Status: Unit 1, Chapter 2

- ✓ Reference Frames and Displacement
- ✓ Average Velocity
- ✓ Instantaneous Velocity
- ✓ Acceleration
- ✓ Motion of Constant Acceleration
- Solving Problems
  - Outline standard approach to problem solving
  - Practice → physical insight

Structure of the Four Equations (assuming $x_0=v_0=0$)

$$v = v_0 + at \rightarrow v = at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \rightarrow x = \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v^2 = 2ax$$

$$\bar{v} = \frac{v_0 + v}{2}$$

Relate velocity, acceleration, time

Relate position, acceleration, time

Relate velocity, acceleration, position

Definition average velocity

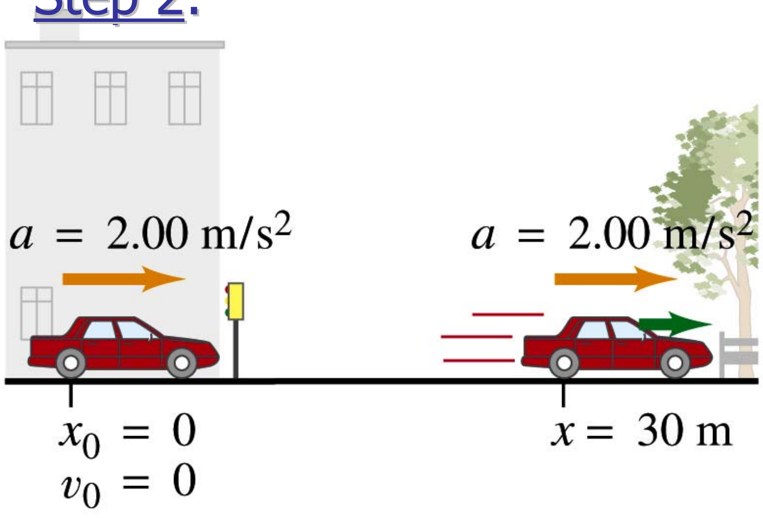
The "8 Step" Way

- Setup
  1. Read & access the problem
  2. Draw a diagram with coordinate system, choose + and - directions
  3. Create a table of known and unknown quantities
- Selection and Manipulation of Equations
  4. Consider the physics involved
  5. Select appropriate equation or equations
  6. Do required calculations
- Check your work
  7. Does it pass the smell test
  8. Do the units make sense?

Memorizing these steps = success!

Start w/ an easy one, for the practice…

- Step 1: How long does it take a car to cross a 30.0m intersection after the light turns green, if the acceleration is 2.00 m/s^2?
- Step 2:



- Step 3:

Known	Unknown
$x_0=0$	t = ?
x= 30.0m	
a=2.0m/s^2	
$v_0=0$	

- Step 4: We have distance and acceleration but an unknown time. We need a relation associating the three, which we can solve for the time.

Continuing with the "8-step" way…

- Step 5: The second equation give us our unknown time! Solving for t we get:

$$x = \frac{1}{2}at^2 \rightarrow 2x = at^2 \rightarrow$$

$$\frac{2x}{a} = t^2 \rightarrow t = \sqrt{\frac{2x}{a}}$$

- Step 6: The calculation gives

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2 \times 30.0m}{2.00m/s^2}} =$$

$$\sqrt{30.0\left(\frac{m}{\cancel{m}/s^2}\right)} = \sqrt{30s^2} = +5.48s$$

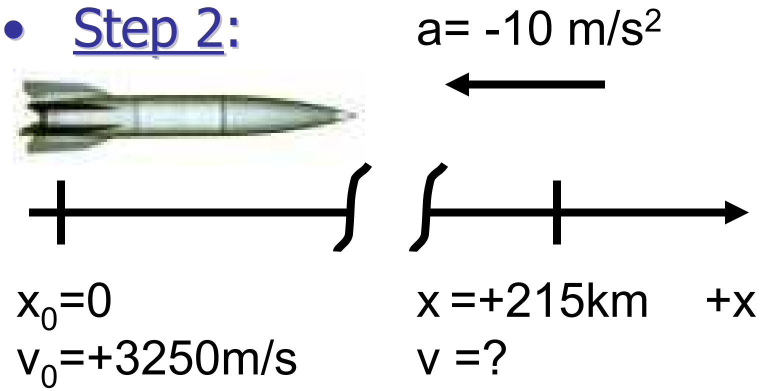
- Step 7: This smells right in that it usually takes a few seconds to get across a large intersection.
- Step 8: Units make sense!

A comment: only the positive root is "physical".

Example 2: A spaceship!

- Step 1: A spaceship is traveling with velocity +3250 m/s. Using retrorockets, the spacecraft begins to slow down with an acceleration of 10 m/s². What is the velocity of the spacecraft when the displacement of the craft is +215 km, relative to the point where the thrust began?

- Step 2:



- Step 3:

Known	Unknown
$x_0=0$	$v=?$
$x=+215km$	
$a=-10 m/s^2$	
$v_0=3250$ m/s	

Example 2: More spaceship

- Step 4: Position and acceleration are known but not final velocity, we need to pick a relationship with the former and solve for the later.
- Step 5: Once again it's the third relationship that cures our ignorance. But this time we use the full relationship

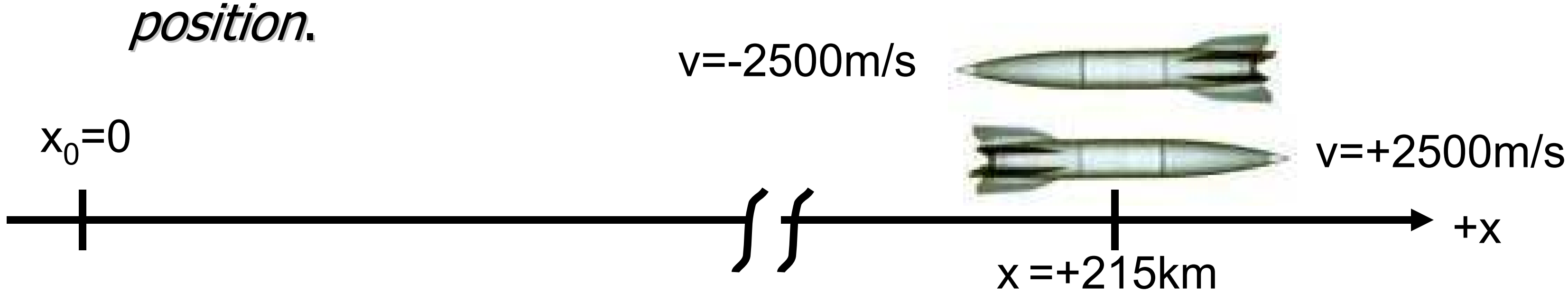
$$v^2 = v_0{}^2 + 2a\left(x - x_0\right)$$

- Step 6: Substituting the known quantities, we find

$$v^2 = v_0{}^2 + 2a\left(x - x_0\right) =$$
$$(3250 m/s)^2 + 2(-10 m/s^2)(215 km)$$
$$= 1.06 \times 10^2 \; m^2/s^2 \text{ - } 4.3 \times 10^6 \; m^2/s^2$$
$$= 6.3 \times 10^2 \; m^2/s^2 \rightarrow$$
$$v = \pm 2500 m/s$$

Example 2: Smart equations

- Step 7 & 8
  - Why two answers, +/- 2500 m/s?
  - Just think of the craft moving along. First it slows until down to +2500 m/s at 215km.
  - Then it turns back or starts moving back to the origin. After a while it reaches a negative velocity of -2500 m/s – *at the same position*.



- Clever how the equation could predict a second scenario even if we hadn't!
- The units certainly work out.

Example 3: Two interrelated objects

- Step 1:
  - A bus has stopped to pick up riders. A women is running at constant velocity of +5.0m/sec in an attempt to catch the bus. When she is 11 m from the bus it pulls away with constant acceleration of 1.0 m/s^2. From this point how much time does it take her to reach the bus?
  - This is rather familiar.
  - You're just behind the bus (11 meters) running to beat the band (5 m/sec) when it starts pulling away.
  - Can she catch it, and if so how long will it take?

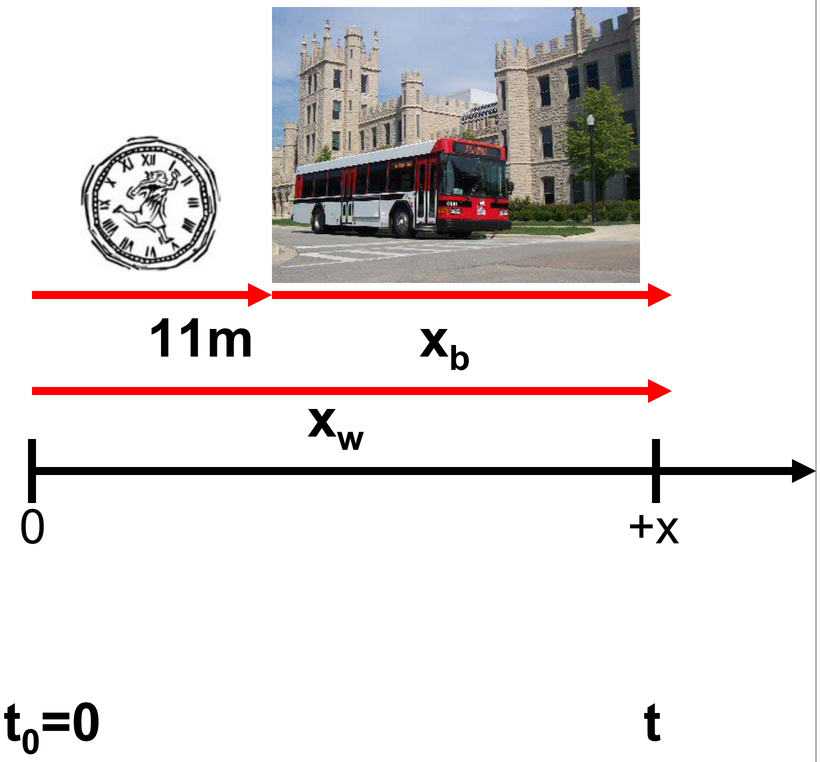
Example 3: Catching a bus

- Step 2: When she catches the bus the distance traveled by the woman, x_w, must equal 11 meters plus the distance traveled by the bus x_b or

  $x_w = x_b + 11m$
- Since she's running at 5m/s the last relation can be written as:

  $x_b + 11m = (5m/s)\ t$
- Ah! We need to find out how far the bus traveled to get the time to catch the bus

Pictorially:



Example 3: Time to catch the bus?

Step 3:

- So, let's look at the data for the bus:

Known	Unknown
x_0= 0 m	x_b=?
v_0= 0 m/s	
a=+1.0m/s^2	
t is related	

- Note that t is the same for the woman and the bus since they both travel the same time interval.

Step 4:

- The problem seems hopeless. If we refer to our list of four equations none of them will give us x_b - they all need two inputs to solve for the unknown.
- When this happens one must find some condition relating one of the unknown variables to the known variables, and combine the equations to eliminate an unknown.

Step 5:

- In this case that would be the two equations:

$$x_b + 11m = (5m/s)\ t$$

$$x_b = (1/2)at^2$$

- There we go! we can just substitute the second into the first and we earn the quadratic equation:

$$(1/2)at^2 + 11m = (5m/s)t$$

- Or in the usual form

$$0.5at^2 - (5m/s)t + 11m = 0$$

- Step 6:
- This has two solutions

$t = 3.3$ s and $t = 6.7$ s

- These are easily explained: First she catches the bus as expected. But then she overruns the bus. As it accelerates it will overtake her a few seconds later. Our answer is 3.3 s.
- Once again, predictive!

Example 3: Checking the bus

- Step 7: The total distance traveled by the woman can now be checked with the second equation:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow$$

$$x = (5.0 m/s)(3.3 s)$$

$$x = 16.5 m$$

Which is reasonable!

- Step 8: And the units certainly check.

Example 4: Segments.

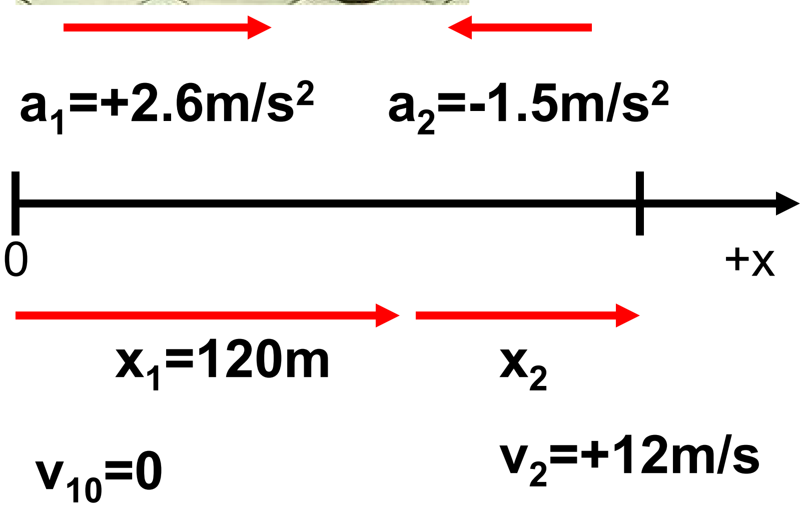
- Step 1: A motorcycle starting from rest has an acceleration of +2.6 m/s^2. After the motorcycle has traveled a distance of 120 m, it slows down with an acceleration of -1.5m/s^2, until its velocity is 12 m/s. What is the total displacement of the motorcycle?

- Well actually we have two problems here, the first segment has positive acceleration and the second negative acceleration.
- We will start by treating each separately but you'll see they are connected.

Example 4: Segment setup.

- Step 2:

1^{st} Cycle: 1885 Daimler



Step 3: Two tables

Seg1: Known	Unknown
x_{10}= 0 m	v_1=?
v_{10}= 0 m/s	t_1=?
a_1=+2.6m/s^2	
x_1= 120 m	

Seg2: Known	Unknown
x_{20}= 120 m	x_2=?
v_2= 12 m/s	t_2=?
a_2=-1.5m/s^2	v_{20}=?

Example 4: Making the connections.

- Step 4:
  - A reminder, we are looking for the total displacement or: $x = x_1 + x_2$.
  - We have x_1 and will need to solve for x_2.
  - But examination of the equations will show the data for segment two is insufficient. We need to find more information for segment two.
  - The key is noting that the final velocity of segment one is the initial velocity of segment two or: $v_1 = v_{20}$
  - In other words, we need to go back to segment one, fill out the table, and transfer that information to segment two.
  - Looking at segment one, we know exactly which equation will give us v_1.

Example 4: Matching the segments.

- Step 5 & 6: For segment one

$$v^2 = v_0{}^2 + 2a(x - x_0) \rightarrow$$

$$v_1{}^2 = v_{10}{}^2 + 2a_1 x_1 =$$

$$0 + 2(2.6m^2/s^2)(120m) =$$

$$624m^2/s^2 \rightarrow$$

$$v_1 = 25m/s = v_{20}$$

- And now for segment two we can use the same eq.:

$$v^2 = v_0{}^2 + 2a(x - x_0) \rightarrow$$

$$v_2{}^2 = v_{20}{}^2 + 2a_2 x_2 \rightarrow$$

$$x_2 = (v_2{}^2 - v_{20}{}^2)/2a_2 \rightarrow$$

$$x_2 = \frac{(12m/s)^2 - (25m/s)^2}{2(-1.5m/s^2)}$$

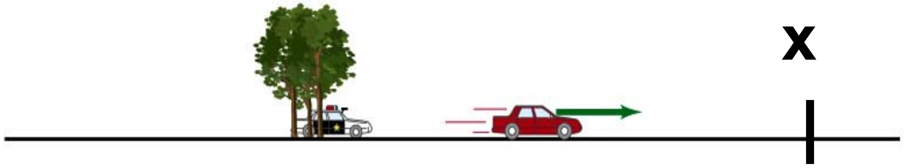
$$x_2 = 160m$$

Example 4: Finishing up

- Step 6:
  - $x = x_1 + x_2 = 120m+160m = 280\ m$
- Step 7:
  - Since the accelerations and velocities for each segment are the same order of magnitude, it's reasonable for this to be true for the distances.
- Step 8:
  - Units checked just fine.

Example 5: More than one object.

- Step 1: A car speeding at a constant 80 mi/hr (36m/s) passes a police cruiser at rest. Assuming the police car gives chase at a constant acceleration of 10 mi/h-s (4.5 m/s^2), what will be the speed of the police car when it overtakes the speeder? Is this reasonable?
- Step 2: In a sense there are two problems here, one dealing with the equation of motion for the offender and one dealing with the equation of motion of the enforcer. A sketch helps a great deal



Citizen:

$X_{0C}=0, V_C=36m/s, a_C=0,$

Law:

$X_{0L}=0, V_{0L}=0m/s, a_L=4.5m/s$

- Step 3: Two tables

Citizen:	Unknown
x_{0C}= 0 m	x_C=?
v_C= 36 m/s	t_C=?
a_C=0 m/s^2	

Law: Known	Unknown
x_{0L}= 0 m	x_L=?
v_{0L}= 0 m/s	t_L=?
a_L = 4.5 m/s^2	

- Step 4: Each table seems to lack sufficient information until we set the unknown final distances and times equal! That is they will "cross paths" at the same time and place.

- Steps 5 and 6: Setting the two positions equal

$$x_C = x_L \rightarrow$$

$$x_{0C} + v_{0C}t + \frac{1}{2}a_C t^2 = x_{0L} + v_{0L}t + \frac{1}{2}a_L t^2 \rightarrow$$

$$v_{0C}t = \frac{1}{2}a_L t^2 \rightarrow$$

$$v_{0C} = \frac{1}{2}a_L t \rightarrow$$

$$t = \frac{2v_{0C}}{a_L} \rightarrow$$

$$t = \frac{2 \times 36m/s}{4.5m/s^2} = 16s$$

- Note:
  - in this case we simultaneously solved the equations of motion!
  - Also t=0 is a valid answer.

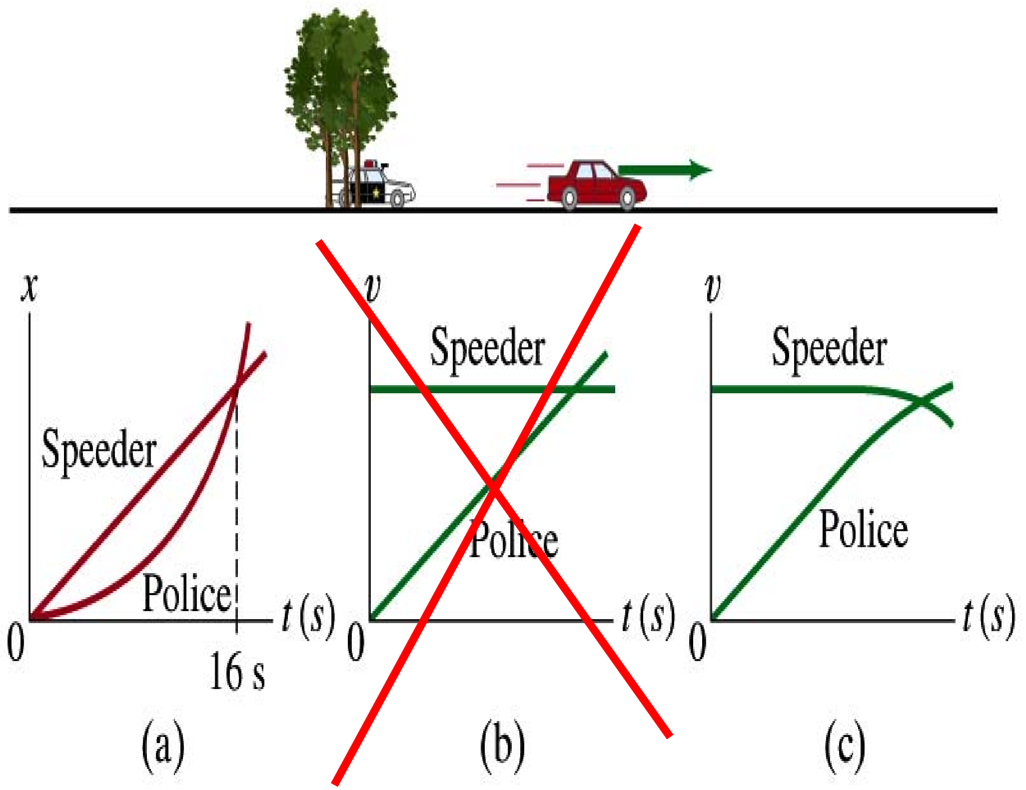
- Steps 7 and 8: Units are certainly correct. To check the reasonableness of the elapsed time we can see how fast the policeman is going after 16 sec:

$$v_L = v_{0L} + a_L t \rightarrow$$

$$v_L = 0 + 4.5 m/s^2 \times 16 s \rightarrow$$

$$v_L = 72 m/s$$

- Which converts to 160 m/hr! A bit fast, likely the cop would instinctively choose a smaller acceleration and also slow down at approach



Some strategic hints for Steps 5 and 6!

- The last three examples showed the impact of additional constraints and relationships between segments, these modify eight-steps.
- Most common case: If sufficient variables are provided find the equation that provides the missing variable.
  - Interrelated: If two objects share a common variable then only two variables need to be specified for each object. The additional equation offers an additional constraint!
  - Segments: Remember if segments are involved the data may be transferred or shared.
- Look for a possible second solution.

Finishing up

- These examples are not all from the book. Please be sure you understand those in the text as well!
- Two things you need to memorize
  – The four equations (or derive!)
  – The modified eight step strategy

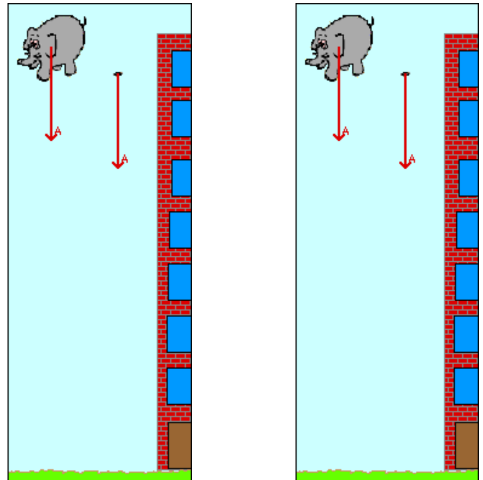
$$v = v_0 + at \rightarrow v = at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow x = \frac{1}{2} a t^2$$

$$v^2 = v_0{}^2 + 2a(x - x_0) \rightarrow v^2 = 2ax$$

$$\bar{v} = \frac{v_0 + v}{2}$$

- Setup
  1. Read & access the problem
  2. Draw a diagram with coordinate system, choose + and - directions
  3. Create a table of known and unknown quantities
- Manipulation of Equations
  4. Consider the physics involved
  5. Select appropriate equation or equations
  6. Do required calculations
- Check you work
  7. Does it pass the smell test
  8. Do the units make sense?



Next lesson we'll be considering objects falling with constant acceleration from the earth's gravitational field.

This was one of Galileo's principal contributions: imagine arguing that a feather falls as fast as an elephant! Even today scientists often receive ridicule, for instance the global warming warnings from the climate scientists.