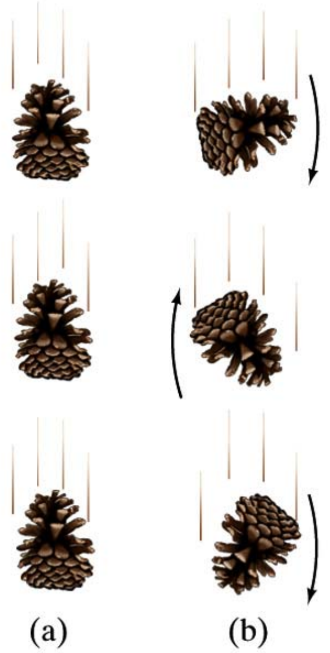
Describing Motion: Kinematics in One Dimension

- Now that we've established our handling of numbers and units, we move onto describing the physical world.
- As usual in physics we start with a simple situation - in this case straight-line translational motion.

On the left the pine cone is falling and undergoing pure translation. Can be described in 1-dimension.

On the right the pine cone is subject to combined translation and rotational motion. Needs 2-D or 3-D for full description.



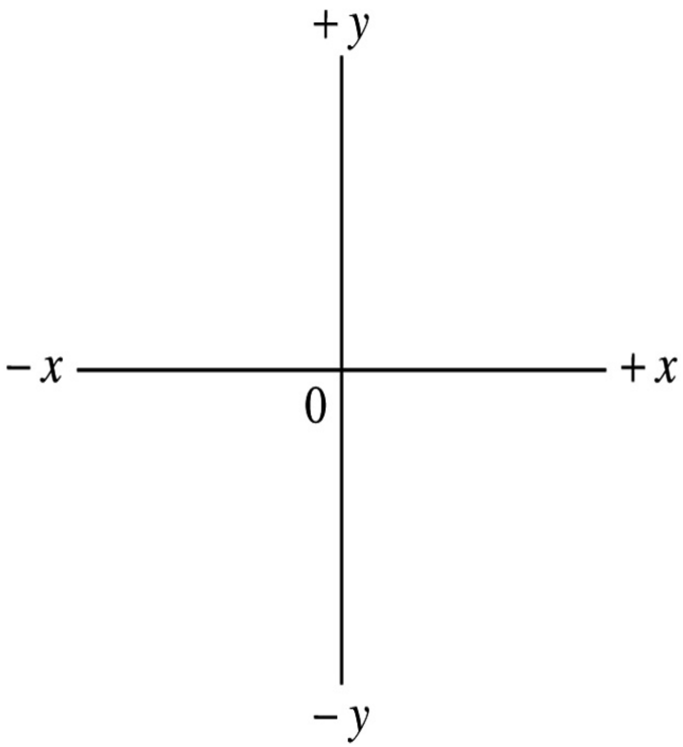
- Some terminology
  - Study of motion of objects and the forces and energies involved is called mechanics.
  - Kinematics involves how objects move: a satellite around the earth.
  - Dynamics involves the origins of movement:
    - the force of gravity causing the satellite to move around the earth or
    - the other three fundamental forces: electromagnetic, weak and strong.
  - We usually consider objects as particles at a mathematical points. This applies to pine cones, people, cars, satellites, planets, and galaxies.
- We will later move onto conceptually equivalent but mathematically more complicated situations
  - Multiple dimensions (not the 11-d string world but 2 and 3-d!)
  - Objects with extent, which requires dealing with rotational motion.

At a distance it seems plausible to consider the galaxies, especially the smaller, as point particles. With calculus it can be shown rigorously.

To fully describe the translational and rotational motion illustrated here we will need to deal with three dimensions of space .

Reference Frames and Displacement

- When you give directions to the local gas station you instinctually refer to a frame (the surface of the earth) with an origin and with directions … "starting here go along this road one mile"
- Formally describing any measurement or motion requires an explicit reference frame for the point particle. The reference frame must have
  - A clear origin
  - Well described directions

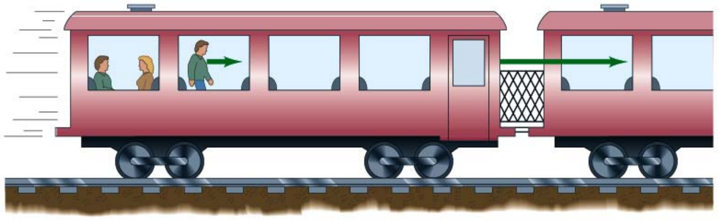


Multiple Reference Frames

- There is no unique reference frame, we live in many
  - The interior of a car or train (not the surface of the earth)
  - Our local patch on the surface of the earth (used for most informal or assumed frames)
  - The center of the earth (not rotating about the axis of the earth)
  - With respect to the center of the sun (not orbiting the sun)

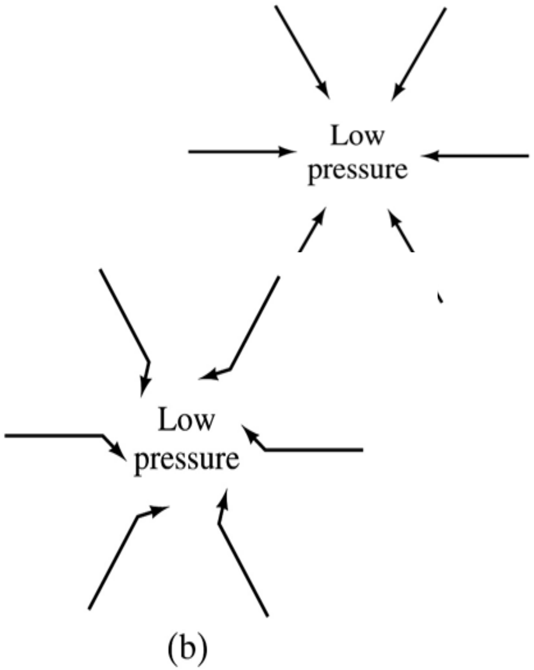
Moving between Reference Frames

- Consider the interior of a train moving at 48 miles/hr with respect to the surface of the earth.
- If you walk down the isle at 3 mi/hr it is with respect to the train.
- But with respect to the surface of the earth your motion is 51 mi/hr.
- It's a trivial example, but clear reference is necessary for any quantitative description



- Actually not fully accounting for the frames of references can lead to interesting artifacts like the Coriolis Effect….

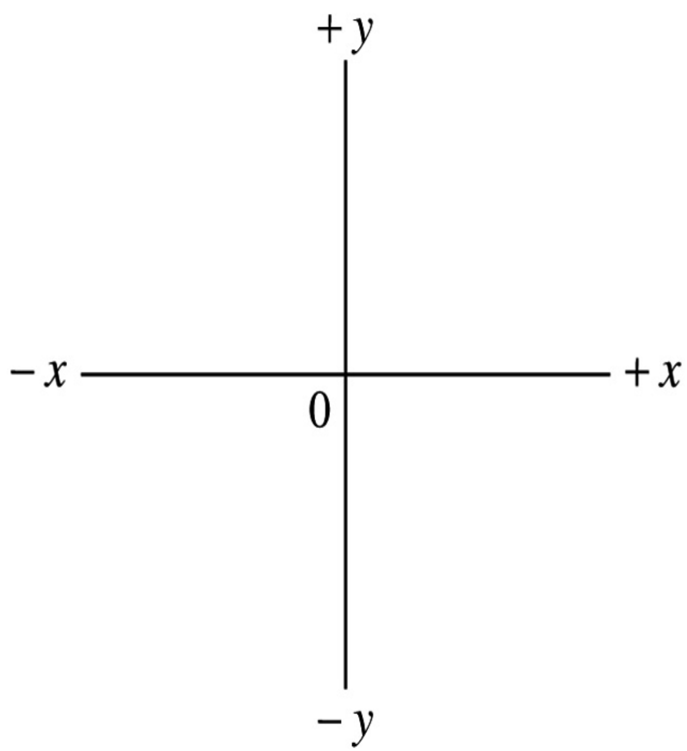
- Apparent cloud motion is greatly influenced by the motion between reference frames
- You might expect clouds to move straight into a low in the northern hemisphere:
- Clouds near the equator are moving faster than near the pole. So the they seem to race ahead as they migrate north to the low:



- There appears to be a mysterious force, called the Coriolis force, but it’s really just due to the relative motion between different reference frames.

A Formal Reference Frame

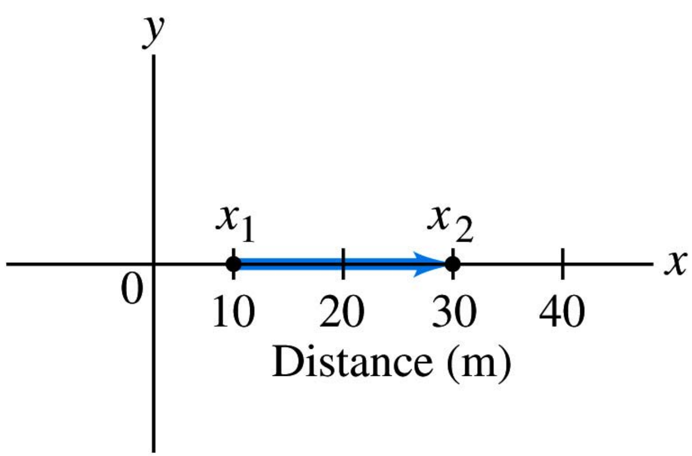
- Uses coordinate axes.
- Places the origin at 0.
- Picks perpendicular directions (important later)
- Usually:
  - Positive or +x to the right
  - Positive or +y up.
- For 1D situations:
  - X axis for horizontal motion
  - Y axis for vertical motions
- 3D: add z axis out of the page.

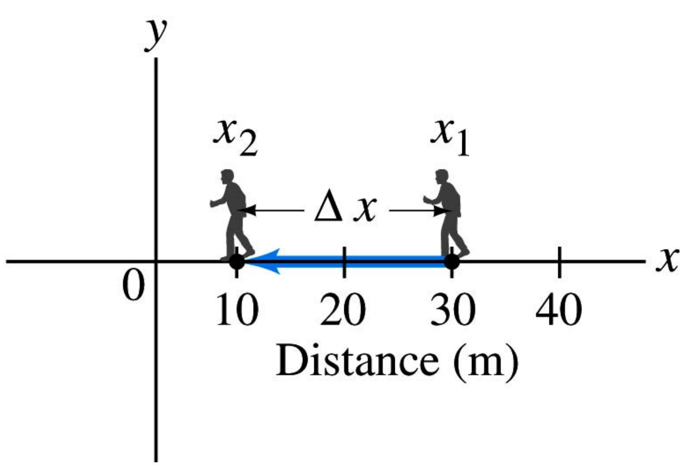


Distance and Displacement

- Distance: a measurement of an object's total travel.
- Displacement: a measurement of an objects change of position
- These are not necessarily the same
  - When you leave class to get some coffee, you might say the distance you've traveled and your displacement are both about 200 meters.
  - However, when you run around the track at the fitness center you may travel a distance of 2 miles but your displacement is 0 miles.

- The crucial difference is that displacement carries information about direction.
- It's said to be a **vector** and as such is quantity with both **magnitude and direction.**
- Consider the history of an object moving along the x axis.
  - At time t_1 it will be at $x_1 = 10$ m
  - At time t_2 it will be at $x_2 = 30$ m
- The **displacement** or change in position is given simply by $x_2 - x_1$ or the final position minus the initial position, and is conventionally written as
  - $\Delta x = x_2 - x_1 = 30\text{ m} - 10\text{ m} = 20\text{ m}$
  - Where the triangle is our first exposure to a Greek symbol (in this case delta) as a placeholder.
- The fact that the displacement is positive means that it is in the +x direction.
- In the second figure our icon has a displacement of – 20m, that is in the –x direction. (The signs are crucial!)





Speed & Average Velocity

- Average speed just refers to the distance traveled over time.
- Average velocity refers to the displacement traveled over time and carries information about direction
  - in other words it's our second vector!
- These are our first derived quantities.
- Informally speed and velocity are taken to be the same concept, but just as with distance and displacement the formal difference involves direction.

Formal Definitions

- Average speed just refers to the distance traveled over time.
- Average Speed:

$$\frac{\text{Distance}}{\text{Elapsed Time}}$$

- Average velocity refers to the displacement traveled over time & carries information about direction our second vector:

$$\frac{\text{Displacement}}{\text{Elapsed Time}}$$

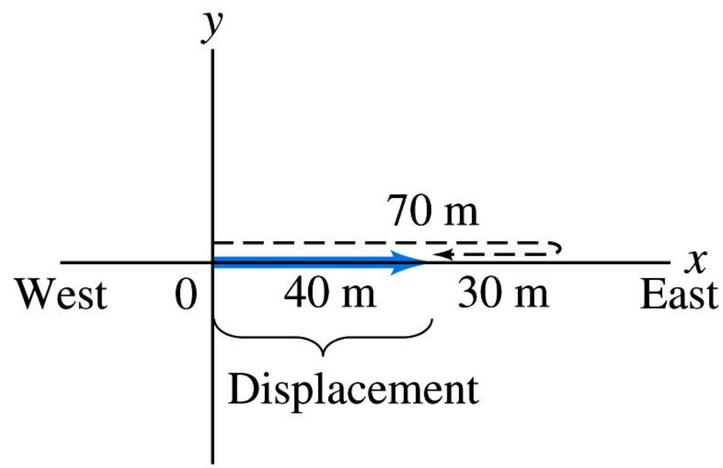
$$= (x_2 - x_1)/\ t_2 - t_1$$

$$= \Delta x/\ \Delta t$$

$$= \bar{V}$$

- Note direction is indicated by the sign of the number

An Example: Consider an object traveling the pictured path in 50 seconds



- **Average Speed:**

$$\frac{\text{Distance}}{\text{Elapsed Time}} = 100\text{m}/50\text{ sec} = 2\text{ m/sec}$$

- **Average velocity:**

$$\frac{\text{Displacement}}{\text{Elapsed Time}} = (x_2 - x_1)/\, t_2 - t_1 = (40\text{m} - 0\text{ m}) / 50\text{ sec} = 0.8\text{ m/sec}$$

An exercise: A car travels at a constant 50 km/h for 100km and then speeds up to 100 km/h for another 100 km. What's the car's average velocity and average speed over the trip?

Average Speed

- Distance traveled
  - 100km +100 km =200 km
- Time:
  - Interval 1: t_1= t/v = 100km/ (50 km/h) = 2 hr
  - Interval 2: t_2 = t/v = 100km/ (100 km/h) = 1 hr
  - Total time = 3 hr
- Average Speed = 200km/3hr = 67 km/hr

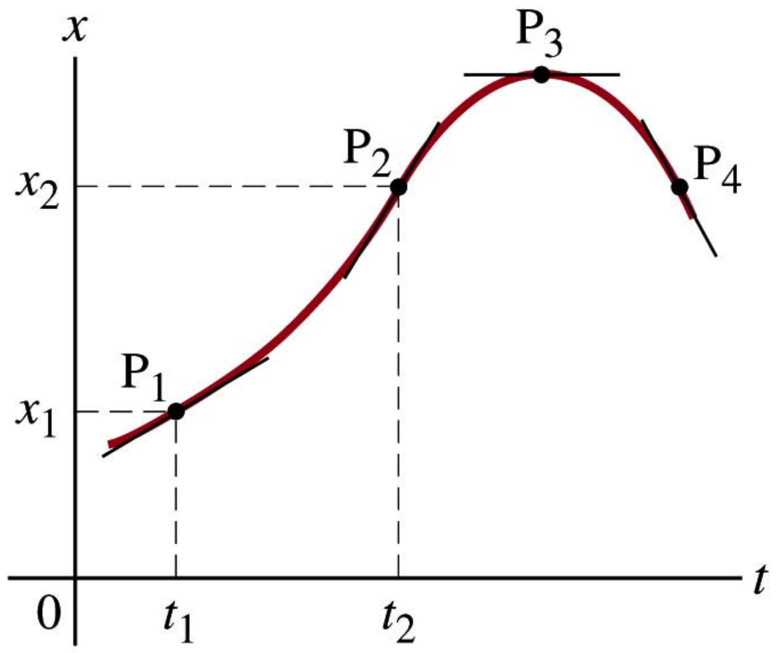
Average Velocity

- Displacement
  - $(x_2 - x_1)$ = 200km – 0km = + 200 km
- Time
  - 3 hr
- Average Velcoity = + 200 km/ 3hr = + 67 km/hr

In this case the same since there was no change in direction.

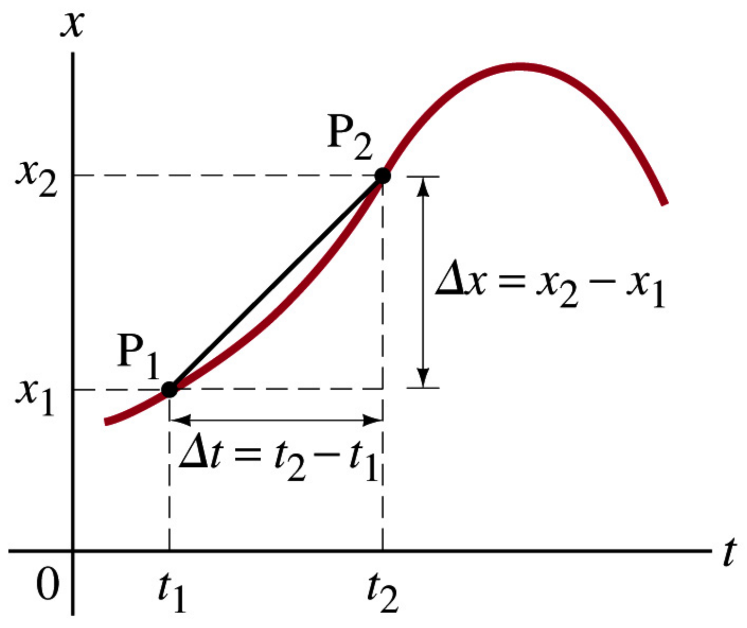
Graphing Motion for One Dimension

- One dimensional motion can be handily described by graphing position as a function of time.
- Helps to intuitively understand kinematics
- Which point is
  - nearest the origin?
  - Farthest from the origin?
- Which intervals have
  - Positive velocity?
  - Negative velocity?
- Which point(s) has the
  - The smallest speed?
  - The greatest speed?



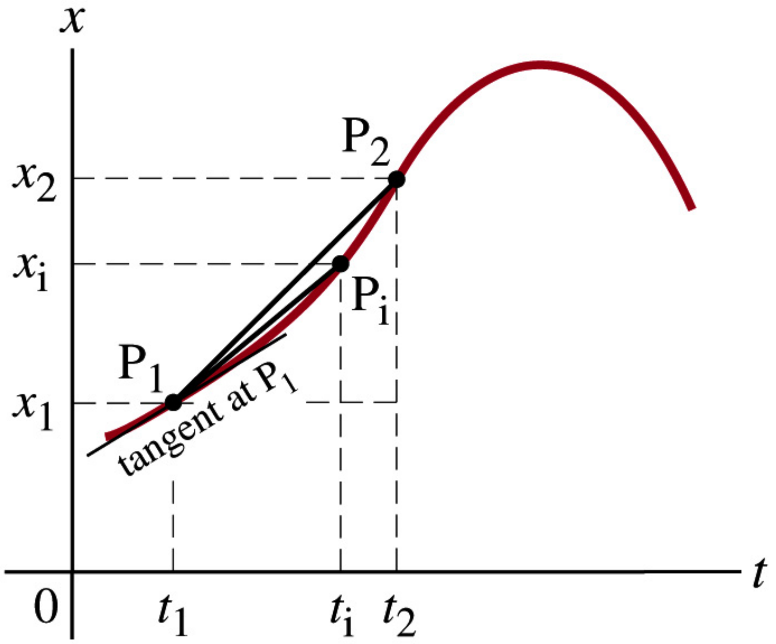
A Graphical Look at Average Velocity

- The ratio $v = \Delta x / \Delta t$ gives the average velocity during the time interval Δt.
- In the graph,
  - $\Delta x = x_2 - x_1$
  - $\Delta t = t_2 - t_1$
- This is the slope of the line



A Closer Look at Average Velocity: Shortening the Time Interval

- Consider the shorter time interval
  - $\Delta t = t_i - t_1 < t_2 - t_1$.
- From the graph
  - $\Delta x = x_i - x_1 < x_2 - x_1$
- And the average velocity for the interval P_1P_i is smaller than for P_1P_2!
- In general if the timescale changes the velocity may change



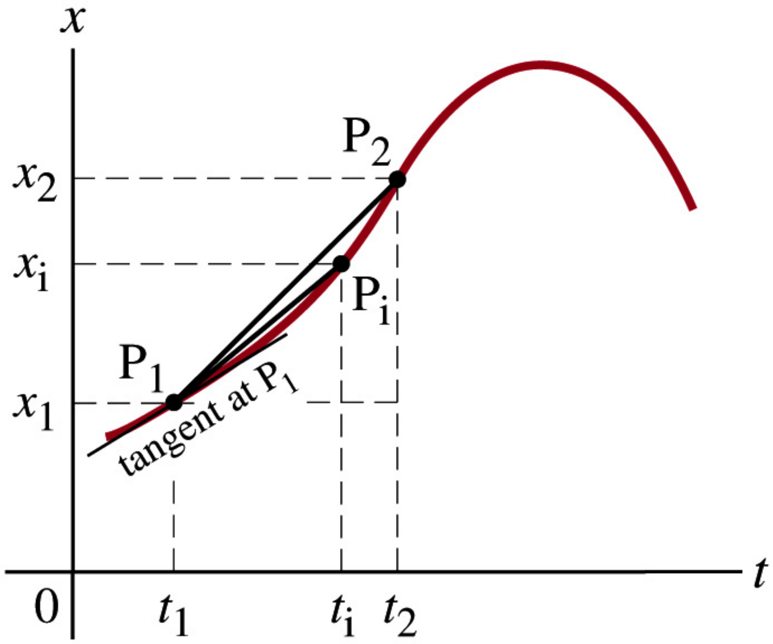
Taking the Limit

- As t_2 gets closer to t_1, Δt gets close to 0.

$$v = \frac{\Delta x}{\Delta t}$$

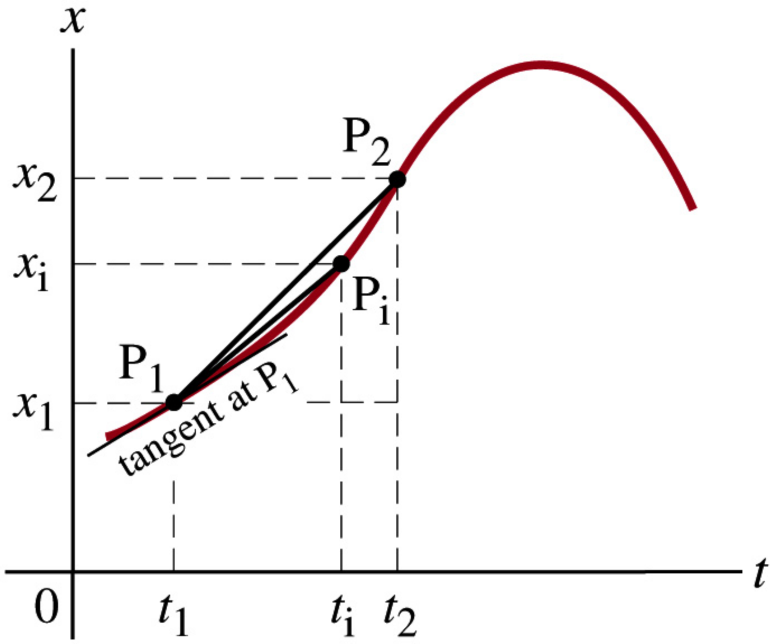
- The above expression cannot be calculated at $\Delta t = 0$.
- The limit is needed.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$



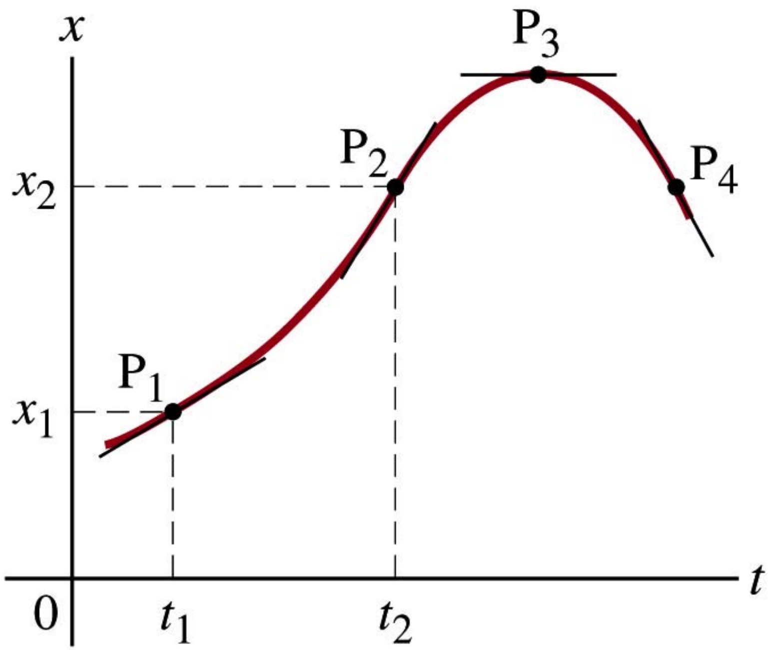
Instantaneous Velocity

- The limit of the velocity for shorter and shorter time intervals defines instantaneous velocity.
- It is essentially the same and should be considered the same as measuring velocity at a point
- This is the tangent at that point.



Returning to Position Versus Time

- The instantaneous velocity appears as the slope of the graph at a point.
- The tangent to the curve at a point is the slope at that point.
- With this formalism much easier to answer the questions about velocity at each point
  - This black lines are tangent to the curve at each point.
  - The slope of those lines are the instantaneous velocity at the points.

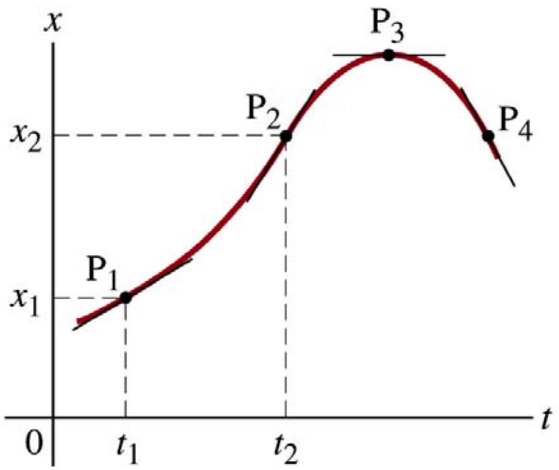


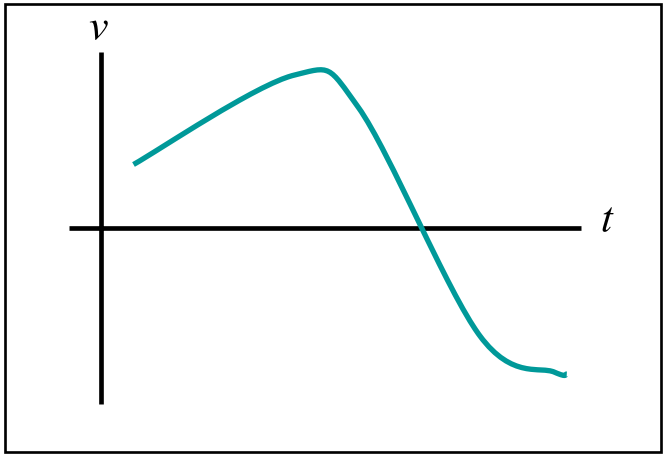
The Derivative and Instantaneous Velocity!

- A derivative measures the limit of the rate of change at each point.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- By now you should have recognized that the instantaneous velocity is given by the derivative of the position with respect to time

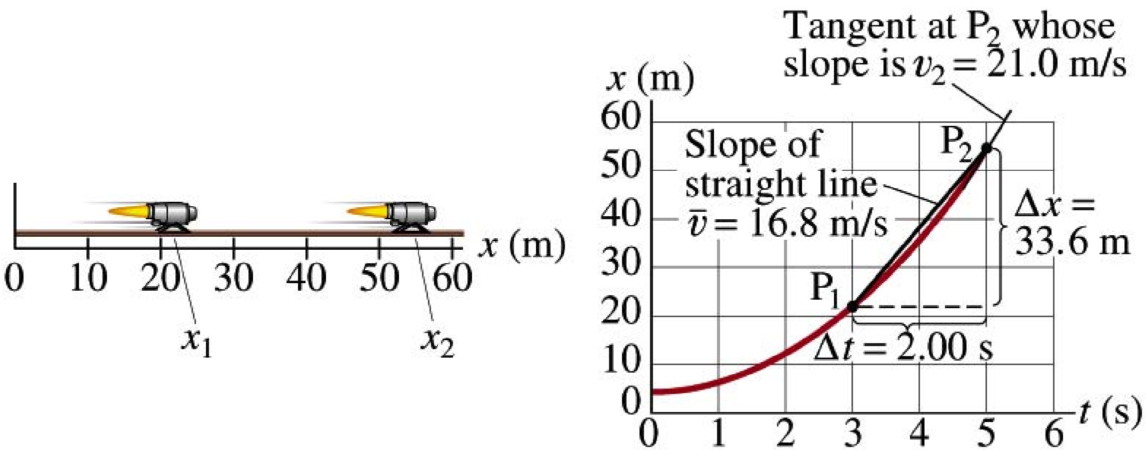




Example 2-3

- The position of a rocket as a function of time is given by $x = At^2 + B$, where $A = 2.10\ m/s^2$ and $B = 2.80$ m.
- Determine
  - The displacement of the particle during the time interval $t_1 = 3.00$ s and $t_2 = 5.00$s.
  - The average velocity during the interval
  - The magnitude of the instantaneous velocity at t = 5.00s.

(a) (b)



Ex 2-3: Displacement

Approximately

- From the graph at

  $t_1 = 3.00s \rightarrow x_1 \sim +20$ m

  $t_2 = 5.00s \rightarrow x_2 \sim +55$ m

- So displacement is approximately

  $x_2 - x_1 \sim$ +55m - 20m =

  + 35 m

Exactly

- From the formula

  $x_1 = (2.10 \text{ m/s}^2)(3.00\text{s})^2 + 2.80\text{m} =$ 21.7m

  $x_2 = (2.10 \text{ m/s}^2)(5.00\text{s})^2 + 2.80\text{m} =$ 55.3 m

- Displacement is then
  - $x_2 - x_1 = 55.3\text{m} - 21.7\text{m} =$ + 33.6m

Example 2-3: Average and Instantaneous Velocity

- By definition the average velocity is just the displacement divided by the time interval

$$\bar{v} = \frac{\Delta x}{\Delta t} =$$

$$\frac{x_2 - x_1}{t_2 - t_1} =$$

$$\frac{33.6m}{2.00s} = +16.8m/s$$

- The instantaneous velocity can be calculated by evaluating the derivative at the time of interest

$$v = \frac{dx}{dt} =$$

$$\frac{d}{dt}\left(At^2 + B\right) =$$

$$2At =$$

$$2(2.10m/s^2)(5.00s) =$$

$$+21.0\text{m/s}$$