Angular momentum dominated electron beam and flat beam generation

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outline

• angular momentum dominated electron beam and its applications

• production and measurements of angular momentum dominated beam

• removal of the angular momentum and generation of flat beam:
  – a bit of theory
  – measurement method
  – data analysis and recent results
  – comparison with simulations

• conclusion
beam dynamics: three different regimes

emittance

canonical angular momentum

space charge

envelope equation in a drift:

$$\sigma'' + \frac{\varepsilon^2}{\sigma^3} - \frac{L^2}{\sigma^3} - \frac{K}{4\sigma} = 0$$

where

$$K = \frac{2I}{I_0 \beta^3 \gamma^3}.$$
Applications of angular momentum dominated beam

• electron cooling for heavy ion

\[
\tau_{mag}^c \approx \frac{\rho}{(v_{z}^\text{ion} - v_{z}^e)}
\]

\[
\tau_{free}^c \approx \frac{\rho}{v_{e}^\perp}
\]

• flat beam for linear $e^+e^-$ collider:
  reduce beamstrahlung

luminosity $\propto \frac{1}{\sigma_x \sigma_y}$

$\delta_E \propto \frac{1}{(\sigma_x + \sigma_y)^2}$

e.g. TESLA 500GeV 10/0.03 mm mrad for 3 nC
Applications of angular momentum dominated beam

- **flat beam for light sources**
  - Linac based ultra-short X-ray pulse (LBNL)
    \[ \Delta l = \Delta y \frac{2 \sin \theta \sin \alpha}{\sin(\theta + \alpha)} \]
  - Smith-Purcell radiator or image charge undulator

U of Chicago, Dartmouth U.

JLab.
Generation of angular momentum dominated e beam

\[ L = \gamma mr^2 \dot{\Phi} + \frac{1}{2} eB_z r^2 \]

On the photocathode: \( \langle L \rangle = eB_0 \sigma_c^2 \)

\( B_z = 0 \): mechanical angular momentum

FNPL 1.625-cell RF gun, 1.3 GHz
Fermilab/NICADD Photoinjector Lab. (FNPL)

- RF gun
- TESLA superconducting cavity
- 4 MeV
- 16 MeV
- Round-to-flat
- Beam transformer
- (skew quadrupoles)
Measurement of canonical angular momentum on the photocathode

\[ \langle L \rangle = eB_0 \sigma_c^2 \]

- \( B_0 \): B-field on cathode
- \( s_c \): RMS beam size on cathode
Measurement of mechanical angular momentum in a drift space

\[ \langle L \rangle = 2 p_z \frac{\sigma_1 \sigma_2 \sin \theta}{D} \]
Measurement of mechanical angular momentum vs B-field

RMS beam size at $z_1 = 3.678$ m

RMS beam size at $z_2 = 5.053$ m

Magnetic field on cathode [Gauss]

Angle $\theta$

Magnetic field on cathode [Gauss]
Demonstration of conservation of canonical angular momentum as a function of magnetic field on cathode

\[ \langle L \rangle = 2 p_z \sigma_1 \sigma_2 \sin \theta / D \ (\text{neV s}) \]

\[ \langle L \rangle = e B_0 \sigma_c^2 \ (\text{neV s}) \]

\( \sigma_c = 0.97 \pm 0.04 \text{ mm} \)

experiment
Parametric dependencies of angular momentum

- Angular momentum versus
  - beam longitudinal position $z$
  - bunch charge
  - beam size on the cathode

\[ \langle L \rangle = eB_0 \sigma_c^2 \]

$B_z = 962 \text{ G}$

$\sigma_c = 0.90 \pm 0.04 \text{ mm}$

$B_z = 683 \text{ G}$

$\sigma_c = 0.82 \pm 0.05 \text{ mm}$
Round-to-flat beam transformation

\[ \Sigma_{\text{round}} = \begin{bmatrix} \varepsilon_{\text{eff}} \beta & 0 & 0 & L \\ 0 & \varepsilon_{\text{eff}} / \beta & -L & 0 \\ 0 & -L & \varepsilon_{\text{eff}} \beta & 0 \\ L & 0 & 0 & \varepsilon_{\text{eff}} / \beta \end{bmatrix} \]

General form of a round beam
(K.-J. Kim)

\[ e_{\text{eff}} = \sqrt{\varepsilon_u^2 + L^2} \]

Un-correlated emittance
“normalized” canonical angular momentum

Transfer matrix
of the round-to-flat beam transformer

\[ \Sigma_{\text{flat}} = M \Sigma_{\text{round}} \tilde{M} \]

Flat beam emittances given by:

\[ \varepsilon_{\pm} = \sqrt{\varepsilon_u^2 + L^2} \pm L \]

e.g. \( L=20 \text{ mm mrad}, e_u=1 \text{ mm mrad} \)
\( e_+=47 \text{ mm mrad}; e_-=0.02 \text{ mm mrad} \)
Round-to-flat beam transformation using skew quadrupoles

Flat beam: large transverse emittance ratio, zero average angular momentum.

\[
\langle \tau \rangle_{\text{total}} = 2 \cdot \sum q_i \langle x_i, y_i \rangle
\]

Two sets of solutions:

\[
q_1 = \pm \sqrt{\frac{-d_2 S_{11} + S_{12} - d_2 d_T S_{21} + d_T S_{22}}{d_2 d_T S_{12}}}
\]

\[
q_2 = -\frac{S_{12} + d_T S_{22}}{d_2 d_3 (1 + S_{12} q_1)}
\]

\[
q_3 = -\frac{q_1 + q_2 + d_2 S_{11} q_1 q_2 + S_{21}}{1 + (d_T q_1 + d_3 q_2) S_{11} + d_2 d_3 q_2 (S_{21} + q_1)}
\]

(D. Edwards)
Position and velocity snap shots at the entrance/exit of the transformer

Round beam

flat beam
Beam evolution through the transformer for the first solution

Right before 1\textsuperscript{st} quad

Right before 2\textsuperscript{nd} Quad

Right before 3\textsuperscript{rd} quad

Right after 1\textsuperscript{st} quad

Right after 2\textsuperscript{nd} Quad

Right after 3\textsuperscript{rd} quad
Removing of angular momentum and generating a flat beam
single slit emittance measurement method

Blue: flat beam at X7; green: H or V slit inserted at X7; red: slit image at X8.

0.6 mm

6 mm
Recent flat beam experiment 1

Data of Jan. 6, 2005
Solenoid setting: main=190A, buck=0A, secondary=75A
UV drive-laser rms size = 0.76 mm, rms pulse length = 3 ps
beam energy = 15.8 MeV       bunch charge = 0.50 ± 0.05 nC
Data analysis: RMS beam size calculation

1. Choose area of interest

   ![Area of Interest Diagram]

2. Decide the background level

   ![Intensity Graph]
Data analysis: RMS beam size calculation

1. area of interest

![Graph showing the relationship between the number of pixels used for rms calculation and the rms beam size.]
Data analysis: RMS beam size calculation

2. background level

![Graph showing RMS beam size calculation](image)

- Red line: using average background level
- Rising background level

Number of pixels used vs. \( \sigma_x \) (pixel)
## Compare experiment 1 with simulation

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Simulation (ASTRA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90%</td>
<td>95%</td>
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<tr>
<td>rms_cathode (mm)</td>
<td>0.71±0.05</td>
<td>0.76±0.06</td>
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<tr>
<td>B_cathode (Gauss)</td>
<td>876</td>
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<tr>
<td>I_Quad1 (A)</td>
<td>-1.92</td>
<td></td>
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<tr>
<td>I_Quad2 (A)</td>
<td>2.40</td>
<td></td>
</tr>
<tr>
<td>I_Quad3 (A)</td>
<td>-2.99</td>
<td></td>
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<tr>
<td>rms_X7y (mm)</td>
<td>0.59±0.03</td>
<td>0.63±0.04</td>
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<tr>
<td>rms_X7x (mm)</td>
<td>0.077±0.005</td>
<td>0.087±0.006</td>
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<tr>
<td>rms_X8_hslit (mm)</td>
<td>1.15±0.02</td>
<td>1.24±0.02</td>
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<tr>
<td>rms_X8_vslit (mm)</td>
<td>0.12±0.01</td>
<td>0.13±0.01</td>
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<tr>
<td>emitx (mm mrad)</td>
<td>0.36±0.04</td>
<td>0.45±0.06</td>
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<tr>
<td>emity (mm mrad)</td>
<td>26±2</td>
<td>30±2</td>
</tr>
<tr>
<td>emit ratio</td>
<td>73±10</td>
<td>68±10</td>
</tr>
<tr>
<td>(emitx· emity)^0.5</td>
<td>3.1</td>
<td>3.7</td>
</tr>
</tbody>
</table>
ASTRA Simulation with 01/06/2005 experiment conditions

emit. ratio $\epsilon$

$\epsilon_x$ (mm mrad)

$\epsilon_y$ (mm mrad)

$z$ (m)

$0.18$ mm mrad

$30$ mm mrad

$165$
Recent flat beam experiment 2

Data of Feb. 25, 2005
Solenoid setting: main=195A, buck=0A, secondary=75A
UV drive-laser rms size = 0.97 mm,  rms pulse length = 3 ps (?)
beam energy = 15.86 MeV     bunch charge = 0.51 ± 0.17 nC
### Compare experiment 2 with simulation

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<th>Experiment</th>
<th>Simulation (ASTRA)</th>
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</thead>
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<td></td>
<td>90%</td>
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<tr>
<td>rms_cathode (mm)</td>
<td>0.97</td>
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<td>B_cathode (Gauss)</td>
<td>898</td>
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<td>I_Quad1 (A)</td>
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<td>I_Quad2 (A)</td>
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<td>I_Quad3 (A)</td>
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<tr>
<td>rms_X7y (mm)</td>
<td>0.58±0.01</td>
<td>0.63±0.01</td>
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<tr>
<td>rms_X7x (mm)</td>
<td>0.084±0.001</td>
<td>0.095±0.001</td>
</tr>
<tr>
<td>rms_X8_hslit (mm)</td>
<td>1.57±0.01</td>
<td>1.68±0.01</td>
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<tr>
<td>rms_X8_vslit (mm)</td>
<td>0.12±0.01</td>
<td>0.13±0.01</td>
</tr>
<tr>
<td>emitx (mm mrad)</td>
<td>0.39±0.02</td>
<td>0.49±0.02</td>
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<tr>
<td>emity (mm mrad)</td>
<td>35.2±0.5</td>
<td>41.0±0.5</td>
</tr>
<tr>
<td>emit ratio</td>
<td>90±5</td>
<td>83±4</td>
</tr>
<tr>
<td>(emitx· emity)^0.5</td>
<td>3.7</td>
<td>4.5</td>
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</tbody>
</table>
conclusion

• A series of experimental investigation of angular momentum dominated electron beam was carried out;

• Simulations show that an emittance ratio >100 is possible when the chromatic effect and space charge are taking into account.

• Recent flat beam emittance measurements are very encouraging
  – at 0.5 nC, normalized emittance of \(0.4 \sim 0.5\) \(\text{mm mrad}\) was measured (LUX design value but at 1nC);
  – emittance ratio of \(70 \sim 90\) was achieved;
  – ideas and simulations of further improving the emittance ratio is underway.

• However as beam size approaches to 10’s of \(\mu\)m, dispersion and camera resolution come to play.

• laser pulse length and temporal profile
acknowledgements

Contributors to the work presented here:

**Guidance and leadership**
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**Laser work:**
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**Cryogenics and vacuum:**
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**RF:**
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**Software:**
Jason Wennerberg

**Discussions:**
Court Bohn, Klaus Flöttmann
Chromatic effects

\[ q_0(1 - \delta + \delta^2), \quad \text{transfer matrix?} \quad M(q_1, q_2, q_3, d_2, d_3) \approx M_0 + \delta \Delta_1 + \delta^2 \Delta_2, \]

\[ \varepsilon_{x,y} = \sqrt{\left(\varepsilon_{eff} + \mathcal{L}\right)^2 + \langle \delta^2 \rangle^2 \left[ |\Delta_{11} \text{ or } 22| + (\varepsilon_{eff} + \mathcal{L})^2 \text{Tr}(T\Delta_{11}^\dagger \text{ or } 22) \right]}. \]

thermal emittance = 1 mm mrad.

space charge force is turned off.
RF asymmetry caused by gun RF coupler kick

- Accelerating mode center is shifted
- Time-dependent dipole kick
- Vertical emittance growth

(Khabiboulline)

1 mm
Quadrupole alignment: rotation and displacement around each axis