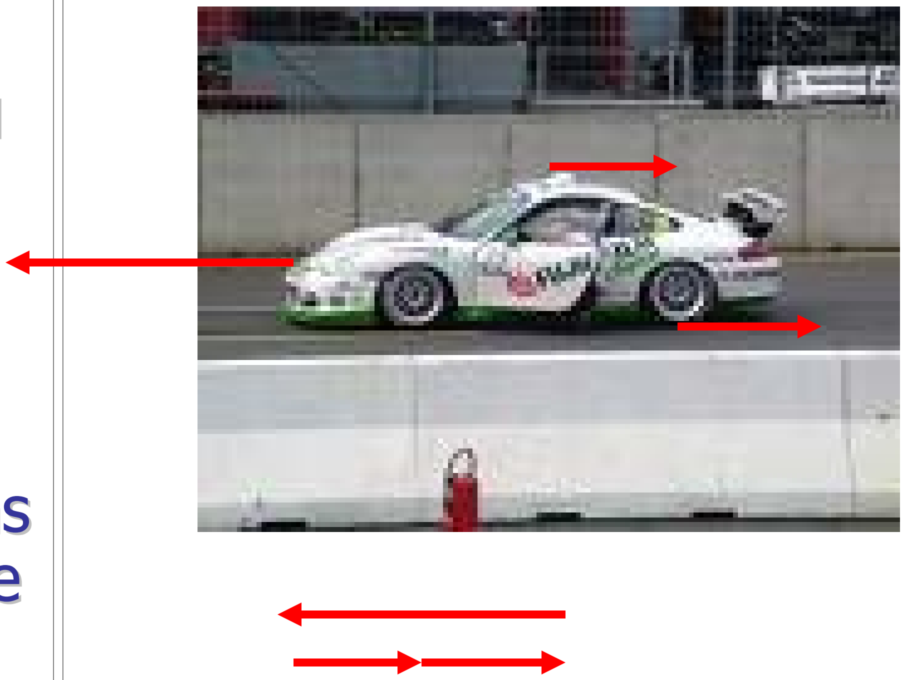
- Mass and Weight are not equal!
- Mass is a measure of an object's inertia or equivalently its response to a force.
- Weight is actually the force of gravity acting on an object.
- Just consider the same standard mass on earth and on the moon. At each location the object is the same so the mass is identical. But the weight on the moon will be 0.16 as much.

http://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html

<u>Newton's 1st Law of Motion</u>:
Every body continues in its state of rest or of uniform speed in a straight line as long as no net force acts on it.

- Although a simple statement the 1st law has a lot of content.
- Consider a hockey puck or an air puck:
  - If it's just sitting there obviously it stays put until struck.
  - But once hit, it moves across the ice with constant velocity - essentially forever, which is practically true since the air and/or ice offer little resistance.
- This was not Aristotle's more limited view which held that an object's natural state was at rest. Galileo's great step was to imagine a ideal frictionless world.

- What's this about "net force"? Often many forces act on an object simultaneously.
- Consider your car:
  - While in motion it feels at least three forces: the engine, air resistance, and resistance from the road.
  - These forces can be represented by a vector sum.
- The car rolls along at constant speed as long as the force from the engine cancels the forces from the air and road for a net force of zero.



- Newton’s law can be rephrased: “A state of constant velocity (zero or otherwise) can only be altered by the application of a nonzero net force.” An imposed force is not required to sustain an object velocity but to change it.
- Compare this to the everyday intuition that a force must be applied to keep an object moving. That's typically because of friction or air resistance.
- Take the car as an example:
- If the engine is off, the car slows because of the resistance. If the engine is on, the car keeps moving. So we automatically think, "well a force is required to keep the car moving at constant velocity.“
- This intuitive view was held by the ancients, "all bodies tend to reach a state of rest" or some such.
- It’s actually more, the force from the car is required to counter-act the friction for a net force of zero.

Inertial Reference Frames

- Newton's first law only holds in reference frames that are not accelerating.
- This would be reference frames that are motionless or moving at constant velocity.
- Does that make sense? Consider an elevator or plane dropping. Our frame of reference is defined by the conveyance. But objects not fastened seem to be accelerating w.r.t. to us even no force is evident. So the first law doesn't hold.
- Most earth-bound frames of reference are inertial or nearly so.

Towards the 2nd Law

- The first law is passive and describes motion in the absence of net force.
- The second law deals directly with the presence of force.
- We can use our intuition to deduce how an object responds to applied forces. Consider the stationary hockey puck.

- If a player hits it, the velocity changes or it accelerates. If a second player hits it twice as hard the acceleration will be twice as large. We already have an important proportionality

$$a \propto F$$

Towards the 2^{nd} Law

- But actually there are always a number of forces acting on an object. Also we know that acceleration is really a vector:

$$\vec{a} \propto \sum \vec{F}$$

- But that can't be all that determines acceleration. Suppose the hockey puck were a Zamboni. Then the acceleration would be quite small.

- Another proportionality suggests itself:

$$\vec{a} \propto \frac{1}{mass} = \frac{1}{m}$$

- Combining these two proportionalities leads to the 2^{nd} Law:

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

<u>Newton's 2nd Law of Motion</u>:
The acceleration of an object is directly proportional to the net force on it and inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.

$$\sum \vec{F} = m\vec{a}$$

The Nature of the 2nd Law

- Note how we've connected our equations of motion through the acceleration to the force.
- This bodes well, not only can we describe the motion we'll now be able to predict the motion for given forces.
- In fact we can redefine force as an action capable of accelerating an object.
- What's more it's a vector quantity and since the perpendicular directions are independent we really have dynamical equations for acceleration in all three dimensions:

$$\sum \vec{F} = m\vec{a}$$

$$F_x = ma_x$$

$$F_y = ma_y$$

$$F_z = ma_z$$

from the right side of the equations the unit of force is kg - m/s^2

Units of Force

- SI units
  - Mass given in kg
  - Acceleration in m/s^2
  - Thus the unit of force is *kg-m/s²* which is called the *Newton*
- CGS units
  - Mass is expressed in grams
  - Acceleration in cm/s^2.
  - Thus the unit of force is *gm-cm/s²* which is called the *dyne*.
- British system (goes at it from the opposite direction)
  - Force is expressed as the *pound* or *lb*.
  - Mass unit is the slug or the mass which accelerates $1 ft/s^2$ when a force of 1 lb is applied.
  - So 1 slug = $1lb/(ft/s^2)$.

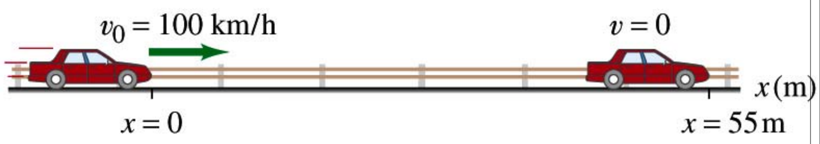
Example: The Force on a fastball or a dropped ball.

- Estimate the force required to accelerate a 0.2 kg baseball from 0 to 100 mi/hr (44m/s) in 0.1 sec?
- The acceleration is given by a_x = 44 m/s / 0.1s = 440 m/s^2
- $F_x=ma_x$ = 0.2kg*440m/s^2 = 88 kg-m/s^2 = 88 Newtons
- This equals 20 lbs.

- Estimate the force with which the baseball feels in free fall.
- Fy = may
  = (0.2kg)*(-9.8m/s^2)
  = 2.0 Newtons
- This is equal to 0.4 lbs.

Combining F=ma and Eqs. of Motion.

- What constant net force is required to stop a 1500kg car moving at 100km/hr in 55 meters?



- We are given
  - v_0=+100km/hr=28m/s
  - v=0
  - $x-x_0$= 55m
- And need the acceleration

$$v^2 = v_0^{\ 2} + 2a(x - x_0) \rightarrow$$

$$v^2 - v_0^{\ 2} = 2a(x - x_0) \rightarrow$$

$$a = \frac{v^2 - v_0^{\ 2}}{2(x - x_0)} \rightarrow$$

$$a = \frac{0 - (28m/s)^2}{2(55m)} \rightarrow$$

$$a = -7.1m/s^2$$

- And

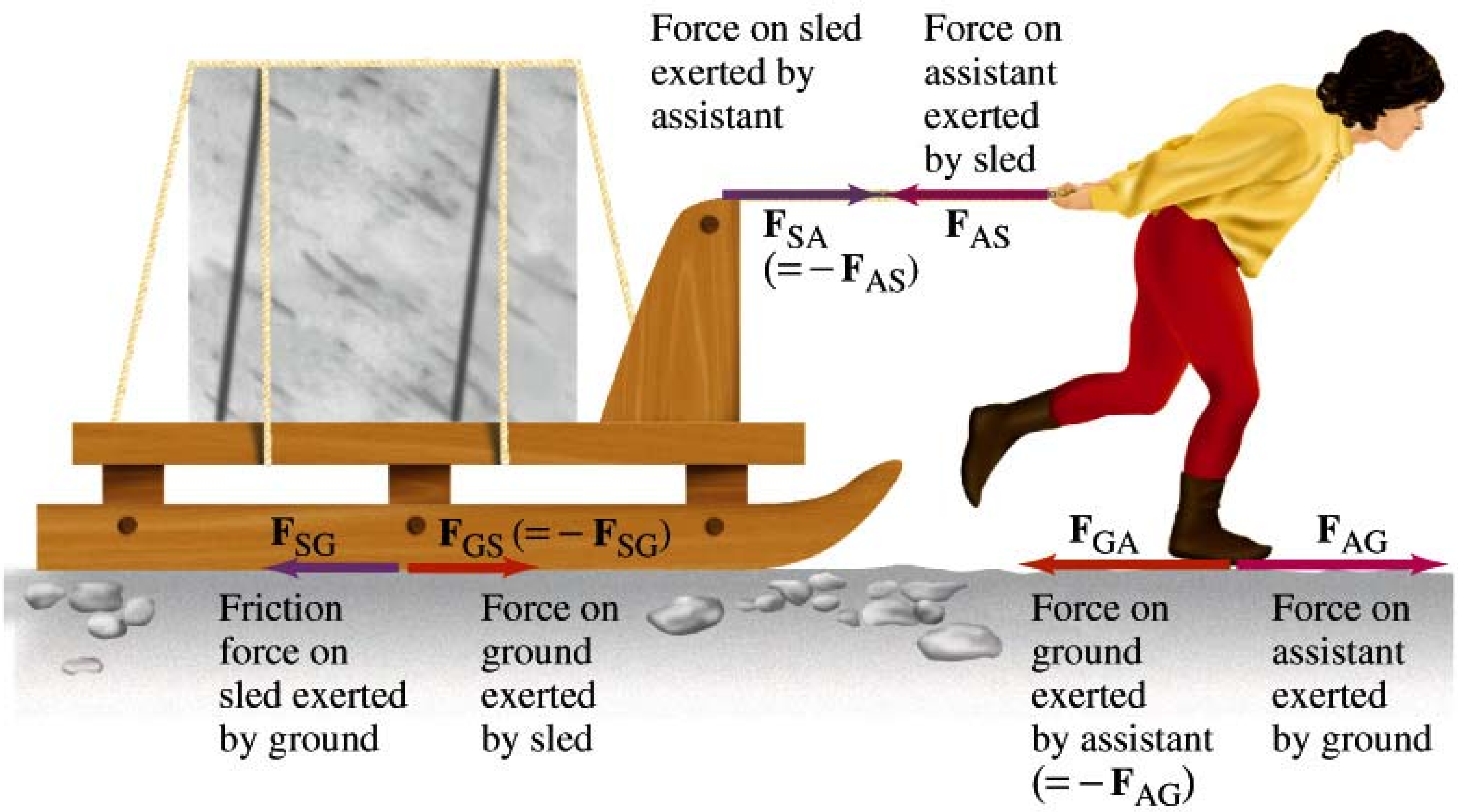
$$F = ma = (1500kg)(-7.1m/s^2)$$

$$F = -11{,}000N$$

Newton's 3nd Law of Motion:
Whenever one object exerts a force on a second object, the second exerts an equal and opposite force on the first

The Third Law

- Intuitively this is actually not a big shocker!
  - When you kick a ball you know you imparted acceleration or a force, but you also feel the force of the ball on your foot.
  - When you are lifting weights you're definitely accelerating the barbell and feeling it press down on you.
- Some care is needed fully analyzing a situation. Consider pulling a sled.



Assistant acceleration occurs when $\Sigma F = F_{AG} + F_{AS} > 0$

Example

- Let's consider an astronaut outside his ship who pushes on his ship with a force **F**. According to third law the ship will push back with force -**F**.
- If the spacecraft has a mass of M_s=11,000 kg and the astronaut M_a=92 kg and the astronaut pushes with a force of +36N, what are the accelerations of the two objects?

- If the astronaut pushes with a force of +36N, the ship pushes on the astronaut with a force of -36 N (the 3rd law).
- We can now derive the acceleration (the 2nd law):

  $\mathbf{a}_s = \mathbf{F} / m_s =$
  $+36\text{N} / 11{,}000\text{kg} =$
  $+0.0033\ \text{m/s}^2$

  $\mathbf{a}_a = -\mathbf{F} / m_a =$
  $-36\text{N} / 92\text{kg} =$
  $-0.39\ \text{m/s}^2$

Summary

1st Law: Body in Motion stays in Motion

2nd Law: $\Sigma\vec{F}=m\vec{a}$

3rd Law: Equal and Opposite

Next we'll start concentrating on the types of forces, how to analyze them, and how they lead to the equations of motion.