Status: Unit 1, Chapter 1

- ✓ Measurement, Uncertainty, & Significant Figures
- ✓ Units, Standards and the SI System
- Converting Units
  - Necessary given the U.S. “system isolation”. Can be tragic: consider the 1999 lose of the $125M Mars Climate Orbiter because of a failed conversion!
  - An essential check of results
- Order of Magnitude
  - A critical life skill
  - Also an essential check
- Dimensional Analysis
  - Ditto!

Converting Units

- Recall every measurement has a number and a unit.
- Often one unit needs to be re-expressed in another unit
  - Inches to centimeters
  - Degrees Celsius to Degrees Fahrenheit
  - Pounds to Newtons
  - Etc...
- Let's do a simple example and then establish the basic technique …

Basic Technique for Conversion

- Let's just convert a length to establish our technique, say my height in inches to centimeters.
- We will need a simple conversion factor between inches and centimeters, by definition

  1 inch = 2.54 cm

- You can divide either side by the other to earn yourself two conversion factors
  - 1 = 2.54 cm/1 in = (2.54/1)*(cm/in) = 2.54 cm/in ➔

    1 = 2.54 cm/in
  - 1 = 1 inch/2.54 cm = (1/2.54)*(in/cm) =0.394 in/cm ➔

    1 = 0.394 in/cm

- Now converting my 68.5 inch height just requires a simple multiplication by 1, which is always free!

68.5 in x 1 =

- Next 1 with a conversion factor

68.5 in x (2.54 cm/in) =

- Now collect numbers and units

(68.5 x 2.54) x (in x cm/in) =

- Calculate final number, cancel units

173.99 cm

- Write in terms of significant figures

174 cm

$\boxed{68.5 \text{ in} = 174 \text{ cm}}$

TECHNIQUE:

Step 1: Multiply by unity!

Step 2: Pick conversion factor(s)

Step 3: Collect numbers & units

Step 4: Do arithmetic and cancel units.

Step 5: Set significant figures

Conversions of only One Dimension (where more than one is present)

- Lets save the Mars Observer! They failed to convert impulse from English to metric units!
  - In this context, impulse is the product of the force provided by the rockets and the time the force is nonzero: Impulse = Force x Time
  - In English units force is given in units of pounds, thus impulse has units lb-sec.
  - In SI units force is given in units of kg-m/sec^2, which is named the "Newton", thus impulse has units N-sec.

- Lets convert 1 lb-sec into N-sec.
- Looking in the text we see that
- 1N = 0.225 lb which leads to two conversion factors:

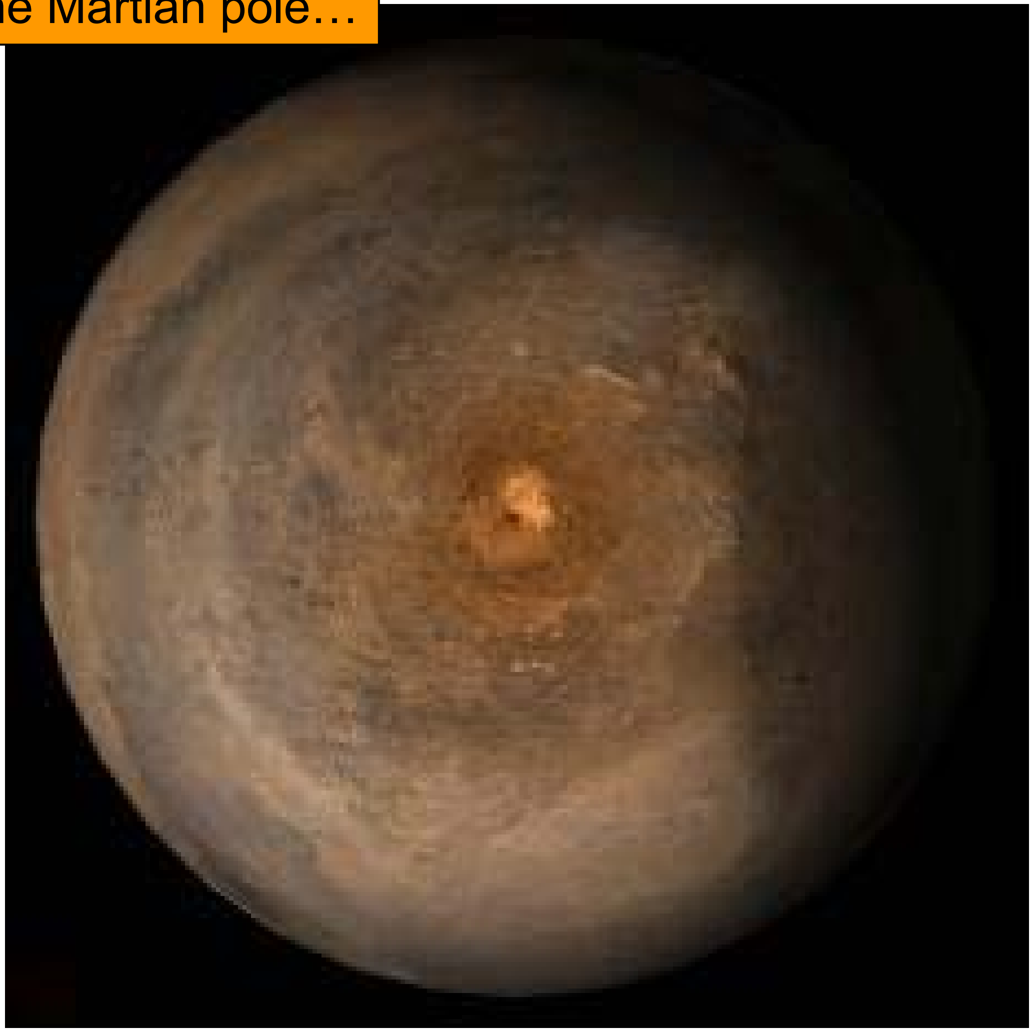
$$1 = 0.225\ \text{lb/N} = 4.45\ \text{N/lb}$$

- Following the five steps for conversion:

$$1 \text{ lb-s} \times 1 =$$
$$(1 \text{ lb-s}) \times (4.45 \text{ N/lb}) =$$
$$(1 \times 4.45)(\text{lb-s} \times \text{N/lb}) =$$
$$4.45 \text{ N-s} \rightarrow$$
$$1 \text{ lb-s} = 4.45 \text{ N-s}$$

- Notice that the seconds unit was just a "bystander".
- From the official NASA report: "…failed translation of English units into metric units in a segment of ground-based, navigation-related mission software…"
- Essentially the rocket received a excess boost upon entry and missed the window…

The next time they got it right…
here's a shot of the Martian pole…



Multiple Conversions 1: Same Dimension

- How about converting 68.5 inches to meters?
- Again we'll need conversion factors
  - First, our already derived 1 = 2.54 cm/in
  - Second, something relating cm and meters

100 cm = 1 m ➔

1 = 1 m / 100 cm =

(1/100) x (m/cm) =

.01 m/cm ➔

1 = .01 m/cm

- Following the five steps for conversion:

68\. 5 in x 1 x 1 =

(68.5 in) x (2.54 cm/in) x (.01 m/cm) =

(68.5 x 2.54 x .01) (in x cm/in x m/cm) =

1.7399 m ➔

1.74 m

68.5 in = 1.74 m

(which I know is right 'cause that's the length of my skis….)

Multiple Conversions 2: Different Dimensions

- As a final example lets convert speed from SI units to English, say the speed of light from meters/s to miles/hr:

$$c = 3.0 \times 10^8 \text{ m/s}$$

- Conversion needed for seconds to hours

$$1 \text{ hour} = 3600 \text{ seconds} \rightarrow$$

$$1 = 3.6 \times 10^3 \text{ s/hr}$$

- Conversion needed for meters to miles

$$1\text{km} = 1000 \text{ m} = 0.62 \text{ miles} \rightarrow$$

$$1 = 6.2 \times 10^{-4} \text{ mile/m}$$

- (Step 1) Multiply by unity:

  3×10^{8} m/s x 1 x 1 =

- (Step 2) Pick conversion factors:

  (3×10^{8} m/s)(3.6×10^{3} s/hr)(6.2×10^{-4} mile/m) =

- (Step 3) Collect numbers and units:

  (3×10^{8} x 3.6×10^{3} x 6.2 $\times10^{-4}$)(m/s x s/hr x mile/m) =

- (Step 4) Do arithmetic and cancel units:

  6.696×10^{8} miles/hr ➔

- (Step 5) Set significant figures:

  6.7×10^{8} miles/hr = 670 million miles/hr

- That's traveling seven times the distance to the sun in one hour…..a pretty fast commute…

Tips for Conversions

- Follow the 5-step program.
- Always be sure your units cancel properly!
- Check to see if the numbers make sense…check with common measures (remember my skis) or make rough estimates…!

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- Order of Magnitude
- Dimensional Analysis

Some Examples of Orders of Magnitude:

- You
- Lecture Hall
- Cole
- NIU

- 5’9” = 1.75 m = 2×10^0 m
- 14 m = 1.4×10^1 m
- 80 m = 0.8×10^2 m
- 2000 m = 2.0×10^3 m = 2.0 km

Each of these lengths is different by about one *order of magnitude*

Map Lengths

- One block
- DeKalb City
- DeKalb County
- Illinois
- United States

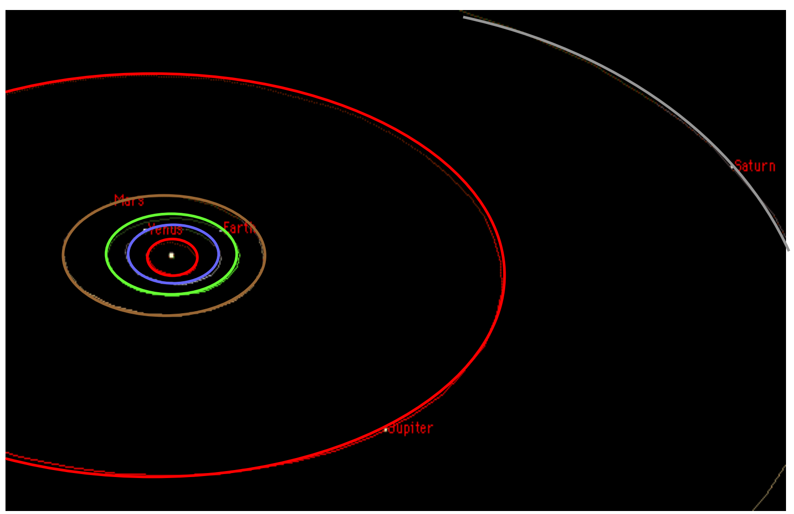
- 300 m = 3×10^2 m
- 5000 m = 5 km = 5×10^3 m
- 30,000 m = 30 km = 3×10^4 m
- 400 km = 4×10^5 m
- 5000 km = 5×10^6 m = 5 Mm

mapquest uses scaling factors, about two steps per order of magnitude

Planetary Lengths

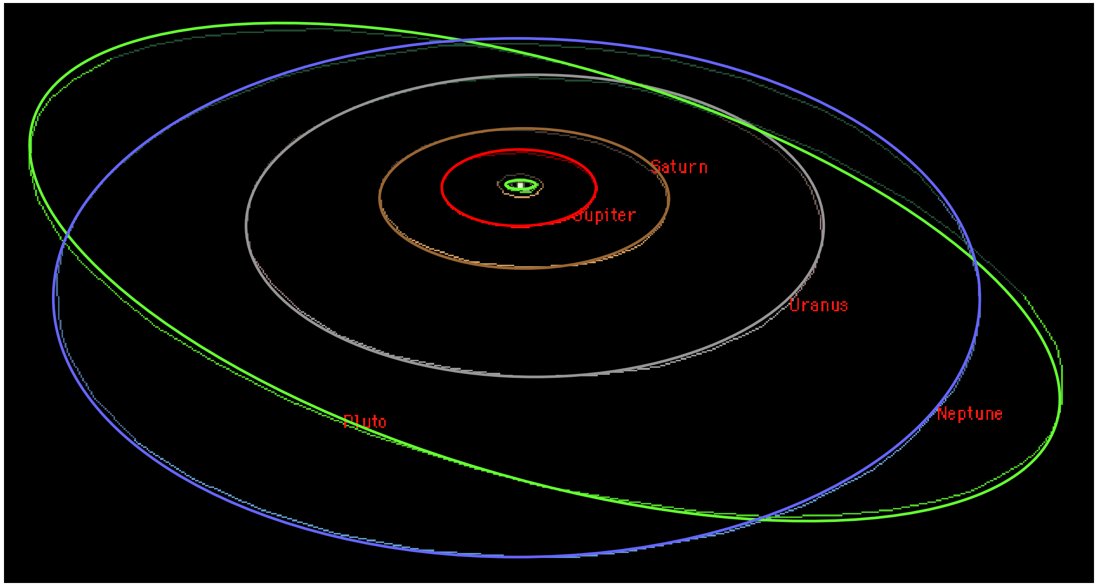
- Earth Diam.
    - 13,000 km = 1.3×10^{7} m
- Earth to Moon
    - 384,000 km = 4×10^{8} m
- Earth to Sun
    - 150,000,000 km = 1.5×10^{11} m

This is 1 Astronomical Unit (1 AU)



Solar-system Lengths

- Earth to the Sun
  - 1 AU = 1.5×10^{11} m
    8 light-minutes
- Solar System
  - 500 AU = 0.8×10^{14} m
    3 light-days



Galactic Lengths

- Nearest Star
- Milky Way
- Universe

- 300,000 AU = 4 x 10^{16} m
  4.3 light-years = 4.3 ly
- 100,000 ly = 10^{21} m
- 15,000,000,000 ly = 1.5 x 10^{26} m

Estimation to an Order of Magnitude

- Estimation:
  - A useful life skill: you can quickly check a contractors estimate, verify press statements, or just impress your pals, etc…
  - Joins dimensional correctness as a second and powerful, check of your work
- Technique:
  - Step 1: Figure out important contributions to an estimate.
  - Step 2: Pick units for which estimates are easy
  - Step 3: Round each input to about one significant figure ➔ The same as a power of ten or an "order of magnitude"
  - Step 4: Do the calculation.
  - Step 5: Give it the smell test! Do the units make sense? Is the value reasonable?

Numeric Estimation Example: How many People to Move a Rock?

- How many people will it take to move a rock with a diameter of 50 centimeters?
- Step 1: What's important?
  - the rock is round, estimate a volume of: $V = (4/3)\,\pi\, r^3$
  - Its density is three times that of water: $\rho = 3$ g/cm^3
  - A hefty person can lift about 100 lbs: *Load* = *L* = 100 lbs

- Step 2: Pick easy, common units
  - Leave the radius in centimeters: $r = 25$ cm
  - Leave the density in grams and centimeters: $\rho = 3\ \mathrm{g/cm^3}$
  - Convert the load a person can take into grams: 100 lbs = 100 lbs x (1kg / 2.2 lb) x (1000 g/kg) = 45,000 g
- Step 3: Round to a single significant digit
  - $V = (4/3)\pi r^3 = 4*(25\ \mathrm{cm})^3 = 62{,}500\ \mathrm{cm}^3 = 60{,}000\ \mathrm{cm}^3$
  - $\rho = 3\ \mathrm{g/cm^3}$ (already done)
  - $L = 50{,}000$ g/person
- Step 4: Calculate!
  - Number of people = $V\rho / L$ = $(60{,}000\ \mathrm{cm^3} \times 3\ \mathrm{g/cm^3}) / (50{,}000\ \mathrm{g/person})$ = 3.4 people ➔ 4 people to be safe!

- Step 5: Smell test, units ok, visual ok

3-4 people or
1 Hulk or
Avery persistent wind

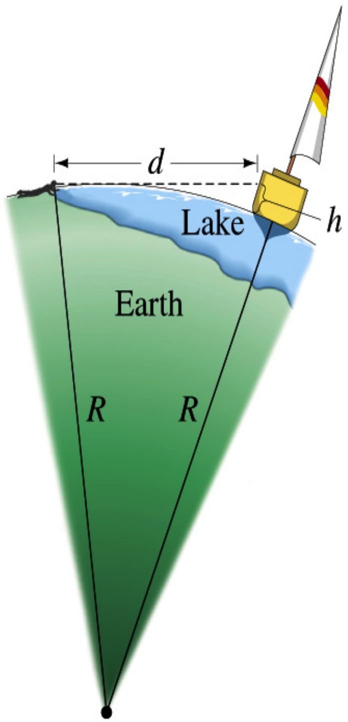
Geometric Estimate Example: What's the Radius of the Earth?

- Step 1 (What's important):
  - Imagine lying on the shore eyeing the boat and seeing only the top of the deck. We'll you can't see the bottom so the water must be in your way, already a signal that curvature is involved.
  - This is just an application of the Pythagorean Theorem and the important quantities are:

Radius of the Earth: *R*

Water to Deck height of the boat: *h*

Distance to the boat: *d*



- Step 2 (Pick Units): In this case let's just use meters
  - H = 1.5 m
  - D = 4.4 km = 4400 m
  - R we'll keep in meters for now
- Step 3 (Rounding): In this case we'll keep two significant figures.
- Step 4 (Calculate): Using the Pythagorean Theorem:
  - $R^2 + D^2 = (R+H)^2 = R^2 + 2RH + H^2$; now subtracting R^2 from both sides we get,
  - $D^2 = 2RH + H^2$; solving for R,
  - $R = (D^2 - H^2)/2H$; Ignoring H^2 as it is very tiny compared to D^2;
  - $R = D^2 / 2H$; substituting the values of H and D
  - R = $(4400m)^2$ / 2x1.5m = (4400*4400/3) m = 6,500,000 m
- Step 5 (Smell Test): this is 6400 km which compares well to our earlier diameter.

Estimate of Number of Civilizations with Intelligent Life in the Milky Way

- Actually it's called the Drake Equation: $N = (N'f_p)x(n_e f_l f_i f_c f_L)$
  - N' is the number of suitable stars (stars like the Sun) in the Milky Way galaxy
  - f_p is the fraction of those stars that have planets
  - n_e is the number of planets capable of sustaining life around each of those stars having planets
  - f_l is the fraction of planets capable of sustaining life that actually evolve life
  - f_i is the fraction of those planets where live has evolved that evolve intelligent life
  - f_c is the fraction of planets with intelligent life that develop the capability to communicate
  - L is the fraction of the planet's life during which the intelligent life can communicate

- Ranges
  - $N' = 100{,}000{,}000{,}000 = 10^{11}$
  - $f_p = 0.1 = 10^{-1}$
  - $n_e = 1 = 10^0$
  - $f_l = 0.1 = 10^{-1}$ (a wild guess)
  - $f_i = 0.01 = 10^{-2}$ (a wild guess)
  - $f_c = 0.1 = 10^{-1}$ (a wild guess)
  - f_L = 1 million yrs/ 1 billion years = 0.001 = 10^{-3}
- $N = (N'f_p)x(n_e f_l f_i f_c f_L) = 1000$
- There is no smell test except it <u>is</u> greater that Zero!

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Dimensional Analysis: Units and Types

- Units are meters, seconds, feet, tons, etc.
- Types of units are length, mass, force, volume, etc.
- The type of unit of a value is called the dimension.
  - A value in square meters has dimensions of an area.
  - A value in kilometers per hour has dimensions of a velocity.

Powers of Units

- It is useful to convert the dimensions of units into fundamental dimensions.
  - Length (L)
  - Time (T)
  - Mass (M)
- Units can be raised to a power, and so can the fundamental dimensions.
  - Area (L^2)
  - Volume (L^3)
  - Force ($M\ L\ /\ T^2$)

Dimensional Expressions

The speed of waves in shallow water depends only on the acceleration of gravity *g*, with dimensions L/T^2, and on the water depth *h*. Which of the following formulas for the wave speed *v* could be correct?

a) $v = \frac{1}{2} g h^2$

b) $v = \sqrt{gh}$

Base Quantities

Acceleration *g*

- dimensions: L/T^2
- length/time^2
- example: m/s^2

Height *h*

- dimensions: L
- length
- example cm

Speed *v*

- dimensions: L/T
- length/time
- example km/h

Checking a Result

$$v = \frac{1}{2} g h^2$$

$$\frac{L}{T} = \frac{L}{T^2} L^2$$

$$\frac{L}{T} = \frac{L^3}{T^2}$$

This doesn't work
Not a candidate formula

$$v = \sqrt{gh}$$

$$\frac{L}{T} = \sqrt{\frac{L}{T^2} L}$$

$$\frac{L}{T} = \frac{L}{T}$$

This does work
A candidate formula

Limitations of Dimensional Analysis

- An excellent check of validity, but only offers candidate relationships need to verify experimentally or theoretically
- Does not provide any information on numeric factor:
  - Also valid $v = \frac{\sqrt{gh}}{3}$
  - Not valid $v = \sqrt{gh} + 4$

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- ✓ Chapter 1 Assignment: Q1.9, P1.8, P1.22, P1.36, P1.45, P1.55