Frequent Questions

- www site is at:

http://nicadd.niu.edu/people/blazey/Physics_253_2006/Physics_253_2006.htm

- Problems, extra credit all due at the time of the test. Next test is Feb 7th, but is in your interests to keep up!

Tips on how to do well redux!

I. Read the material before lectures

II. Listen in class, take notes sparingly

III. Do the problems, these are key to understanding

IV. Read the material again after doing problems, this locks down the concepts

V. Take advantage of the extra credit

VI. If you don’t understand, ask!

How is lim(Δt→0) Δx/Δt a derivative?

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} =$$

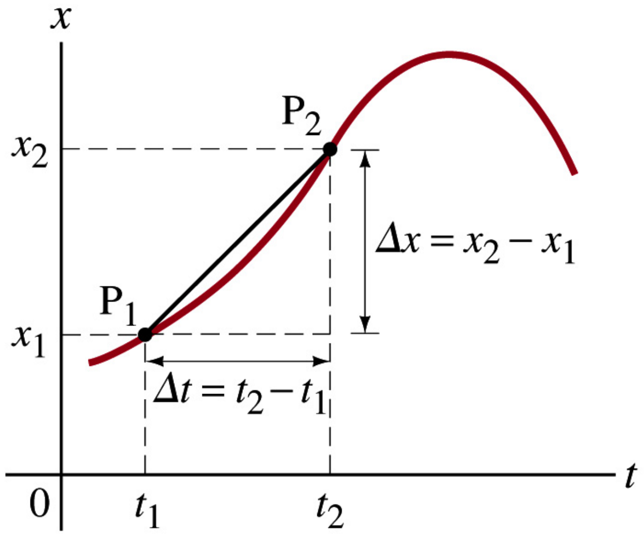
$$\lim_{\Delta t \to 0} \frac{x_2 - x_1}{\Delta t} = \text{but x is just a function of t} =$$

$$\lim_{\Delta t \to 0} \frac{f(t_2) - f(t_1)}{\Delta t} =$$

$$\lim_{\Delta t \to 0} \frac{f(t_1 + \Delta t) - f(t_1)}{\Delta t} =$$

$$\lim_{h \to 0} \frac{f(t_1 + h) - f(t_1)}{h} =$$

$$\frac{df}{dt} = \text{but f is just x as a function of time} = \frac{dx}{dt}$$



Status: Unit 1, Chapter 2

- ✓ Reference Frames and Displacement
- ✓ Average Velocity
- ✓ Instantaneous Velocity
- Acceleration
  - Extending our tool kit to include change of velocity with respect to time
- Motion of Constant Acceleration
  - A mathematical description of simple linear motion
  - Contains many of the elements necessary to describe the kinematic world!
- Solving Problems
  - Technique applies to many arenas
  - Shows predictive aspect of physics.

Acceleration

- Acceleration is a quantitative statement about how fast the velocity of an object is changing. Complete analog to velocity
  - Instead of change of distance with respect to to time: $$v = \frac{dx}{dt}$$
  - We consider change of velocity with respect to time: $$a = \frac{dv}{dt}$$
- The equation says it all: an object with changing velocity is accelerating.

Average and Instantaneous Acceleration

- Average Acceleration
- Change in velocity divided by the time taken for the change

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- A vector, for 1-D motion indicated by a plus or minus sign.

- Instantaneous Accel.
- Limiting value of the ave. accel. as time interval goes to zero.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- Again a derivative: this time of the velocity wrt to time!

Example 1: Average Acceleration

- A car accelerates along Lincoln Highway from rest to 75m/hr in 5.0s. What is the average acceleration?
- Plug-and-chug!

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} =$$

$$\frac{75km/hr - 0km/hr}{5.0s} =$$

$$\frac{75}{5}\frac{km/hr}{s} =$$

$$15\frac{km/hr}{s}$$

- Which reads:
  "15 km/hr per second"
- Or on average the velocity changes 15 km/hr every second.
- Which is a statement of how fast the velocity changes with time.
- The mixed units are non standard much prefer

$$\left(15\frac{km/hr}{s}\right)\left(\frac{1000m}{km}\right)\left(\frac{1hr}{3600s}\right) =$$

$$(15\times1000/3600)\times\left(\frac{km/hr\times m\times hr}{s\times km\times s}\right) =$$

$$4.2\frac{m}{s^2}$$

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$$(15 \times 1000/3600) \times \left(\frac{km/hr \times m \times hr}{s \times km \times s}\right)$$

- We'll see later if you'd get a ticket

Example 2: Average Acceleration

- An auto is moving to the right (+x) and the brakes are applied. If the initial velocity is 15.0 m/s and 5.0 s are required to slow down to 5.0 m/s what is the average acceleration?

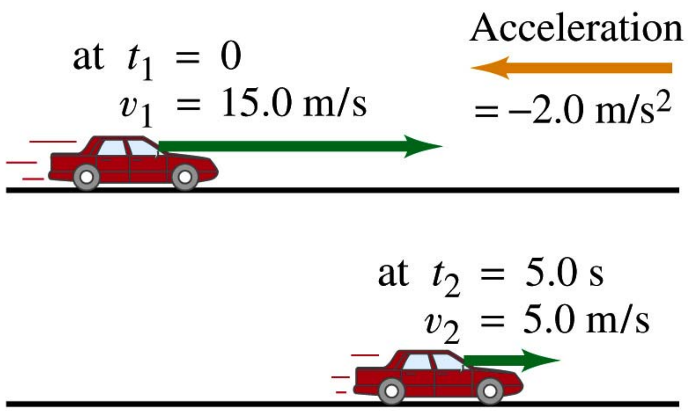
$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} =$$

$$\frac{5.0m/s - 15m/s}{5.0s - 0s} =$$

$$\left(\frac{5.0 - 15.0}{5.0}\right) \times \left(\frac{m/s}{s}\right) =$$

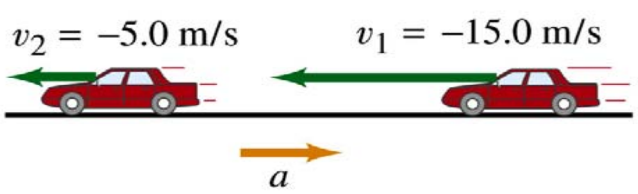
$$-2.0\frac{m}{s^2}$$

- Note the negative sign appears because the final velocity is less than the initial velocity.
- The green arrows or vectors indicate velocity and the orange acceleration.



Three comments on deceleration

- First, spelled with one "c".
- Second, occurs whenever an object is slowing down.
- Third, occurs whenever velocity and acceleration point in opposite directions
- Consider, the following observations 5 seconds apart



$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{-5.0m/s - (-15m/s)}{5.0s - 0s}$$

$$= \left(\frac{5.0 + 15.0}{5.0}\right) \times \left(\frac{m/s}{s}\right) = +2.0\frac{m}{s^2}$$

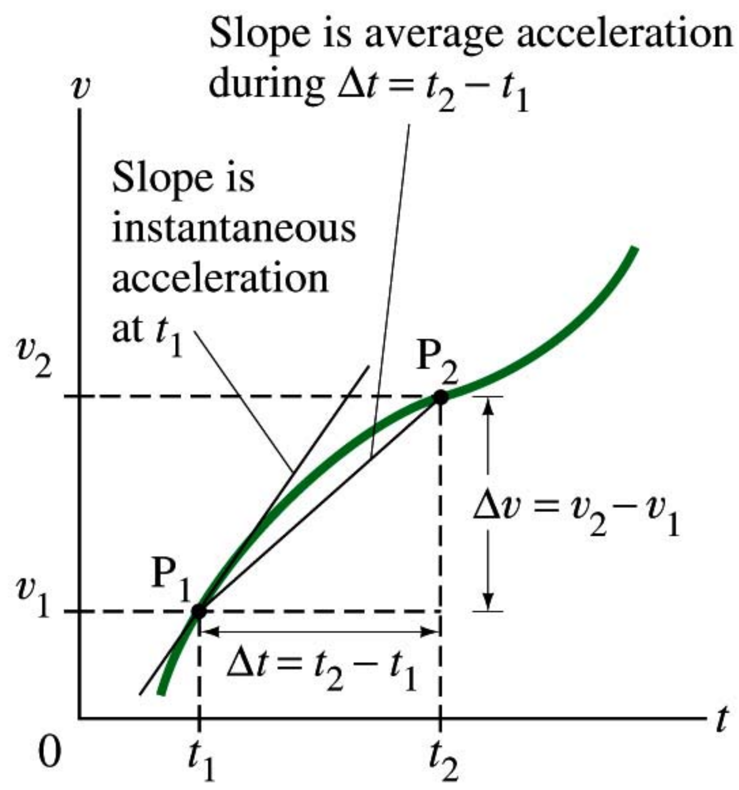
- Even though the car is moving to the left because it's slowing down it has positive acceleration

Instantaneous Acceleration

- Defined as Limiting value of the ave. accel. as time interval goes to zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

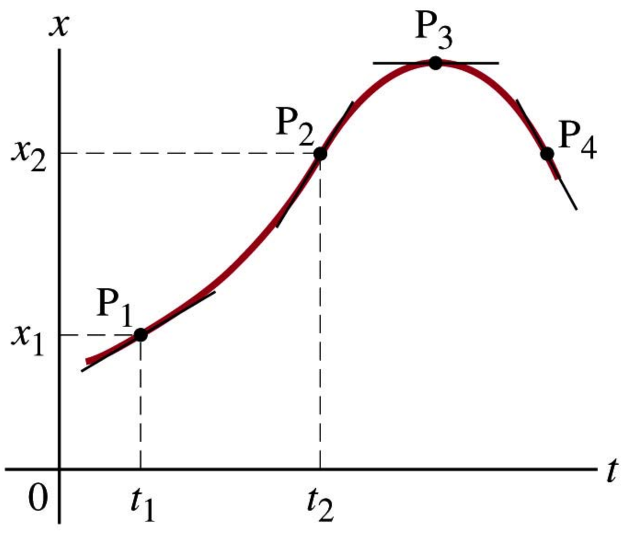
- Average acceleration is just the slope of the line connecting the points in the graph.
- As the time interval vanishes the line becomes the tangent to the curve.



Instantaneous Acceleration

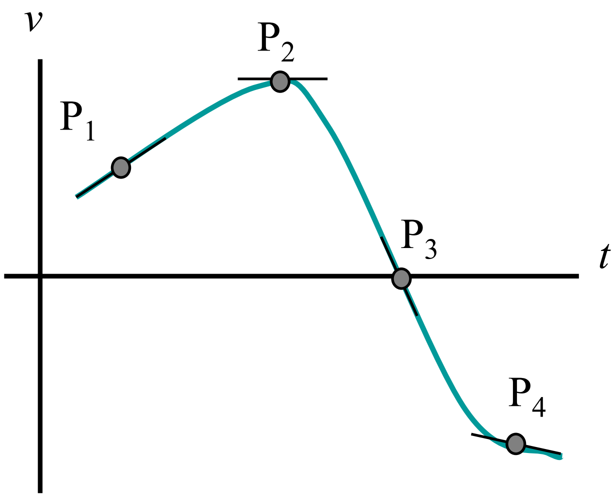
- Recall velocity was the derivative of position wrt time

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



- Similarly acceleration is the derivative of velocity wrt time

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$



Acceleration as a second derivative

$$x = x(t)$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(\frac{dx}{dt}) = \frac{d^2 x}{dt^2}$$

Example: Acceleration

- A particle position along a straight line is given by:

  $x = (2.10\ m/s^2)t^2 + 2.80\ m.$
- Calculate its
  - average acceleration between $t_1 = 3.00$ s and $t_2 = 5.00$
  - instantaneous acceleration as a function of time.

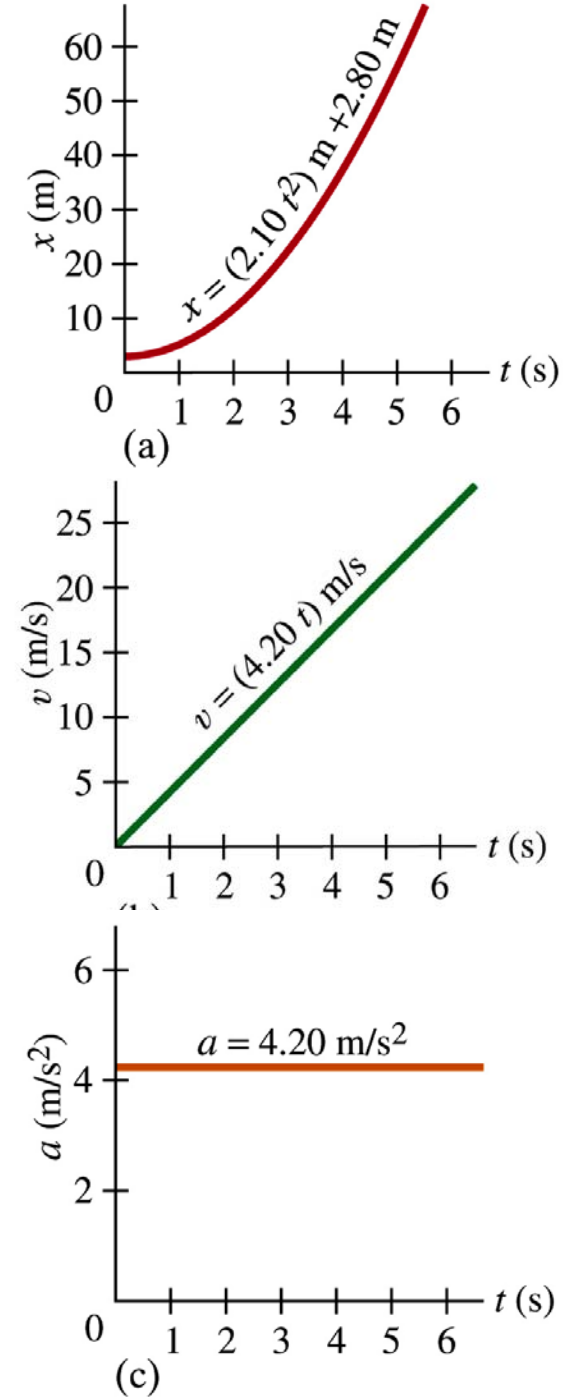
- The velocity at time t is given by $v = dx/dt = (4.20\ m/s^2)t$, thus
  - At $t_1 = 3.00s$, $v = (4.20 m/s^2)(3.0s) = 12.6$ m/s
  - At $t_2 = 5.00s$, $v = (4.20 m/s^2)(5.0s) = 21.0$ m/s

And, the average acceleration,

$$\bar{a} = \frac{21.0m/s - 12.6m/s}{5.0s - 3.0s} = +4.20\frac{m}{s^2}$$

- Since velocity is
  - $v = dx/dt = (4.20\ m/s^2)t$

  the acceleration is
  - $a = dv/dt = 4.20\ m/s^2$
- Note the acceleration is constant and has no time dependence.
- The three plots at the left show the time dependence of x, v and a.



Average Speed
Distance/Elapsed Time

Velocity

- Average Velocity (Displacement/Elapsed Time)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- Instantaneous Velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Acceleration

- Average Acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

- Instantaneous Accel.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Motion at Constant Acceleration

- Ok now that we've got general (and actually quite rigorous) definitions of displacement, velocity, and acceleration we can derive equations of motion.
- Initially we'll limit ourselves to constant acceleration, although not general, the resulting equations of motion are still extremely useful.
- For instance near the surface of earth the acceleration of gravity can be taken as a constant, thus we can easily describe the basic kinematics of falling objects.
- We'll begin with a derivation of 1-dimensional equations and then move onto illustrative problems and later 2-dimensions.

The Setup

- Assumptions:
  - Motion in a straight line
  - Constant acceleration → average and instantaneous acceleration are equal.
- Notation:
  - Set initial time to zero.
    - That is, $t_1 = 0$.
  - Start with "t-naught" not "t-one"
    - $x_1 \rightarrow x_0$ and $v_1 \rightarrow v_0$
  - And we can simplify notation
    - $t_2 \rightarrow t$, $x_2 \rightarrow x$, and $v_2 \rightarrow v$

Starting Point

- As a result

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

- And

$$\bar{a} = a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t}$$

Since acceleration is assumed constant

First Equation

- The last result can be easily rewritten:

$$a = \frac{v - v_0}{t} \rightarrow$$

$$at = v - v_0 \rightarrow$$

$$at + v_0 = v \rightarrow$$

$$\boxed{v = v_0 + at}$$

- Which gives the velocity of an object after a given time and constant acceleration

- Remember accelerating along Lincoln Highway from rest to 75 m/hr in 5.0 s? Would you get a ticket?
- We determined the average acceleration was 4.2 m/s^2.
- Assuming the acceleration was constant

  v = 0 + (4.2 m/s^2)(5.0 s)
  = 0 + 21.0 m/s
  = 21.0 m/s

  Converting to mph we see
  v = (21.0m/s)(3600s/hr)
  (1km/1000m)(0.6km/mi)=
  45.4 mi/hr

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  = 0 + 21.0 m/s
  = 21.0 m/s

  Converting to mph we see
  v = (21.0m/s)(3600s/hr)(1km/1000m)(0.6km/mi)= 45.4 mi/hr

BUSTED!!!

Second and Third Equations

- Rewriting the first result:

$$\bar{v} = \frac{x - x_0}{t} \rightarrow$$

$$\bar{v}t = x - x_0 \rightarrow$$

$$x = x_0 + \bar{v}t$$

- Not so helpful since it doesn't give us the position in terms of velocity and acceleration. So, substitute for the average velocity with:

$$\bar{v} = \frac{v_0 + v}{2}$$

- And use our first equation of motion to inject acceleration

$$\bar{v} = \frac{v_0 + v}{2} = \frac{v_0 + (v_0 + at)}{2}$$

- And now using this to eliminate the average velocity from the expression for position:

$$x = x_0 + \bar{v}t \rightarrow$$

$$x = x_0 + \left(\frac{v_0 + (v_0 + at)}{2}\right)t \rightarrow$$

$$x = x_0 + \left(\frac{2v_0 t + at^2}{2}\right) \rightarrow$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

- Which gives us position in terms of velocity and acceleration.
- Note the quadratic dependence on time

Fourth Equation

- The three equations derived relate position, velocity, and acceleration as functions of time.
- Useful to relate the three variables alone. To do so we use the three equations just derived to eliminate time.
- Starting with

$$x = x_0 + \bar{v}t$$

- Substituting for the average velocity:

$$x = x_0 + \bar{v}t \rightarrow$$

$$x = x_0 + \left(\frac{v + v_0}{2}\right)t$$

- Substituting for t using

$$v = v_0 + at \rightarrow$$

$$t = \frac{v - v_0}{a}$$

Fourth Equation

- We get

$$x = x_0 + \left(\frac{v + v_0}{2}\right)t \rightarrow$$

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) \rightarrow$$

$$x = x_0 + \frac{v^2 - v_0^{\ 2}}{2a}$$

- Now solving for the square of the velocity

$$x = x_0 + \frac{v^2 - v_0^{\ 2}}{2a} \rightarrow$$

$$x - x_0 = \frac{v^2 - v_0^{\ 2}}{2a} \rightarrow$$

$$2a(x - x_0) = v^2 - v_0^{\ 2} \rightarrow$$

$$\boxed{v^2 = v_0^{\ 2} + 2a(x - x_0)}$$

- Which gives a result independent of time

The Four Equations (constant acceleration):

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^{\ 2} + 2a\left(x - x_0\right)$$

$$\bar{v} = \frac{v_0 + v}{2}$$

Memorize these, put them on your mirror, write a song about them, whatever!

First Example: 1-D Equations

- A jet plane has a takeoff speed of 250 km/h. The plane starts from rest, and has a constant acceleration of 1.25 x 10^4 km/h^2
- What is the length of the runway needed?
- We are given final velocity and acceleration and asked for the distance traveled. The third equation seems appropriate.

- We need to recast the equation, by solving for the distance traveled:

$$v^2 = v_0{}^2 + 2a(x - x_0) \rightarrow$$

$$x - x_0 = (v^2 - v_0{}^2)/2a$$

- Note the initial position and velocity are zero

$$x = (v^2)/2a \rightarrow$$

$$x = \frac{(250 km/h)^2}{2(1.25 \times 10^4 km/hr^2)} \rightarrow$$

$$x = 2.5 km$$

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Passes the smell test as well!

2nd Example: Air bag estimate

- How fast must an airbag inflate to protect a driver within a 1 m crumple zone at a collision of 100 km/hr?
- First lets get everything into SI units:

$$v = 100\frac{km}{hr} \times 1000\frac{m}{km} \times 1\frac{hr}{3600s}$$

$$v = (\frac{100 \times 1000}{3600})(\frac{km}{hr}\frac{m}{km}\frac{hr}{s})$$

$$v = 28m/s$$

- Now we have
  - the initial velocity Vo=28m/s
  - the final velocity V=0
  - The distance traveled x = 1m

- Looking at the equations we see to get time we need accel.

$$v^2 = v_0^{\ 2} + 2a(x - x_0) \rightarrow$$

$$0 = v_0^{\ 2} + 2ax \rightarrow$$

$$a = -\frac{v_0^{\ 2}}{2x} = \frac{(28m/s)^2}{2m} \rightarrow$$

$$a = -390m/s$$

- And a 3rd eq gives the time

$$v = v_0 + at \rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{0 - 28m/s}{-390m/s^2} \rightarrow$$

$$t = \frac{28}{390}(\frac{m/s}{m/s^2}) = 0.07(\frac{1}{1/s}) = 0.07s$$

They do deploy in milliseconds

http://nicadd.niu.edu/people/blazey/Physics_253_2006/Physics_253_2006.htm