DIAGNOSTICS OF THE WAVEFORM OF ELECTRON BUNCHES USING THE DISTRIBUTION OF COHERENT DIFFRACTION RADIATION

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Transition / diffraction radiation

Field of electron, Fourier transform

\[ E_r(r, \varphi, z, \omega) = E(r, \omega) \exp(i\omega z / V) \]
\[ E(r, \omega) = e\alpha K_1(\alpha r) / \pi V \]
\[ \alpha = \omega / V \gamma \quad \alpha^{-1} = \beta \gamma \lambda / 2\pi \approx \gamma \lambda \]

Huygens’s and other principles

\[ E_{\perp}(p) = \frac{\mathbf{k}}{2\pi i} \int_S a_{\perp} \cos \varphi \cdot \exp(ikR) \frac{\exp(ikR)}{R} dS \]
\[ a_{\perp} = E(r, \omega) \cdot \exp(ikz / \beta) \cdot \sin \varphi \]
\[ a = E(r, \omega) \cdot \exp(ikz / \beta) \cdot \cos \varphi \]

Intensity in point p. Depends on frequency energy and the size of target.
Coherent radiation. Longitudinal coherency.

Longitudinal distribution of the bunch charge

$$\rho(z) = \rho_0 \exp\left(-z^2 / 2\sigma^2\right)$$

$$E_\rho(\omega, p) = E(\omega, p) \int \rho(z) \exp(ikz / \beta) dz$$

$$J_\rho(\omega, p) \propto |E_\rho(\omega, p)|^2 = J(\omega, p) S(\omega)$$

Bunch form factor

$$S(\omega) = \left| \int_{\omega_1}^{\omega_2} \rho(z) \exp(i\omega z / V) dz \right|^2.$$  

Energy distribution

$$W(p) = \int_{\omega_1}^{\omega_2} J(\omega, p) S(\omega) d\omega$$

14 MeV beam

Formfactor of a relativistic Gaussian bunch. Peak of TR distribution.

DR/TR from a disc D=8mm, \(\Psi=45^0\)

Small beam spot size is required
100 MeV pre-injector LINAC of the Swiss Light Source (SLS) with bunch lengths between 0.75 ps and 1 ps, depending on the setting of the pre-buncher phase. PBU-0 corresponds to the shortest bunches and PBU+3 corresponds to a 3° phase offset from this optimum setting.

Energy density distribution of CTR for tune PBU-0.

Energy density distribution of CDR for tune PBU-0.

<table>
<thead>
<tr>
<th>Target</th>
<th>Tune</th>
<th>$T$(ps)</th>
<th>RMS, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTR</td>
<td>PBU-0</td>
<td>0.69</td>
<td>5</td>
</tr>
<tr>
<td>CTR</td>
<td>PBU+3</td>
<td>1.1</td>
<td>9.1</td>
</tr>
<tr>
<td>CDR</td>
<td>PBU-0</td>
<td>0.78</td>
<td>8.6</td>
</tr>
<tr>
<td>CDR</td>
<td>PBU+3</td>
<td>1.07</td>
<td>10.7</td>
</tr>
</tbody>
</table>

Table I. shows the fitted bunch lengths $T$, for CTR and CDR targets for beam tunes PBU-0 and PBU+3.
Coherent Diffraction Radiation.
Flat detector plane at distance 200mm.
16 MeV, Gaussian Bunch 2 and 4 ps,

Form Factor

\[ S_L(\sigma_L, \omega) \]

Frequency dependent DR, foil tilt angle = 41 deg

\[ J(x, y, \omega) \]

Wide band DR

\[
\frac{dW(\sigma_L, x)}{dx dy} = \int_{\omega_1}^{\omega_2} J(x, y, \omega) S_L(\sigma_L, \omega) d\omega
\]

2ps (narrow, intensity=40 units), 4 ps (wide, intensity=8.2 units), target 25 x 25 mm, 41 deg, horizontal scan, distribution on the window L=200mm (observ. angle =-90deg)
Intensity of light on the foil of radiator.
Gaussian Bunch: 4.3 GeV, $\lambda = 750$ nm, $10^4$ particles
$\sigma_x = 0.2$ mm, $\sigma_y = 0.2$ mm, $\sigma_z = 10$ nm, $\sigma_E = 4$ MeV

X, Y beam density distribution

X, Y incoherent intensity $\Sigma E^2 = 10^5$

X, Y coherent intensity $(\Sigma E)^2 = 10^8$
Analytical calculations

coherent

incoherent

red, blue – radiation with X, Y polarization
black – no polarization, green – beam distribution
yellow – gradient of distribution
EXAMPLE: LCLS ELEGANT INPUT: $2 \times 10^6$ particles

XY Coherent, $I = 3.4 \times 10^{10}$

$\lambda = 550 - 650 \text{ nm}$

Incoherent, $I = 2.23 \times 10^{10}$

The analysis of fluctuations can possibly let to