Clarifications

- Extra Credit
  - There are two assignments for each unit.
  - The total credit is 10 points/ unit
  - To be precise the score for each unit equals the number of questions answered correctly divided by the total number of questions times 10.
  - Do them all! They'll give you a boost and help understanding.
- Last lesson I used some integral calculus. This is off course new stuff to about half of you. Don't worry, they'll be no such stuff on any test or in any problems. Just sit back and consider it cultural exposure.

Unit 2: Vectors and 2D Motion
Kinematics in Two Dimension: Vectors

- We've pretty much explored one-dimensional motion under constant acceleration and a bit with variable acceleration.
- To go any further and consider multi-dimensional motion we'll need to add **vectors** to our tool box.
- Unfortunately this can be pretty dry, but it's also a key element to understanding motion. Bear with me.
- By the way such mathematical tools don't stop here. For instance, more advanced mechanics requires tensors, particle physics requires pseudo-scalars and pseudo-vectors, and so on…

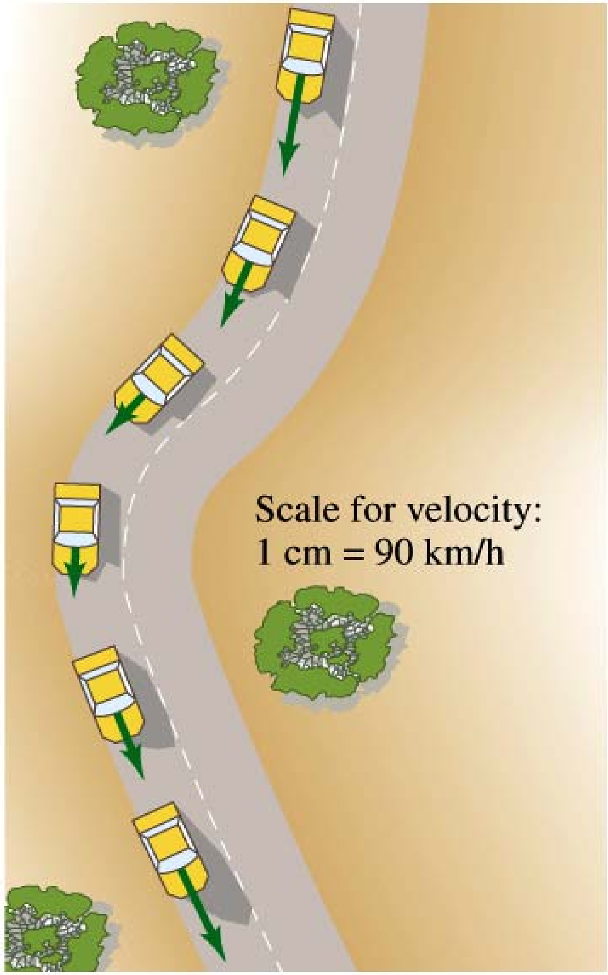
Vectors and Scalars: Some Definitions

- The simplest physical quantity is a **scalar**. It a quantity specified completely by a number and unit.
  - Examples: Mass, temperature, time, voltage potential…
- A bit more complicated is the **vector** which has direction as well as magnitude and units.
  - Examples: Displacement, velocity, electric field, quantum spin
- Vectors have two main representations:
  - Graphical
  - Algebraic, with standard references or unit bases
- We'll start with graphical methods to improve our intuition and then move to the more rigorous vector algebra and vector kinematics.

- Graphically
  - Direction = Arrow
  - Magnitude = Length
- Print notation:
  - Boldface: **v**
  - Arrow: $\vec{v}$
- Examples

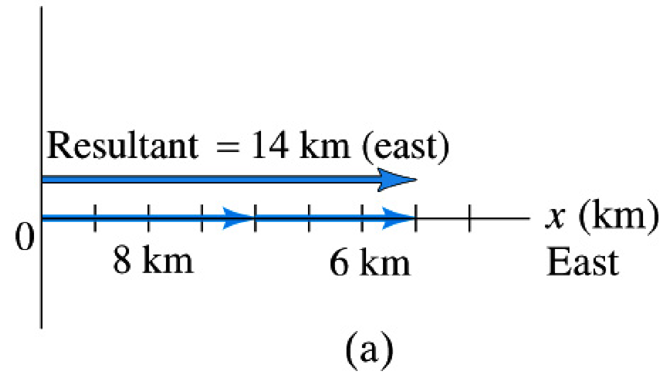
  Displacement: $\vec{D}$

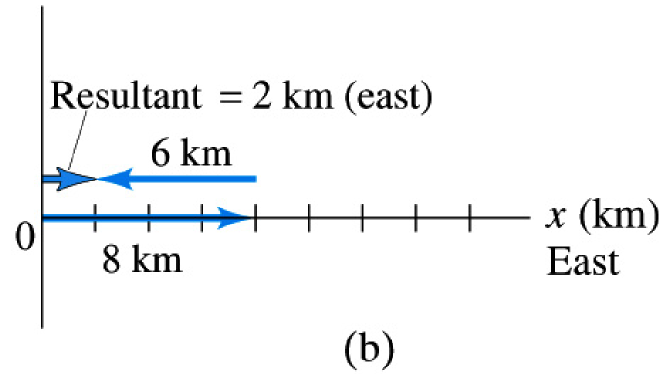
  Acceleration: $\vec{a}$



Graphical Vector Addition

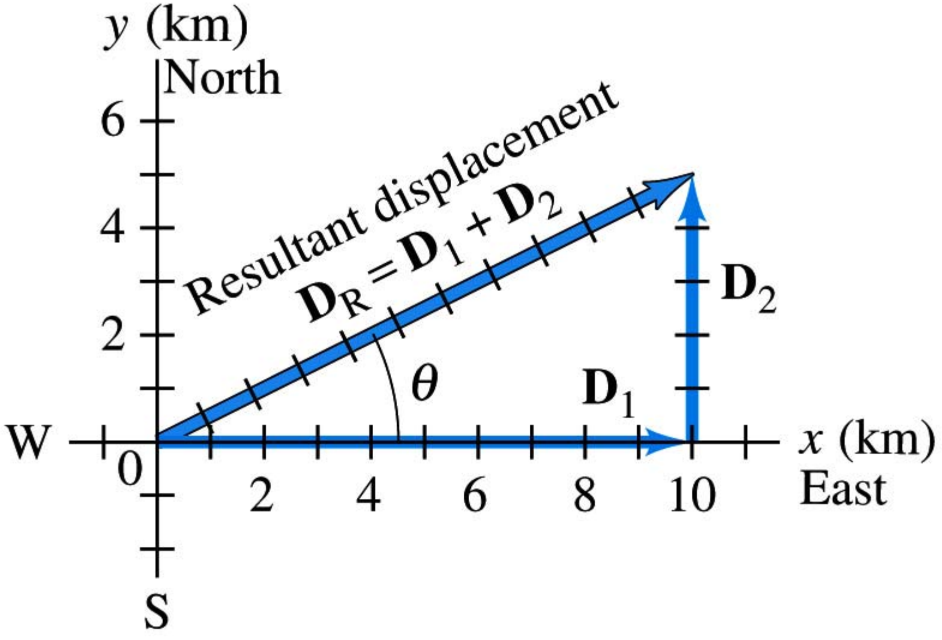
- Because of the direction involved, vector addition more complicated than scalar addition.
- But the easiest example of vector addition, the addition of two coincident or anti-coincident vectors, is almost identical to simple scalar addition.
- **<u>We start with a coordinate system!</u>**
- As can be seen in the example, we simply add magnitudes to get the final or resultant displacement.
- Direction is still involved but in the form of a minus or positive sign.





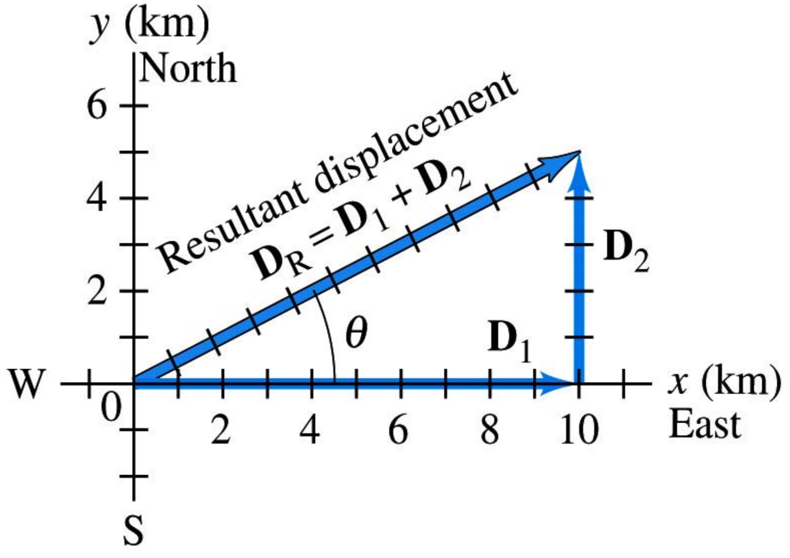
More general addition: normal or perpendicular vectors

- Consider addition of vectors not coincident
- We explicitly extend the coordinate system to x and y.
- Add the two vectors
  - $\vec{D}_1$ = 10 km east
  - $\vec{D}_2$ = 4 km north
- Resultant displacement vector drawn is from "start-to-finish" is $\vec{D}_R$
- Our first vector equation:
  - $\vec{D}_R = \vec{D}_1 + \vec{D}_2$



Specifying Vectors

- Graphical image of $\vec{D}_R$ not enough.
- More precise, but awkward, descriptions
  - $\vec{D}_R$ = 11.2km long at an angle of 27° wrt to x-axis.
  - $\vec{D}_R$ = (11.2km, 27° NE)
- Already a hint that we will need something more precise

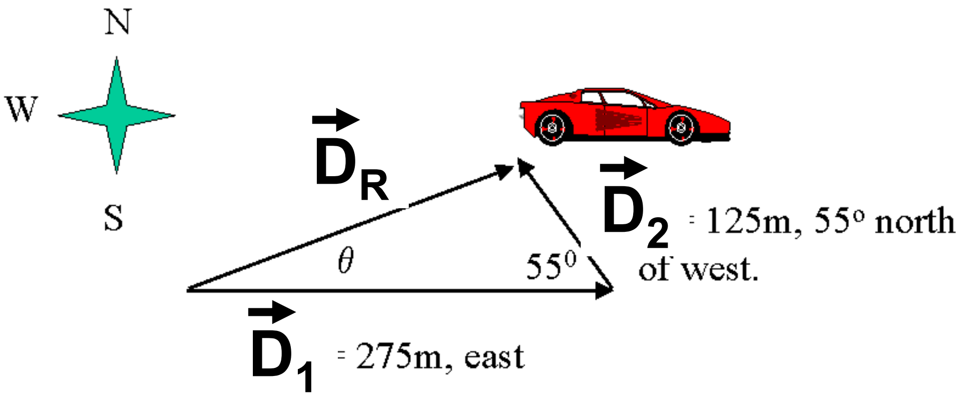


General Addition

- The following vector relation is always true

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

- This is a general relation in the sense that the two initial vectors can be at any angle.
- At this point we lack tools, can't be precise, and rely on an estimate using a protractor or ruler:
  $\vec{D}_R$ = (228 meters, 27° North of East)



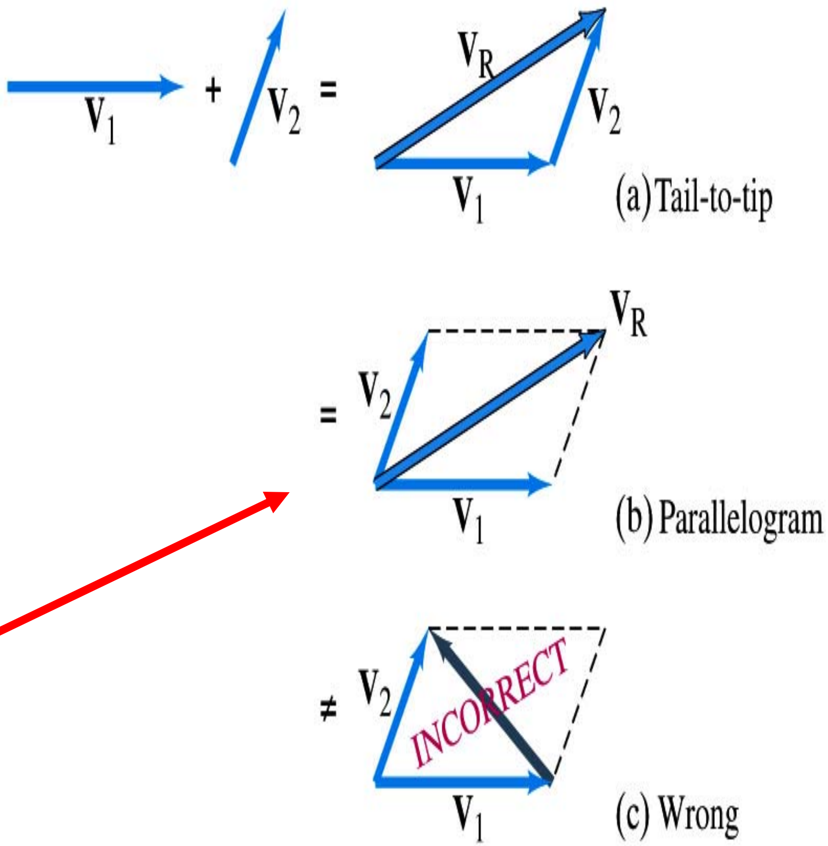
Collecting our results:

Addition of Collinear Vectors	Just Add Magnitude	Direction Unchanged or Opposite
Addition of Perpendicular Vectors	Use Pythagorean Theorem	Use Trig Functions
Addition of General Vectors	Ruler and Scale!	Use Protractor

- The latter choice is unappealing and a dead-end which will later yield to an exact treatment.
- Let's use the graphical approach a bit longer to explore vector properties.

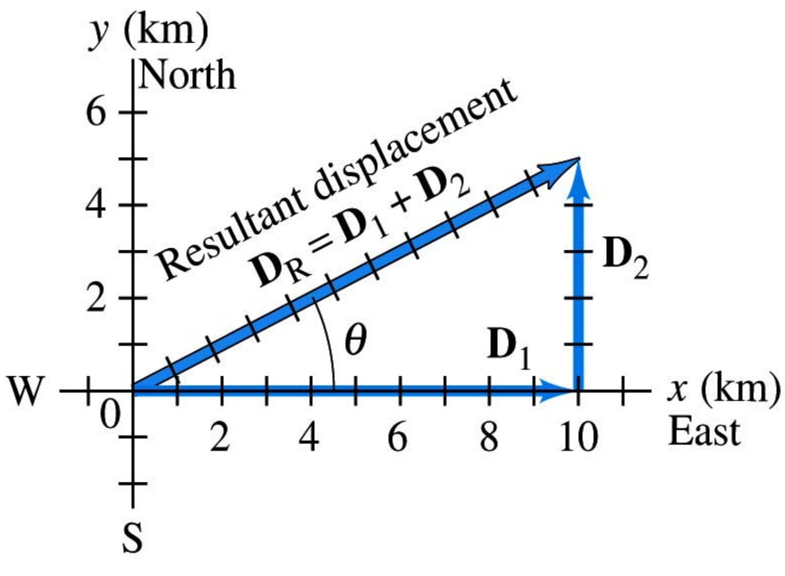
General Rule for Graphical Addition

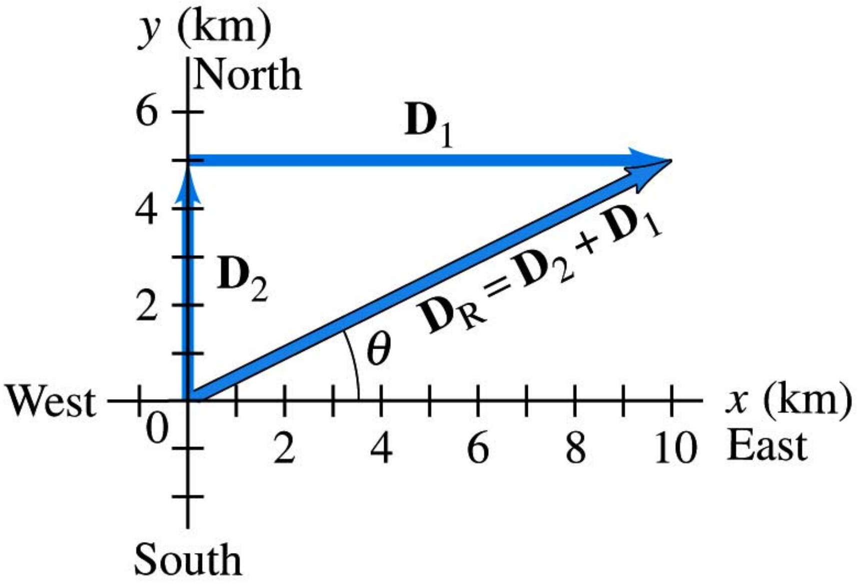
1. Tail-to-tip Method
   a. Draw the 1st vector, $\vec{V}_1$, to scale.
   b. Draw the 2nd vector, $\vec{V}_2$, to scale putting it's tail a the tip of the first vector and with the proper direction.
   c. Draw the resultant vector, $\vec{V}_R$, from the tail of the first vector to the tip of the second.
2. Parallelogram Method
   a. Draw both vectors from a common origin.
   b. Make a parallelogram
   c. Diagonal from the origin is the resultant.



Vectors are Commutative

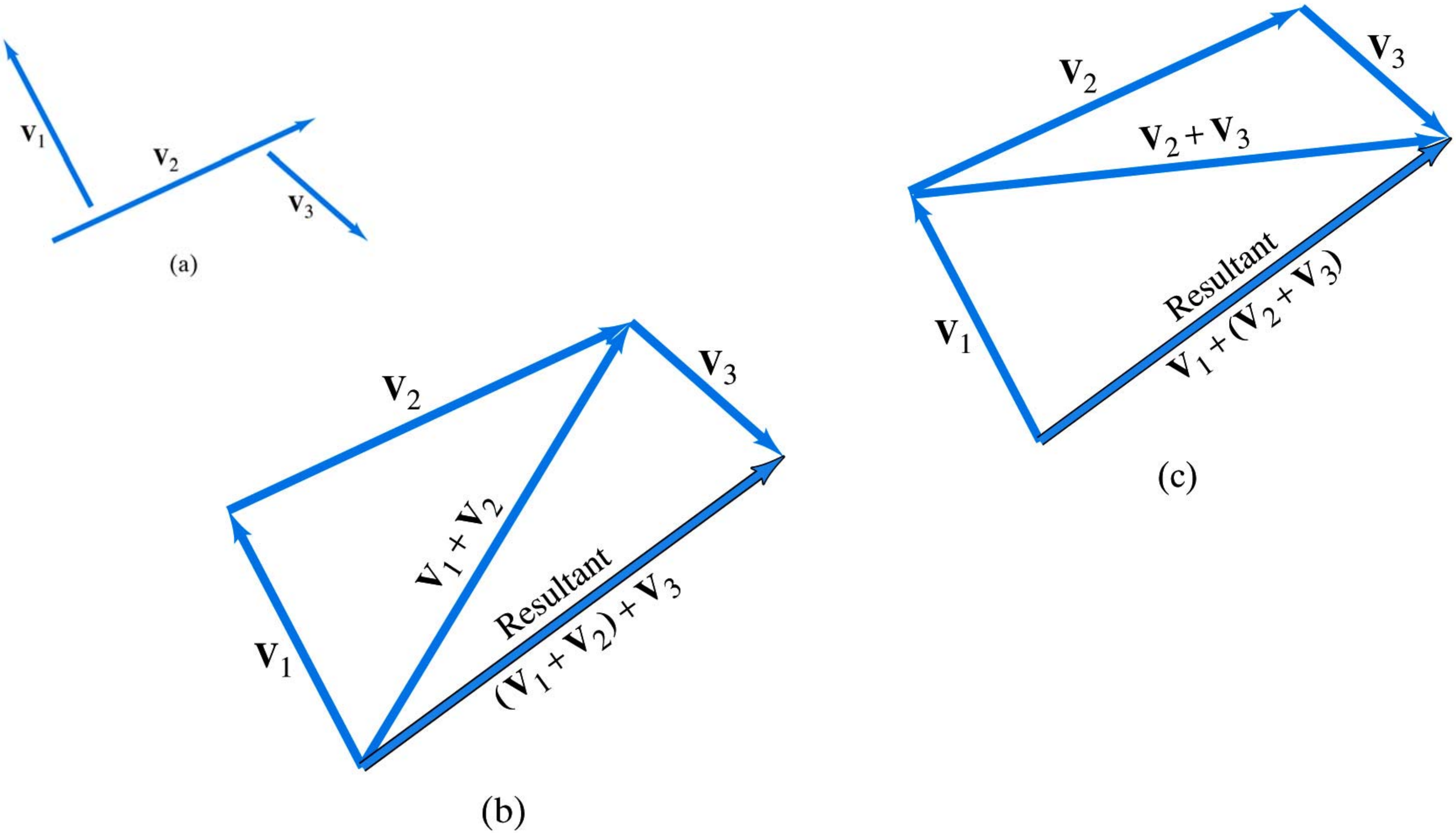
$$V_1 + V_2 = V_2 + V_1$$





Vectors are Associative

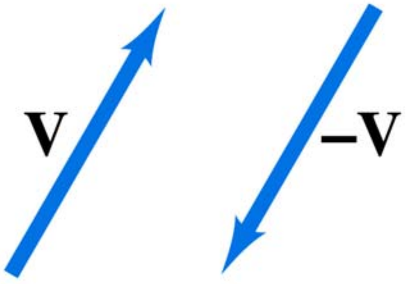
$$V_1+(V_2+V_3)= (V_1+V_2) +V_3$$

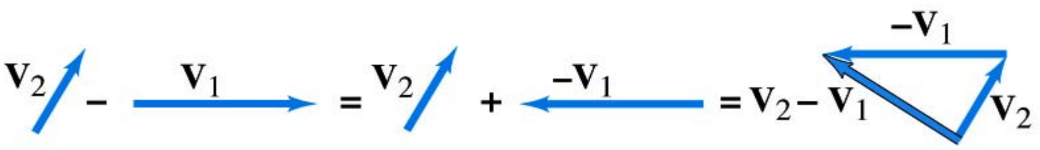


Vector Subtraction

- Actually just addition in disguise
- First need to discuss the negative of a vector, that is going from: $\vec{V} \rightarrow -\vec{V}$
- Note this doesn't change the magnitude of the vector - just the direction
- Subtraction is the addition of a negative:

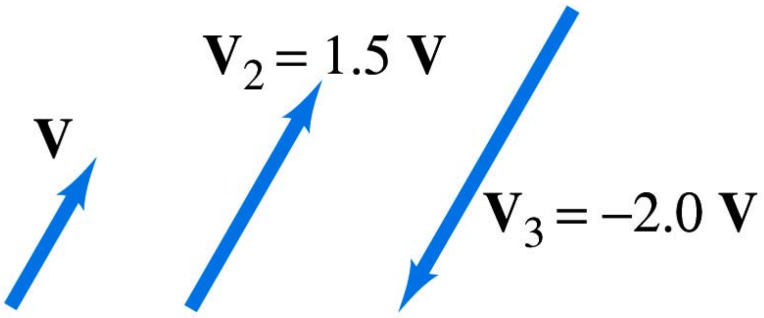
$$\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1)$$





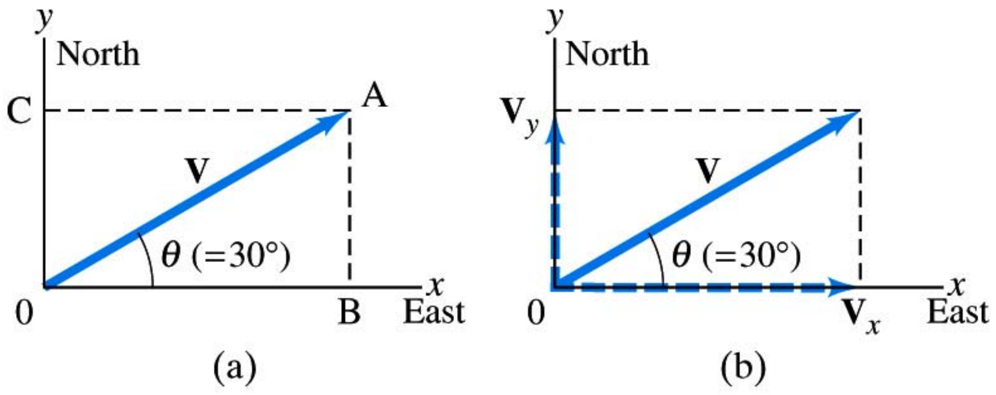
Scalar Multiplication

- Multiplication by a positive factor, c, changes magnitude from V to cV but does not change the direction.
- If c is negative the magnitude changes from V to cV, and the direction also changes.



Resolving Vectors into Components

- To really go any further and prepare for 2-D motion we'll need a much more powerful, exact algebraic approach to vector manipulation.
- Vector addition shows that any vector can be expressed as a sum of two other vectors commonly called "components".
- The key is to choose these components along two perpendicular axes.



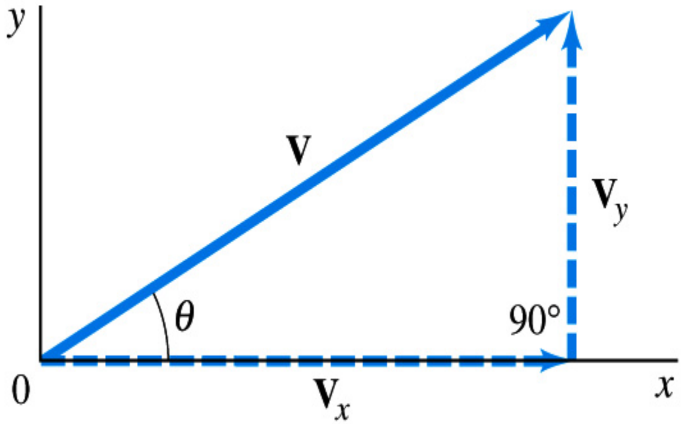
- In this case we resolve the vector $\vec{V}$ into perp. components along the x and y axes:

$$\vec{V} = \vec{V}_x + \vec{V}_y$$

- Easily generalizes into three or more dimensions.

Magnitude of Components.

- Once a coordinate system is established:
  - Given a vector's length and it's angle with respect to an axis, trig can be used to find the magnitude of the perpendicular components
  - Likewise given the magnitude of the components, the components, the Pythagorean theorem and trig identifies the vector



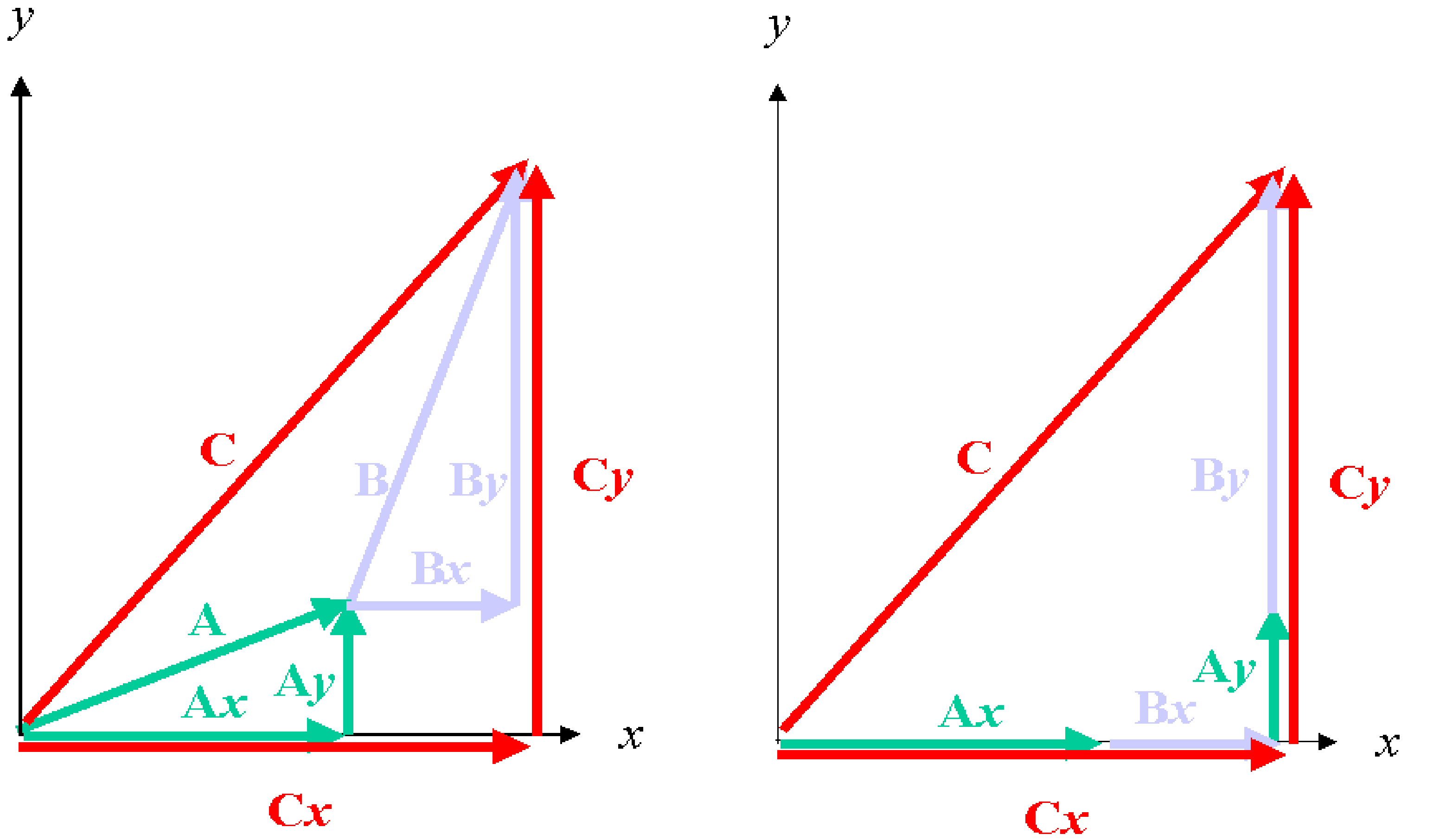
$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

Graphically Adding Vectors A and B by Components to Derive C



Bottom Line:

$\vec{C} = \vec{A} + \vec{B}$

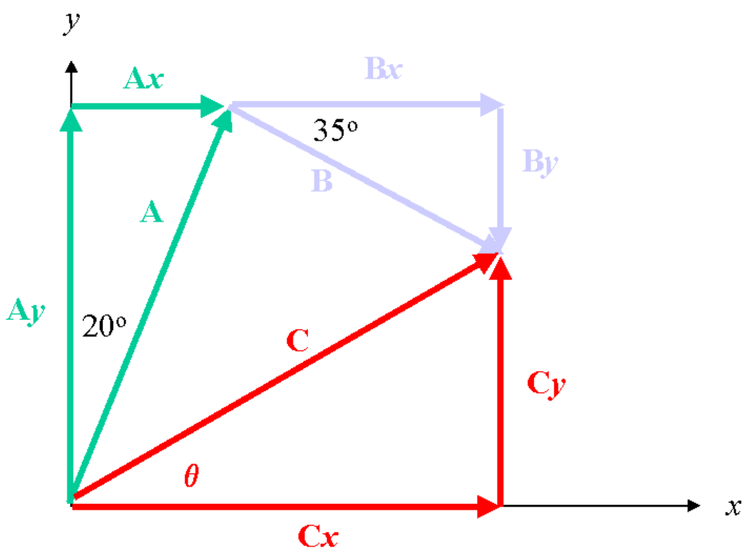
$C_x = A_x + B_x$ and $C_y = A_y + B_y$

Prescriptively Adding Vectors By Components

1. Draw the situation.
2. Find a convenient and perpendicular set of x and y coordinate axes.
3. Resolve each vector into x and y components
4. Calculate the component of each vector. Keep track of signs!
5. Add the x components and add the y components (DON'T MIX!)
6. Calculate final vector magnitude and direction.

An example: taking a jog.

- A jogger runs 145m 20 degrees east of north and then 105 m 35 degrees south of east. Determine her final displacement vector.
- Our first two steps are to draw the situation with convenient axes:



- It's easy to see that Cx=Ax+Bx and Cy=Ay+By, but next we need to resolve the components quantitatively.
- Followed by addition of the independent components.

Vector	X component	Y component
A	$A_x = 145m*\sin 20^o = 49.6m$	$A_y = 145m*\cos 20^o = 136m$
B	$B_x = 105m*\cos 35^o = 86.0m$	$B_y = -105m * \sin 35^o = -60.2m$
C	$C_x = A_x+B_x = 135.6m$	$C_y = A_y+B_y = 76m$

- Note that B_y decreases the "y-position" and so it is negative.

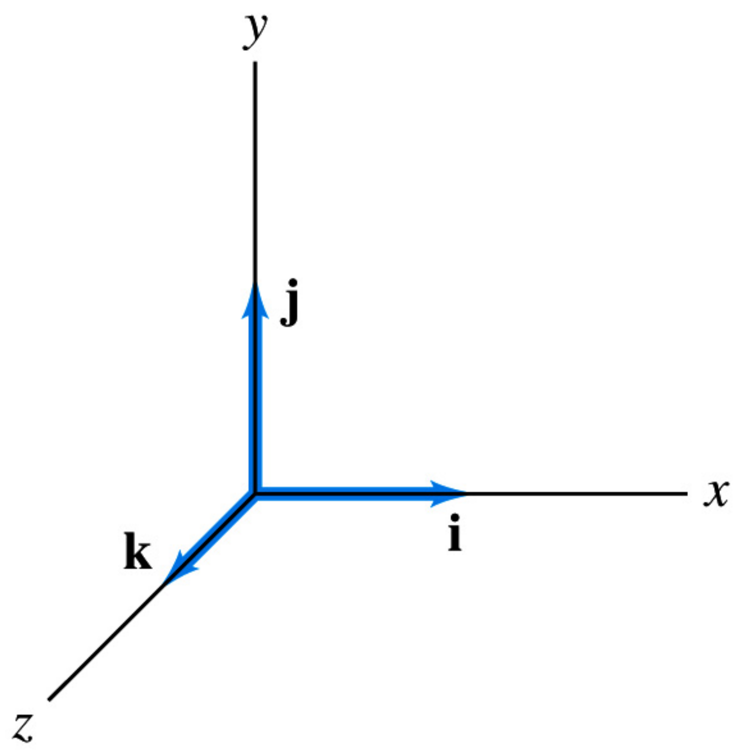
- We can finish up by calculating the magnitude and direction of the resultant vector

$$C = \sqrt{(C_x^2 + C_y^2)} = \sqrt{(134.6m)^2 + (76m)^2} = 155m$$

$$\theta = \tan^{-1}(C_y / C_x) = \tan^{-1}(76m / 135.6m) = 29^0$$

Unit Vectors

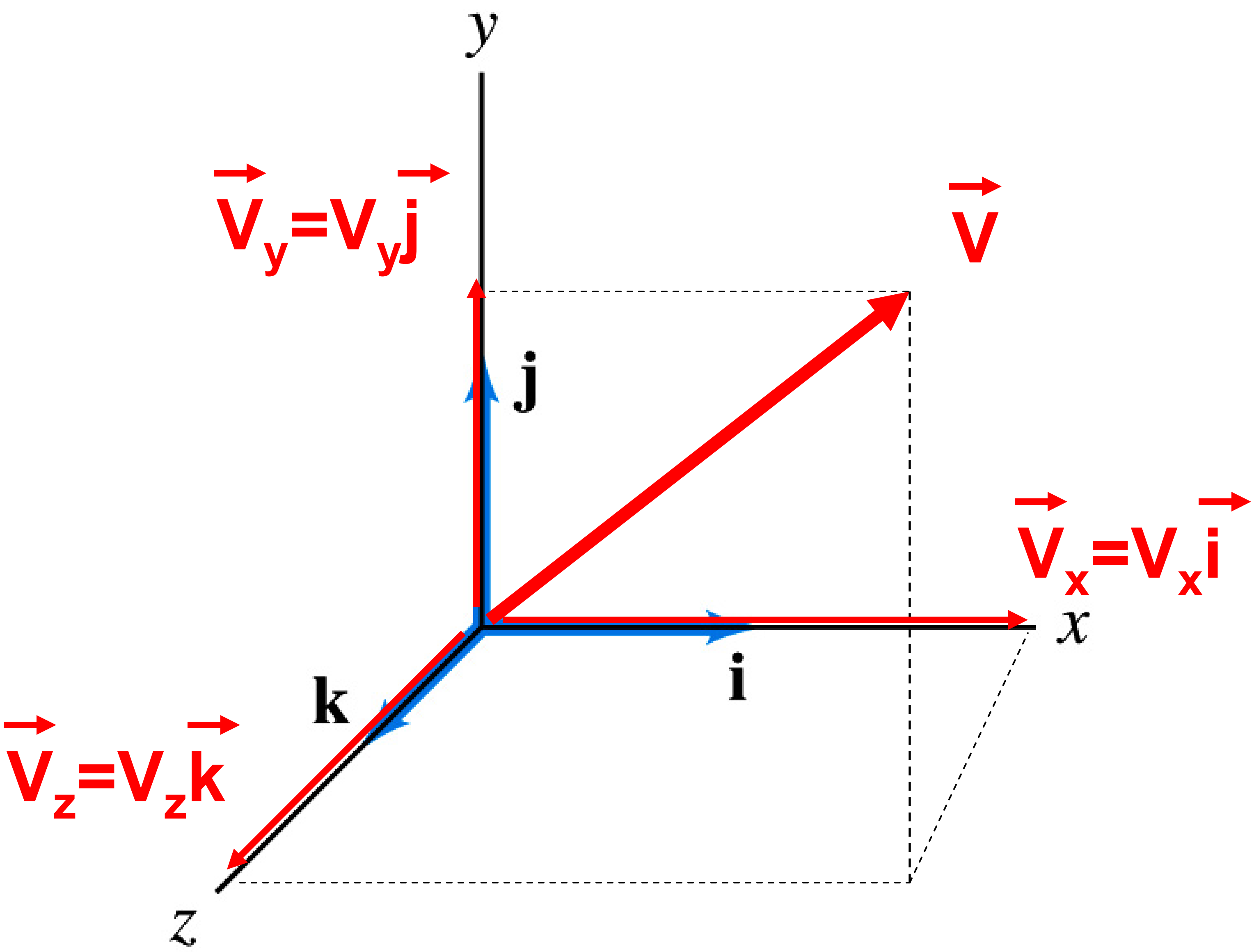
- A nifty device or convention simplifying treatment of vectors.
- Properties:
  - Magnitude equal to 1 or unity
  - Usually perpendicular and point along the coordinate axes
  - Commonly named $\vec{i}, \vec{j}, \vec{k}$ and point along the x, y, z axes, respectively.



Utility of Unit Vectors

- Any vector can be broken down into component vectors.
- Any vector can be re-expressed as a scalar times a vector in the same direction
- Thus, in general

$$\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$$



$$\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$$

Power of Unit Vectors

- Now it's a snap to add and subtract vectors!
- One just adds the coefficients of the units vectors.
- Later on we'll learn about other key operations that are facilitated by unit vectors such as dot and cross products.

$$\vec{V} = \vec{V}_1 + \vec{V}_2$$

$$= V_{1x}\,\vec{i} + V_{1y}\,\vec{j} + V_{2x}\,\vec{i} + V_{2y}\,\vec{j}$$

$$= (V_{1x} + V_{2x})\,\vec{i} + (V_{1y} + V_{2y})\,\vec{j}$$

$$= V_x\,\vec{i} + V_y\,\vec{j}$$

or

$$V_x = V_{1x} + V_{2x} \text{ and } V_y = V_{1y} + V_{2y}$$

Example: Vector Addition

- Remember the jogger?
- We can now quickly write the displacement vectors in terms of unit vectors quickly and do addition.

$$\vec{A} = 49.6m\ \vec{i} + 136m\ \vec{j}$$

$$\vec{B} = 86.0m\ \vec{i} - 60.2.m\ \vec{j}$$

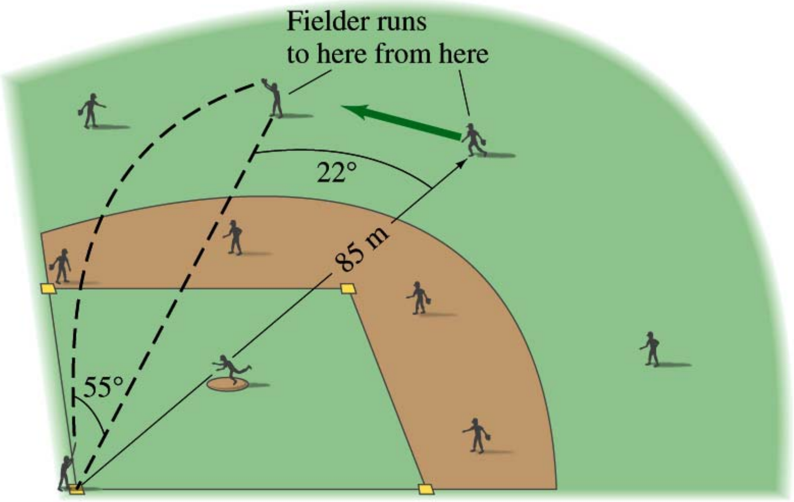
$$\vec{C} = 135.6m\ \vec{i} + 76m\ \vec{j}$$

so the components of $\vec{\mathrm{C}}$ are

$$C_x = 135.6m \text{ and } C_y = 76m$$

To sum it up…

- Well now we've got a good handle on vectors! Thanks for your patience.
- You may not have noticed but the treatment presages an important physical observation.
- The vector components can be treated independently!
- Likewise motion in perpendicular directions is independent
- This all leads to projectile motion.



http://www.lon-capa.org/~mmp/kap3/cd060.htm

http://webphysics.davidson.edu/course_material/py130/demo/illustration2_4.html