EXPRESSION OF INTEREST: Space-Charge-Induced Phase Mixing and Related Evolutionary Time Scales

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SUMMARY OF IDEA

The dynamics of nonequilibrium beams under the influence of space charge is a fundamentally important concern for the production of high-brightness beams. An experiment that is under consideration concerns the identification and measurement of fast evolutionary time scales. The idea is to superpose a relatively localized density enhancement at a desired location in a beam bunch as it is formed from the cathode. Then, by using view foils and longitudinal diagnostics to monitor the size and location of this enhancement as the beam proceeds through the machine, one can hope to extract information on the time scale by which the enhancement mixes through the bunch. This tactic would parallel an ongoing line of investigation involving simulations to illuminate evolutionary mechanisms in nonequilibrium beams in general, and to study the physics of phase mixing of initially localized density irregularities in particular.

STATEMENT OF THE PROBLEM

We adopt the viewpoint that, under the influence of space charge, the evolution of beams, and of confined nonneutral plasmas in general, may be understood in terms of phase mixing of the constituent particle orbits. For example, linear Landau damping is merely phase mixing of regular orbits [1], a process by which initially neighboring orbits diverge secularly, i.e., as a power law in time [2]. A given space-charge potential may or may not support a population of globally chaotic orbits, i.e., orbits that wander over a large portion of their accessible phase space. Initially neighboring globally chaotic orbits fill their accessible phase space exponentially, a process known as "chaotic mixing" that was initially conceived in the astrophysical context of galactic dynamics [3,4]. When a substantial population of globally chaotic orbits exists, it dissipates correlations irreversibly. In beams the consequence is an irreversible emittance growth. Inasmuch as chaotic mixing is irreversible and acts exponentially, it is essential to identify conditions for its presence in beams, and to quantify the time scale of the associated dynamics.

A semianalytic theory exists that relies on assumptions of ergodicity and a microcanonical distribution to estimate the largest Lyapunov exponents, i.e., the chaotic-mixing rates, in lower-dimensional, e.g., fully coarse-grained, time-independent Hamiltonian systems [5]. Chaos arises generically from a parametric instability that can be modeled by a stochastic-oscillator equation; linearized perturbations of a chaotic orbit satisfy a harmonic-oscillator equation with a randomly varying frequency. The underlying assumptions are, strictly speaking, invalid, yet the theory commonly yields estimates that are good to within a factor ~2 [6].

Applied to space-charge potentials, the theory yields an estimate of the chaotic-mixing rate λ as [5]:

$$\lambda(\rho) \simeq (\kappa/3)^{1/2}\,[L^2(\rho) - 1]/L(\rho);$$

in which

$$\kappa = (\omega_f^{\,2} - \omega_{po}^{\,2}\langle \nu\rangle)/2,$$

$$L(\rho) = \{T(\rho) + [1+T^{\,2}(\rho)]^{1/2}\}^{1/3},$$

$$T(\rho) = (3^{3/2}\pi\rho^2)/\{8[2(1+\rho)^{1/2} + \pi\rho]\},$$

$$\rho = [2(\langle \nu^2\rangle - \langle \nu\rangle^2)]^{1/2}/[(\omega_f/\omega_{po})^2 - \langle \nu\rangle];$$

$\omega_f = (\omega_x^{\,2} + \omega_y^{\,2} + \omega_z^{\,2})^{1/2}$ and ω_{po} refer to the external focusing frequency and the plasma frequency at the system's centroid, respectively, ν is the density normalized to the centroid density, and "$\langle q\rangle$" denotes a phase-space average of quantity q weighted by the microcanonical ensemble. In a system that is moderately out of equilibrium, one would expect to have $\rho \sim 1$ typically, for which $\lambda/f \sim 0.82$, with $f \equiv \kappa^{1/2}/(2\pi)$ representing the "dynamical frequency", i.e., the average orbital frequency. Thus, in such systems, the chaotic-mixing time scale is roughly one dynamical time. Thus, for one to be reasonably sure of its efficacy, a process of emittance compensation, i.e., removal of correlations within the beam, should be completed within a plasma period as measured from the source of the correlations.

Results thus far suggest the notion that chaotic orbits are common in space-charge configurations that are out of equilibrium, and they are present in both nonequilibrium and equilibrium configurations to a degree that tends to increase with increasing asymmetry. One example, taken from Ref. [7] and corresponding to a time-independent potential, appears in Fig. 1, which plots the rate of mixing of chaotic orbits versus the individual particle energy in a space-charge-dominated thermal-equilibrium potential corresponding to a harmonic-oscillator external focusing potential. The mixing rate is normalized to the orbital frequency, indicating that for this configuration mixing proceeds over an e-folding time comparable to an orbital period.

A second example is the well-known accelerator-physics experiment of Martin Reiser and collaborators [8] concerning the propagation of five beamlets in a 5-m-long periodic solenoidal transport channel. The beam is nonrelativistic and subject to considerable space-charge forces. The relaxation time via two-body collisions in this beam corresponds to a propagation distance ~1 km. Yet, regardless how well the beam was root-mean-square (rms) matched to the transport channel, the beamlets were seen to reappear only once, at a point ~1 m from the source. Their failure to reappear again would seem to reflect a collisionless process that, in effect, causes the particle orbits to

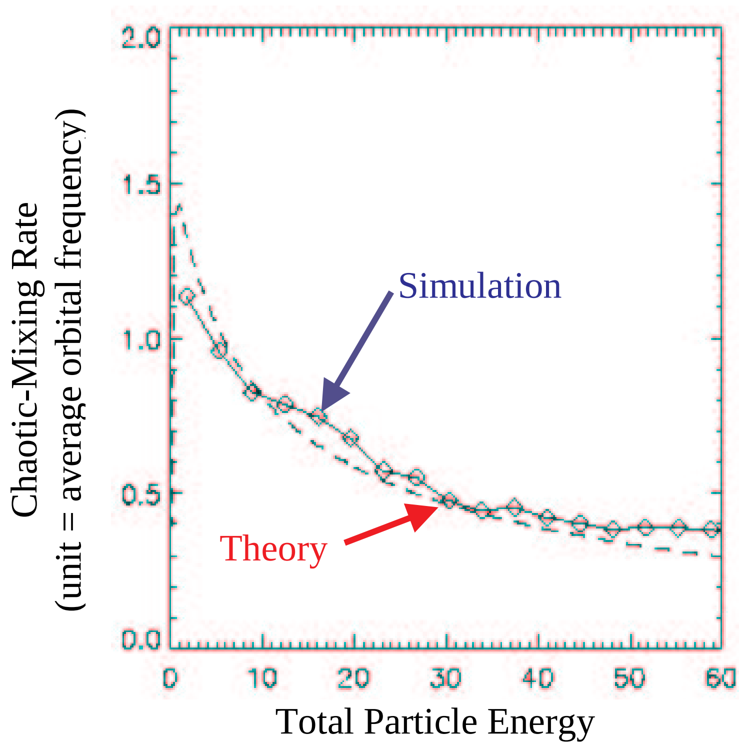

Figure 1. Mixing rate of chaotic orbits in a thermal-equilibrium potential of dimensionless form $\Phi(\boldsymbol{x}) = (\Omega^2/2)[(a/b)^2x^2+y^2+(c/b)^2z^2] + \Phi_{SC}(\boldsymbol{x})$, in which $\Omega^2 = 1.0002/3$, $(a/b)^2 = 4/5$, $(c/b)^2 = 4/3$, and $\Phi_{SC}(\boldsymbol{x})$ is the space-charge potential found from Poisson's equation. The simulation results reflect statistics from ~2000-particle samplings of orbits that were started at various locations in configuration space and at zero velocity. They agree well with analytic theory.

lose memory of their initial conditions. Simulations with a particle-in-cell code well reproduced the measurements. This is an example of evolution involving a strongly time-dependent potential.

A simulation of phase mixing in the 5-beamlet experiment recently done by Rami Kishek (University of Maryland) is depicted in Fig. 2 [7]. One sees that typical ensembles that are initially localized in phase space grow exponentially to fill much of their respective accessible regions of phase space. Meanwhile the five beamlets lose their identity. Plans for simulations include further explorations, especially in connection with designing laboratory experiments to decipher rapid evolutionary time scales in nonequilibrium beams, such as the experiment discussed herein.

PROCEDURAL PLAN

One can assess the influence of space charge on the photoinjector beam by comparing the Debye length to the full beam width:

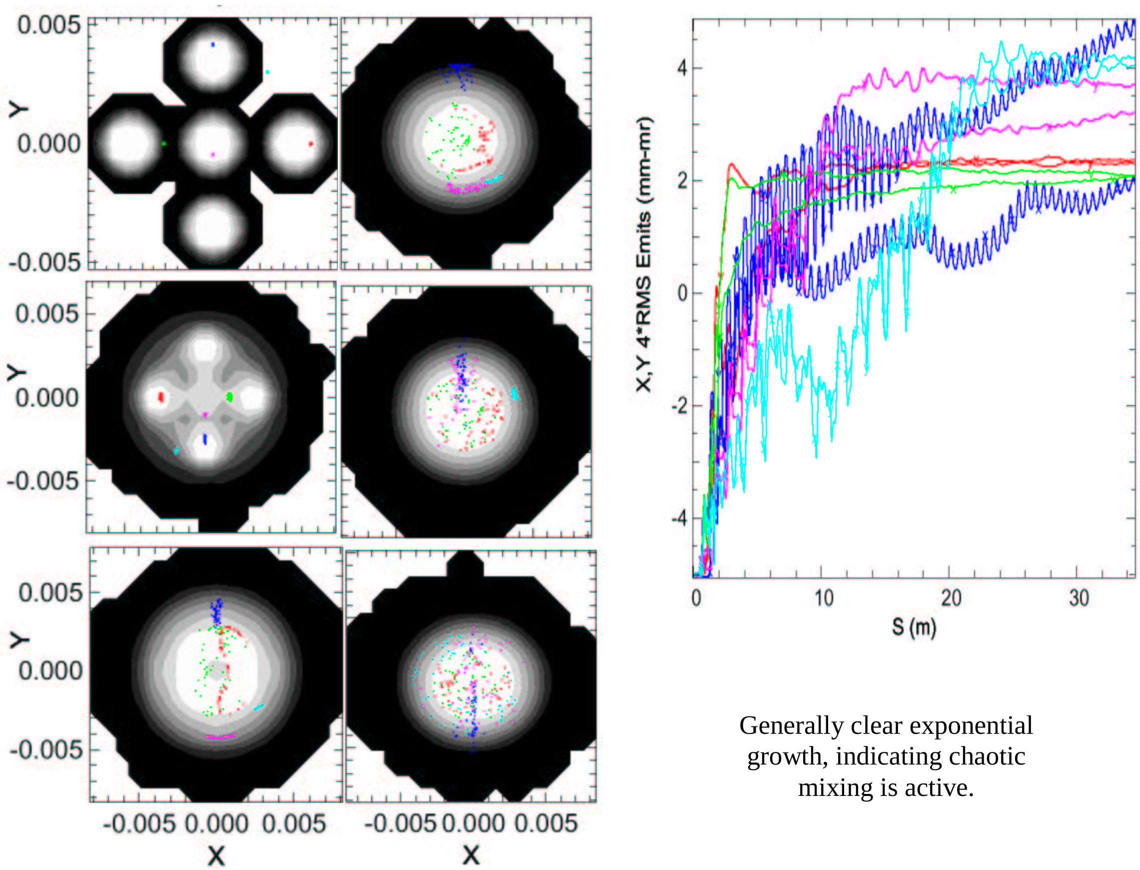

Figure 2: Evolution of five representative ensembles of test particles in the five-beamlet simulation. Beam parameters are: 5 keV energy, 44 mA current, 4.6 mm radius, and 64.8 μm full (90%) emittance. The left panel shows snapshots at (top-to-bottom left column) 0 m, 0.98 m, 2.88 m and (top-to-bottom right column) 5.24 m, 11.52 m, 31.68 m. The right panel shows the evolution of the natural logarithm of the *x* and *y* "emittance" moments of the ensembles.

$$\frac{\bar{\lambda}_D}{2X} \cong \left[\frac{5\sqrt{5}}{24}\frac{I_A}{c}\frac{\gamma\sigma_z}{q}\left(\frac{\varepsilon_x}{\sigma_x}\right)^2\right]^{1/2} .$$

This expression is obtained by using the properties of a beam bunch corresponding to its "equivalent uniform ellipsoid" [7,8]; I_A is the Alfven current (17 kA), c is the speed of light, γ is the beam kinetic energy in units of the electron rest mass, q is the bunch charge, σ_x and σ_z are the rme transverse and longitudinal sizes of the beam bunch, and ε_x is the rms transverse emittance. Space charge is important if this ratio is less than unity. Using representative parameters for the photoinjector beam, i.e., $\gamma \sim 30$, $q \sim 1$ nC, $\sigma_x \sim 1$ mm, $\sigma_z \sim 3$ mm, and $\varepsilon_x \sim 3$ μm, one finds the Debye length is ~ 0.15 of the full beam width, indicating that space charge remains active downstream of the 9-cell booster cavity.

The total potential that drives the beam dynamics is strongly time-dependent in the beam frame, and it is difficult to ascribe an "equilibrium configuration" to this beam. Nonetheless, inasmuch as space charge is active, the plasma period enters as a dynamical time scale. An estimate of the plasma period derives by first using Ref. [11] to relate the plasma period (measured in the lab frame) to the beam perveance, and then using Table 1 of Ref. [12], a paper that concerns configurations of thermal equilibrium for beams as functions of space charge, to relate the perveance to σ_x and ε_x. Upon doing so, one finds that a (correspondingly crude) estimate of the length of FNPL beamline spanning one plasma period is ~ $4\sigma_x^2/\varepsilon_x$, or ~1 m. Accordingly, evolution associated with rapid space-charge-induced phase mixing would seem to be accessible for observation with FNPL.

A key to the experiment is the purposeful formation of a density enhancement at a select location in a beam bunch as the bunch is formed at the cathode. The essential idea is to implement a delay line, as pictured in Fig. 3, to superpose a localized spatio-temporal laser pulse onto the main laser pulse during formation of the beam bunch at the photocathode. *Perfecting this technique of "pulse stacking" is a prerequisite for these experiments, and doing so is a nontrivial matter.* Yang Xi, a laser/optics specialist, will be a key contributor to the success of this experiment.

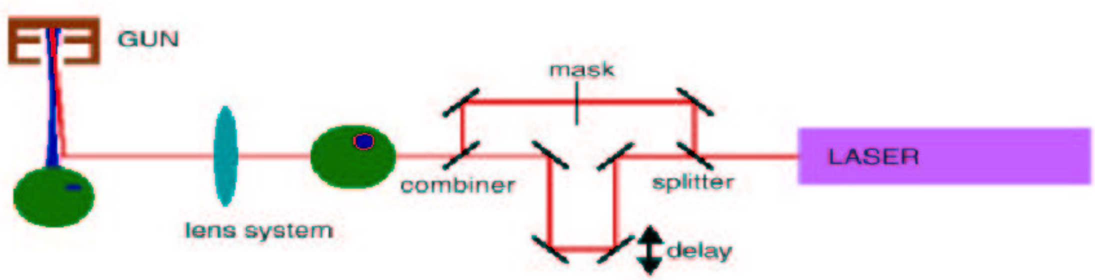

Figure 3. Using time delay to produce a density enhancement in a beam bunch.

Another key to the experiment is adequate control of both the longitudinal and transverse laser-pulse profile [13]. For example, to do the control experiment, one needs to produce a stable pulse with a smooth profile, i.e., one that is free of hot spots. This may be challenging with the existing system.

One possibility for producing the density enhancement is to use the existing laser-beam optics so that "intensified spot" is unstacked and the main beam goes through the existing 4-way pulse stacker, so the FWHM of the intensified spot is 4 times shorter than the main beam. Makng a longitudinally uniform pulse-stacked beam has thus far proven challenging and has met with limited success. Accordingly, the successful conduct of a phase-mixing experiment using the photoinjector hinges on improving the control and stability of the drive laser itself, and on perfecting the pulse-stacking procedure.

Another possibility for producing the intensified spot is to use a separate telescope rather than use the mask. The advantage would be preservation of intensity because scraping would be avoided.

The FNPL photoinjector is equipped with an array of viewers both at low energy (i.e., at the exit of the electron gun) and at high energy (i.e., at the exit of the 9-cell booster cavity). With these viewers the transverse evolution of the beam, and of the density enhancement, can in principle be monitored. One uncertainty is the signal-to-noise ratio of the viewers, which will become especially critical as the density enhancement mixes away. Work to improve this signal-to-noise ratio may prove necessary.

Monitoring the longitudinal density distribution requires different instrumentation. A streak camera is available and can be used to measure the temporal properties of the laser pulse and the longitudinal distribution of the beam at one location downstream of the 9-cell cavity. Interferometric and electro-optic diagnostics, discussed under a separate EOI [14], would yield better spatio-temporal resolution of the beam properties.

Ideally, the intensified spot would be localized within the bunch and be sufficiently small to constitute a collection of test particles. In practice this clearly will not be possible, and the evolution of the density enhancement will proceed according to self-consistent phase mixing in general. Accordingly, simulations will be necessary to interpret the experimental results. The force acting on a particle as a function of position and time can be generated with a suitable injector simulation code, such as ASTRA, a code developed by Klaus Floettmen of DESYwhich is used routinely to model the performance of the photoinjector. Given the force tabulation, Ioannis Sideris has on hand all the necessary tools for simulating phase mixing of particle orbits, from which evolutionary time scales can be inferred. Accordingly, this EOI offers the prospect of a self-contained research effort.

The essential goal is a fundamentally correct, microscopic understanding of the dynamics of nonequilibrium beams. Work done to date suggests the underlying physics is generally applicable to self-interacting many-body systems governed by a long-range force, such as self-gravitating systems. Accordingly, in that it is an analog of the dynamics of galaxies that are far from equilibrium, the outlined effort comprises a form of laboratory astrophysics.

ESTIMATED RESOURCE REQUIREMENTS

Optics for delay line: 5 k$ (firmer estimate TBD)
Time to set up the delay line: 140 hours (160 person-hours)
Beam time: 480 hours (12 weeks of single-shift operation)

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