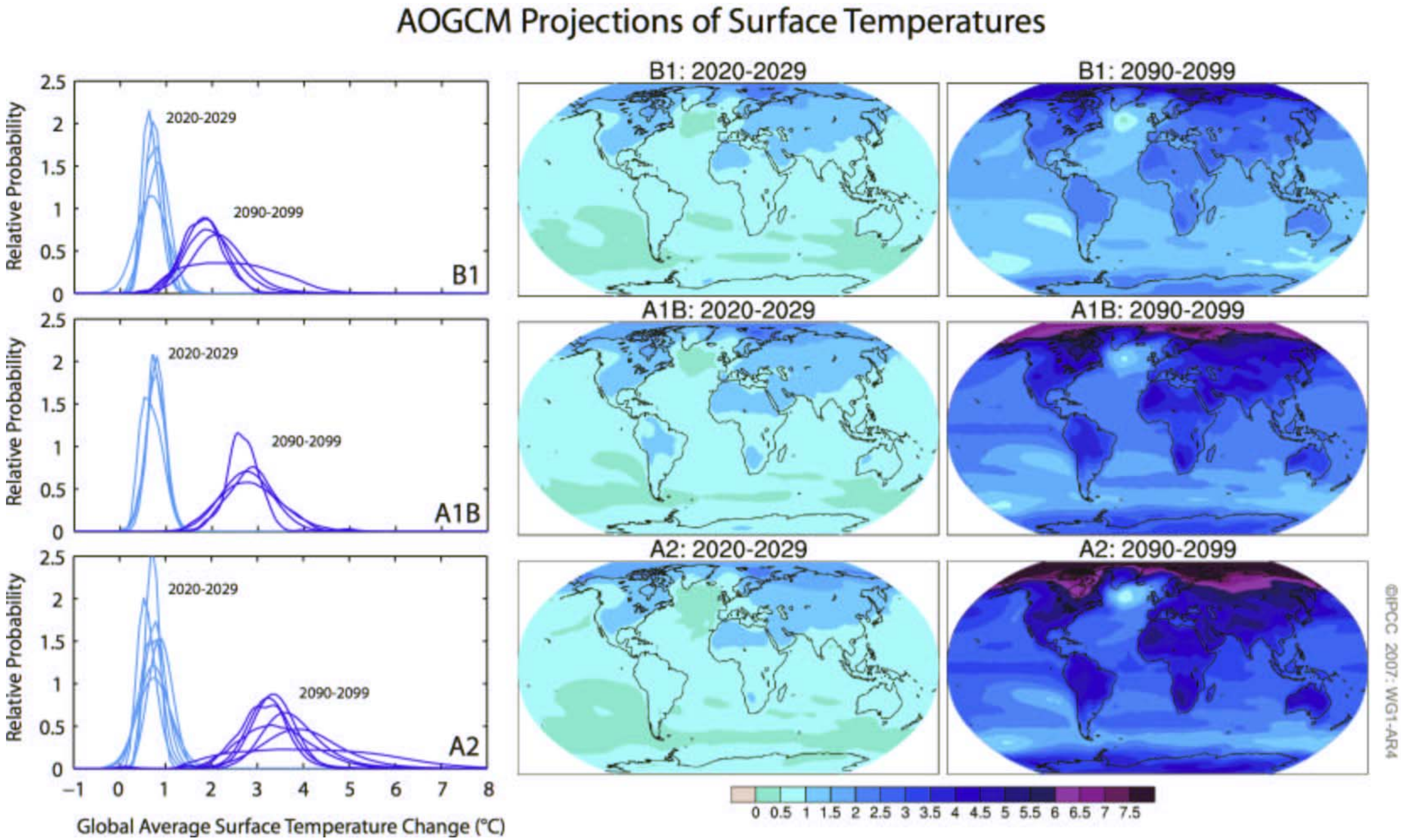
Science Advertisement
Intergovernmental Panel on Climate Change:
The Physical Science Basis
http://www.ipcc.ch/SPM2feb07.pdf



http://www.foxnews.com/projects/pdf/SPM2feb07.pdf

Status: Unit 2, Chapter 3

- ✓ Vectors and Scalars
- ✓ Addition of Vectors – Graphical Methods
- ✓ Subtraction of Vectors, and Multiplication by a Scalar
- ✓ Adding Vectors by Components
- ✓ Unit Vectors
- Vector Kinematics
- Projectile Motion
- Solving Problems in Projectile Motion
- Relative Velocity

Section Two Problem Assignment

- Q3.4, P3.6, P3.9, P3.11, P3.14, P3.73
- Q3.21, P3.24, P3.32, P3.43, P3.65, P3.88

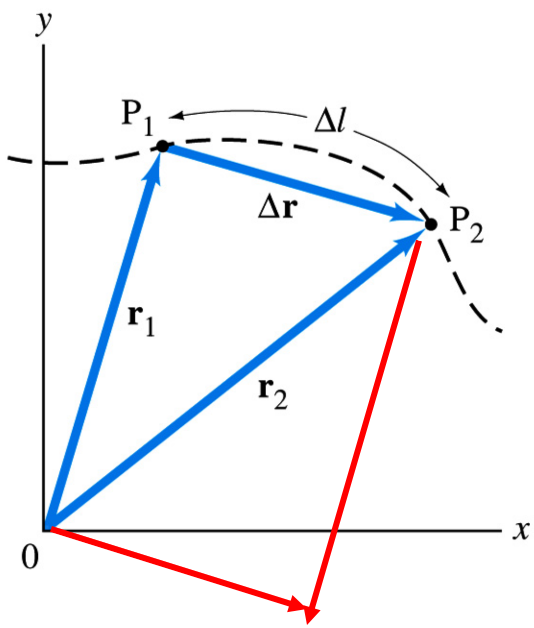
Vector Kinematics: Displacement, Velocity, Acceleration

- Now that we have vectors well described we can focus on the general description of motion in multiple dimensions.
- Each of the quantities displacement, velocity, and acceleration, which we discussed in Chapter 2, have a more general vector representation
- As shown in the figure the displacement:

$$\Delta x = x - x_o \rightarrow \Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

Occurs in the time interval

$$\Delta t = t_2 - t_1$$



$$\vec{r}_1 = x_1\,\vec{i} + y_1\,\vec{j} + z_1\,\vec{k} \text{ and } \vec{r}_2 = x_2\,\vec{i} + y_2\,\vec{j} + z_2\,\vec{k}$$

so

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_1 - x_2)\,\vec{i} + (y_1 - y_2)\,\vec{j} + (z_1 - z_2)\,\vec{k}$$

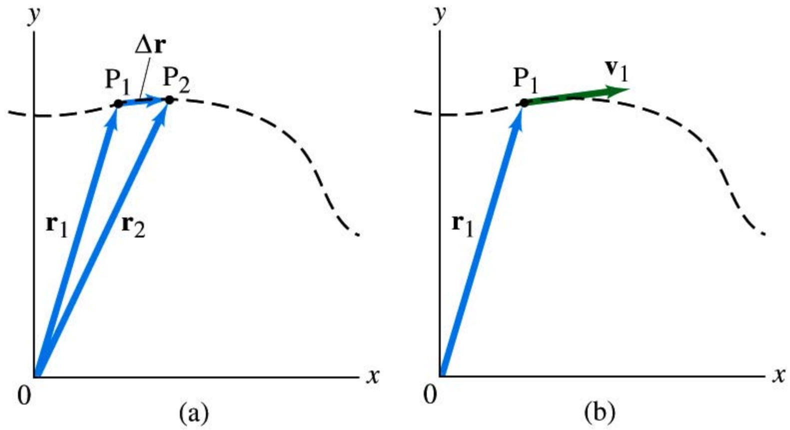
Average and Instantaneous Velocity Vectors

- The average velocity vector is the obvious extension of average 1-D velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow \bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

- Note that the direction of the average velocity and displacement are identical
- As Δt approaches zero we have the instantaneous velocity vector:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



- Taking the derivative of the vector equation we see

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

Average and Instantaneous Acceleration Vectors

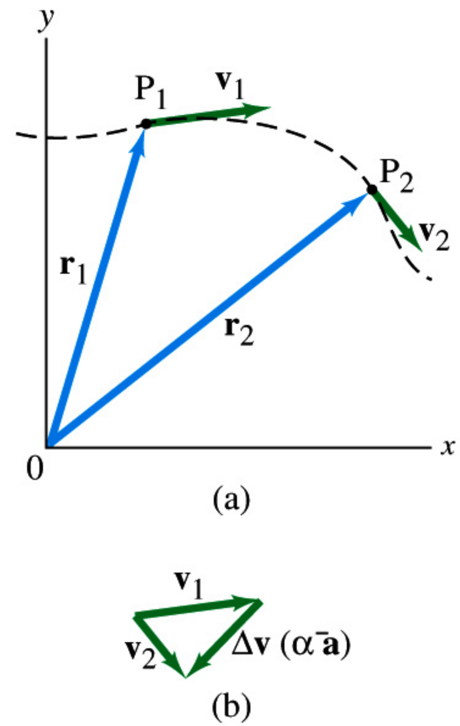
- The average acc. vector is the extension of ave. 1-D acc:

$$\bar{a} = \frac{\Delta v}{\Delta t} \rightarrow \bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- As Δt approaches zero we have the instantaneous acc. vector:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

- Notice that
  - 1) acceleration may be in a different direction than vel.
  - acceleration may be due to a change of velocity magnitude, direction, or both



$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} + \frac{dv_z}{dt}\vec{k}$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

Summary of Generalization

$$\Delta x = x - x_0 \rightarrow \Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$v = \frac{dx}{dt} \rightarrow \vec{v} = \frac{d\,\vec{r}}{dt}$$

$$a = \frac{dv}{dt} \rightarrow \vec{a} = \frac{d\,\vec{a}}{dt}$$

Vector Generalization of Eq. of Motion.

- If we have a constant acceleration vector, then the equations derived for 1-D apply separately for the perpendicular directions.

$$a_x = \text{constant}, a_y = \text{constant}$$

$$v_x = v_{x0} + a_x t \qquad v_y = v_{y0} + a_y t$$

$$x = x_0 + v_{xo} t + \frac{1}{2} a_x t^2 \qquad y = y_0 + v_{yo} t + \frac{1}{2} a_y t^2$$

$$v_x{}^2 = v_{x0}{}^2 + 2a_x\left(x - x_0\right) \qquad v_y{}^2 = v_{y0}{}^2 + 2a_y\left(y - y_0\right)$$

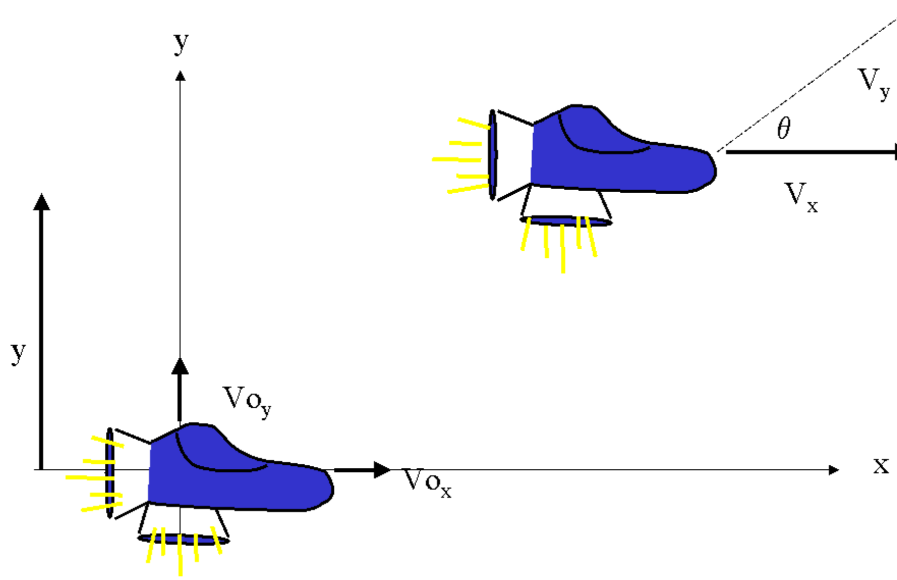
$$\bar{v}_x = \frac{v_{x0} + v_x}{2} \qquad \bar{v}_y = \frac{v_{y0} + v_y}{2}$$

- Some of these can be recast as vector equations, though the component form is more practical.

$$\vec{v} = \vec{v}_0 + \vec{a} t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

Example: A 2D Spacecraft



- The spacecraft has an initial velocity of
  - V_{ox} = +22 m/s and
  - V_{oy} = +14 m/s
- and an acceleration of
  - a_x= +24m/s^2 and
  - a_y= +12m/s^2.
- The directions to the right and up have been chosen as positive components.
- After a time of 7.0 s find
  - a) x and V_x,
  - b) y and V_y, and
  - c) the final velocity.

- Since the directions are independent we simply follow the 1-D drill from Chapter 2.
- x-Direction:

Known	Unknown
t = 7.0 s	x=?
v_{ox}=+22m/s	v_x=?
a_x=+24m/s^2	

- The eqs. we need:

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{xo} t + \frac{1}{2} a_x t^2$$

- Substituting

$$v_x = v_{x0} + a_x t$$
$$= 22\ m/s + 24\ m/s^2 \times 7.0\,s$$
$$= +190\ m/s$$
$$x = x_0 + v_{xo} t + \frac{1}{2} a_x t^2$$
$$= 0 + 22\ m/s \times 7.0\,s + \frac{1}{2}(24\ m/s^2)(7.0\,s)^2$$
$$= +740\ m$$

- y-Direction:

Known	Unknown
t = 7.0 s	y=?
v_{oy}=+14m/s	v_y=?
a_y=+12m/s^2	

$$v_y = +98\ m/s \text{ and } y = +380\ m$$

- The two velocity components can be combined using the Pythagorean Theorem to find the magnitude of the final velocity:

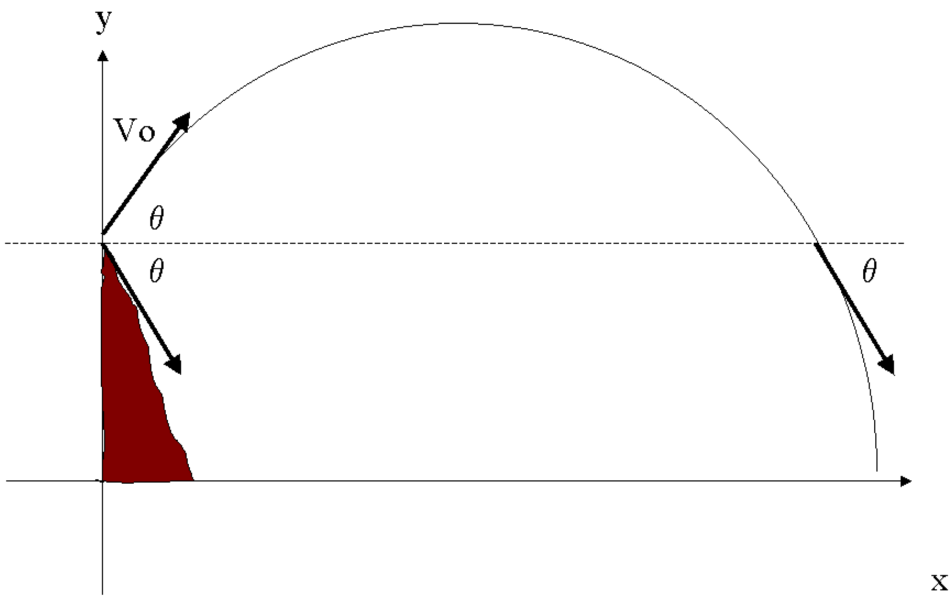
  $V^2 = V_x^2 + V_y^2 = (190\ m/s)^2 + (98\ m/s)^2$ or $V = +210\ m/s$

  (We keep only the positive solution as it's the only physical one.)
- The direction is given by

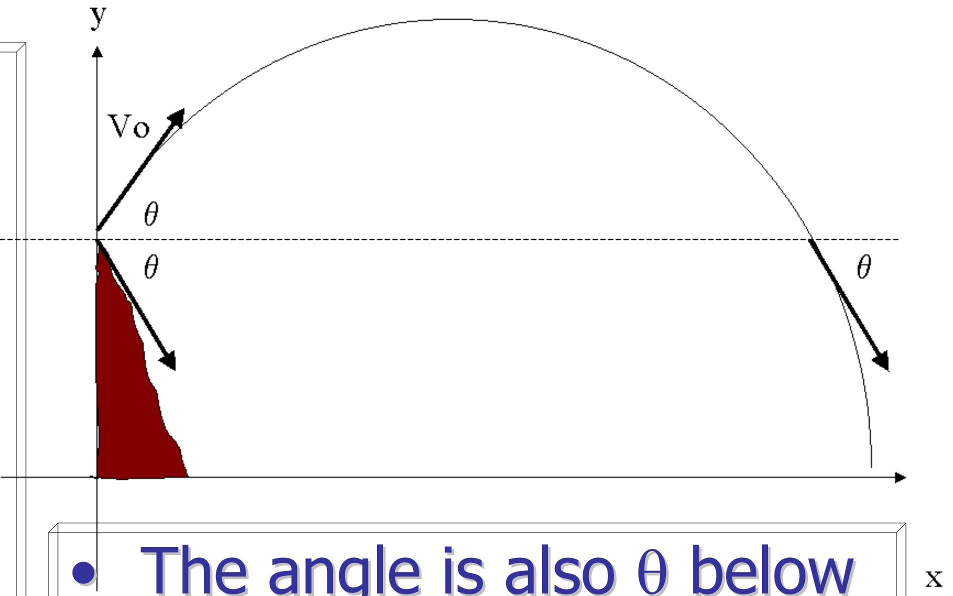
  $\theta = \tan^{-1}(Vy/Vx) = \tan^{-1}(98\ m/s\ /\ 190\ m/s) = 27^o$
- Thus, after 7.0 s the spacecraft is moving with a speed of 210 m/s above the positive x axis. **Note how we treated the two directions independently. This is a crucial point.**

Thought Experiment One:

- From the top of a cliff overlooking a lake, a person throws two stones. The stones have identical speeds V_o, but stone 1 is thrown downward at an angle θ and stone 2 is thrown upward at the same angle above the horizontal.
- Which stone, if either, strikes the water with greater velocity?



- My naive guess is that the downward thrown stone will have the greater velocity, actually that's not true.
- Consider the upwardly thrown stone. First it rises to its maximum height and then falls back to earth.
- When the stone returns to its initial height it has the same speed horizontal and vertical speed as when thrown. (We discussed the vertical speed symmetry in one dimensional motion.)



- The angle is also θ below the horizon. This is exactly the speed and direction the downward thrown stone had when it left the cliff.
- From this point on, the two stones have identical velocity. So both stones strike the water with the same velocity.

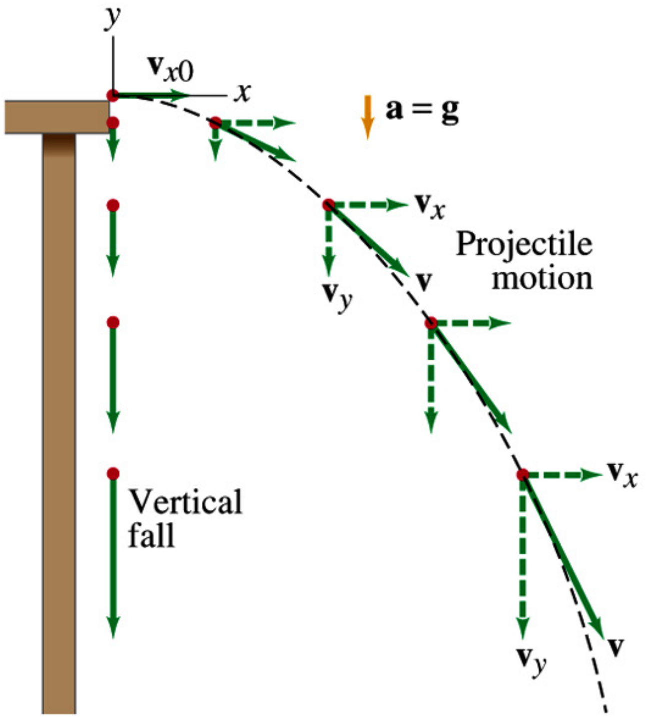
Projectile Motion

- Generally: Any object moving freely through air in two dimensions near the earth's surface
- Only vertical acceleration involved, $g=9.80\ m/s^2$ downward.
- Galileo was the first to analyze projectile motion
  - The two dimensions independently
  - The horizontal component has no acceleration
  - The vertical subject to the acceleration of gravity.

http://webphysics.davidson.edu/course_material/py130/demo/illustration2_4.html

More Elements of Projectile Motion

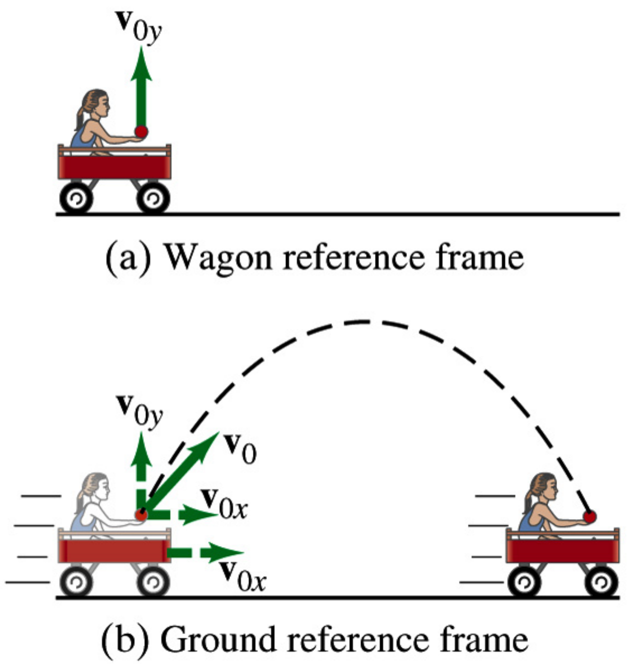
- The key: The individual components or dimensions can be analyzed separately.
- Consider a ball moving in two dimensions: The horizontal component of the motion, which is acceleration free, is independent of the vertical component of the motion which is subject to acceleration!
- Vertical direction: Vy is zero but increases linearly with time due to g.
- Horizontal Direction: no acceleration and constant velocity



- Note in this figure a dropped ball and a thrown ball fall at the same rate and reach the ground at the same time.

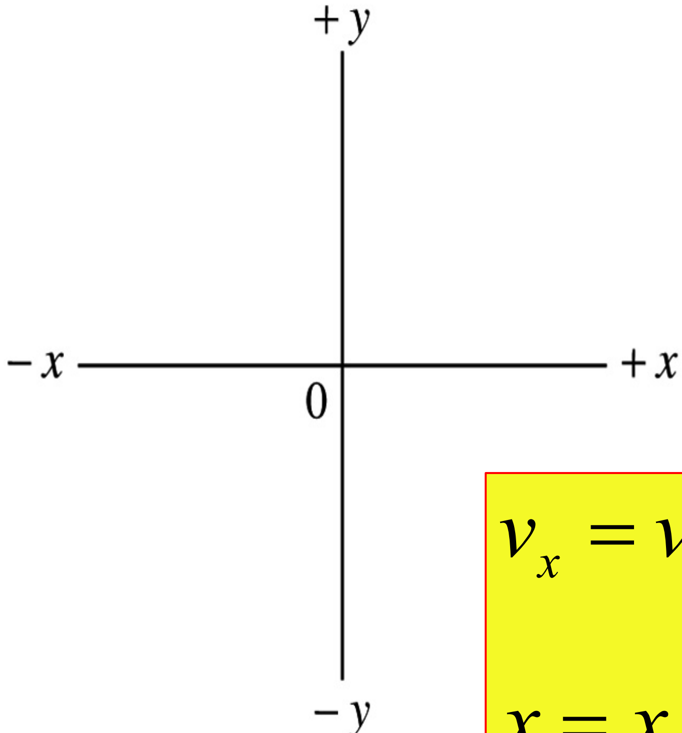
Thought Experiment Two:

- A child sits upright in a wagon which is moving to the right at constant speed. The child tosses up an apple while the wagon continues to move forward.
- Ignoring air resistance will the apple land behind, in or in front of the wagon?



- Well we could do a full blown analysis calculating how much time the ball is in flight and how far it would carry and how far the wagon would move.
- But that’s unnecessary once we realized both the ball <u>and</u> the wagon have the same, unchanging horizontal velocity.
- No matter how long the ball is in flight both travel the same distance during that time.
- The ball will land in the wagon.
- By the way this is why a tossed ball in your car always lands in your lap! There’s no air resistance involved inside the car and you and the ball have the same constant velocity.

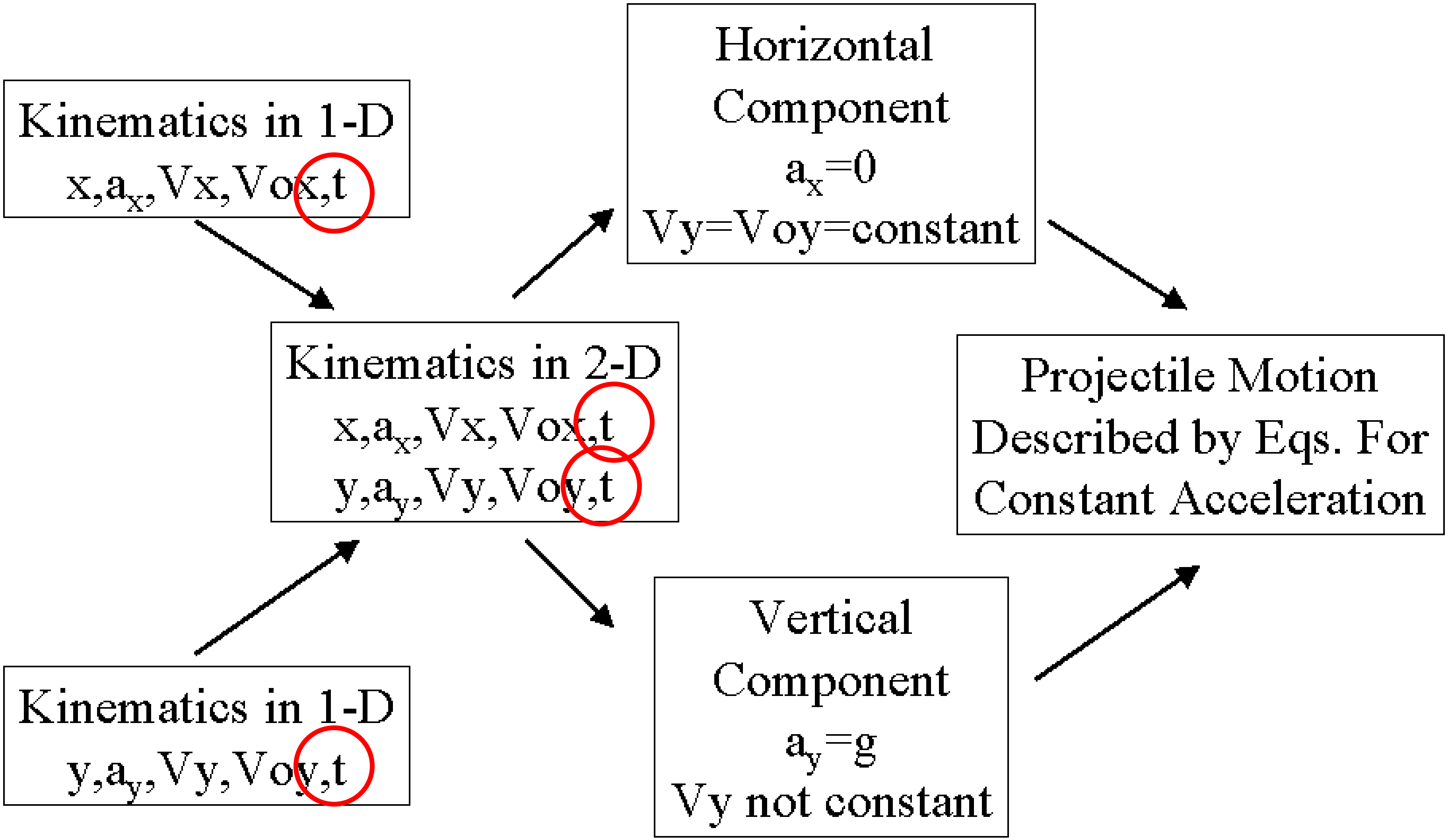
Kinematic Equations for Projectile Motion (+y up, a_x =0, a_y=-g= -9.8m/s^2



$$v_x = v_{x0} \qquad v_y = v_{y0} - gt$$

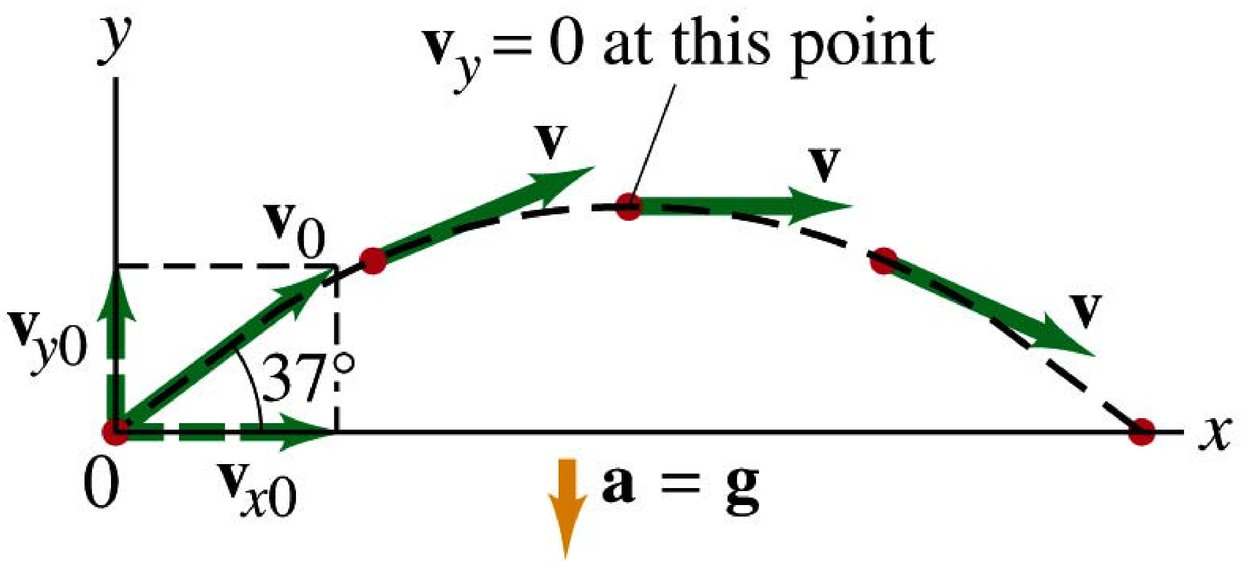
$$x = x_0 + v_{xo}t \qquad y = y_0 + v_{yo}t - \frac{1}{2}gt^2$$

$$v_y^{\;2} = v_{y0}^{\;2} - 2gy$$


Kinematics in 1-D
x,a_x,Vx,Vox,t
Kinematics in 1-D
y,a_y,Vy,Voy,t
Kinematics in 2-D
x,a_x,Vx,Vox,t
y,a_y,Vy,Voy,t
Horizontal Component
a_x=0
Vy=Voy=constant
Vertical Component
a_y=g
Vy not constant
Projectile Motion Described by Eqs. For Constant Acceleration

Finding Final Variables Given Initial Variables: A kicked football

- A football is kicked at an angle $\theta=37.0^o$ with an initial velocity of 20.0m/s.
- What will be
  - Maximum height?
  - Time of travel?
  - Final displacement?
  - Velocity at apex?
  - Acceleration at apex?
- From just the initial conditions the projectile equations provide all subsequent history of the trajectory



- Well what do we know? the initial velocity and initial position and acceleration.
  - $x_o = 0$
  - $y_o = 0$
  - $v_{xo} = v_o\cos\theta_o$ = (20.0m/s)cos37.0° = +16.0m/s
  - $v_{yo} = v_o\sin\theta_o$ = (20.0m/s)sin37.0° = +12.0m/s
  - $a_x=0$
  - a_y=-9.8m/s
- The first unknown quantity is the maximum height. Well, we get this by considering the y dimension. You've done this before! Filling out the table:

Known	Unknown
yo = 0	y=?
v_{oy}=+12m/s	
v_y = 0	
a_y=-9.8m/s^2	

- The third y-equation does the trick!

$$v_y^2 = v_{y0}^2 - 2gy \rightarrow$$

$$y = \frac{v_{y0}^2 - v_y^2}{2g} =$$

$$\frac{(12.0m/s)^2 - 0}{2 \times 9.8m/s^2} =$$

$$7.35m$$

- Next comes the time of travel. If we just consider the y dimension we see a very familiar problem:

Known	Unknown
yo = 0	v=?
v_{oy}=+12m/s	t=?
y = 0	
a_y=-9.8m/s^2	

- And we use the 2nd y-equation which as shown on the right has two roots corresponding to the initial kick and to the return to earth.

$$y = y_0 + v_{yo}t - \frac{1}{2}gt^2 \rightarrow$$

$$0 = 0 + (12m/s)t - \frac{1}{2}(9.8m/s^2)t^2 \rightarrow$$

$$\frac{1}{2}(9.8m/s^2)t^2 = (12m/s)t \rightarrow$$

$$\frac{1}{2}(9.8m/s^2)t^2 - (12m/s)t = 0 \rightarrow$$

$$(\frac{1}{2}(9.8m/s^2)t - (12m/s))t = 0 \rightarrow$$

$$((4.9m/s^2)t - (12m/s))t = 0 \rightarrow$$

$$t = 0 \text{ and } t = 2.45s$$

- Now that we have the time of travel we simply turn to the x dimension equations to get the final displacement

$$x = x_0 + v_{xo}t \rightarrow$$
$$x = 0 + 16.m/s \times 2.45s \rightarrow$$
$$x = 39.2m$$

- At the apex v_y=0 so there is only horizontal motion so $v=v_x=v_{xo}=+16.0m/s$
- The question at the acceleration at the apex is a trick question. Acceleration is always -9.8m/s down!

Schedule

- Projectile motion is quite rich, we'll continue to explore the consequences.
- Review Feb 7
- No class Feb 9
- Test Feb 12
- First two problem sets due Feb 12
- If you need help see me soon!