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RF Devices – Lecture 2

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Review of Pillbox Cavity

- You can repeat all this for TE modes, but we want longitudinal electric fields for acceleration!
- Pick the lowest frequency, simplest mode: TM₀₁₀

•
$$B_{\rho} = E_{\rho} = E_{\phi} = 0$$
 and $j_{m,n} = 2.405$

•
$$E_z = E_0 J_0 \left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

•
$$H_{\phi} = \frac{E_0}{\eta} J_1\left(\frac{2.405\rho}{R}\right) e^{-i\omega t} e^{\frac{i3\pi}{2}}$$
 with $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \cong 376.7 \Omega$ is the

impedance of free space.

• $\omega_{010} = \frac{2.405c}{R}$ Note: only depends on radius, not length!

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Review of Pillbox Cavity 2

•
$$T = \frac{2\beta c}{\omega_0 L} Sin\left(\frac{\omega_0 L}{2\beta c}\right) <-$$
 always less than one

- $\frac{dT}{dL} = 0$ when $L = \beta \lambda/2$
- This length factor is also important when synchronizing many cells together with the beam



Length and Radius are the only free parameters



Summary

- We went through an overview of how bound RF waves could be used to build accelerators
- We got to a pillbox cavity, the most simple work-horse cavity topology that's used almost everywhere
- Today we will finish the pillbox calculations and dive into how we characterize and optimize these cavities and into the zoo of topologies actually used in the real world

Gap Synchronism

Plotted is the synchronism factor for 20% error in β for gaps ranging from 4 to 20.

Larger number of gaps have smaller velocity acceptance.

Machine parameters drive design here, heavy ion v electrons, for instance.

For wide range of β , multiple cavity types may be needed.

FRIB is accelerating anything from carbon to uranium, so the acceptance has to be huge, SLC was only electrons



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PIP-II Cavity Choices

- Optimization of cavity styles, number, etc. is a very complex process
- Ultimately it comes down to complexity and cost
- This also shows the limited usefulness of geometric beta
- Look at the difference made by having one module of HWRs!
- Largest cost savings comes from reducing number of cavity types
- Electron machines don't worry about this

Table 3.6: Accelerating cavities in the PIP-II Linac and their operating ranges in the Linac. ($\beta_g = \beta_G$ for the HWR, SSR1 and SSR2 cavities, β_g for the elliptic cavities is defined as the ratio of regular cell length to half-wavelength. Fitting to Eq. 3.3 for the elliptic cavities yields: $\beta_G = 0.64$ for LB650 and $\beta_G = 0.947$ for HB650.)

Cavity name	β_{g}	β_{opt}	Freq. (MHz)	Cavity type	Energy gain at eta_{opt} per cavity (MeV)	Energy range (MeV)
HWR	-	0.112	162.5	Half wave resonator	2	2.1 - 10.3
SSR1	-	0.222	325	Single-spoke resonator	2.05	10.3 - 35
SSR2	-	0.475	325	Single-spoke resonator	5	35 - 185
LB650	0.61	0.65	650	Elliptic 5-cell cavity	11.9	185 - 500
HB650	0.92	0.971	650	Elliptic 5-cell cavity	19.9	500 - 800



Figure 3.18: Variation in the transit time factor with beam velocity for the PIP-II cavities. Red dots mark the position of β_{G} , and blue dots the position of β_{opt} .

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Stored Energy

- We stated earlier: $u = \frac{1}{2} \left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2 \right)$
- So it follows that U = $\int_{V} \frac{1}{2} \left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2 \right) dV$
- While this is generally true, we can chose a time where this calculation is easier. Choose time such that the electric fields are zero and magnetic fields are maximized.

• So, U =
$$\int_V \frac{1}{2} \left(\frac{1}{\mu_0} \vec{B}^2 \right) dV$$

 Generally, this is done for you in simulation. For a pillbox, this can be done analytically.

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•
$$U = E_0^2 \pi L \epsilon_0 \int_0^R \rho J_1^2 \left(\frac{2.405\rho}{R}\right) d\rho = \frac{\pi \epsilon_0 E_0^2}{2} J_1^2 (2.405) LR^2$$

Peak Surface Fields

- We want to calculate the peak surface fields.
- $E_{pk} = E_0$ is easy.
- Maximizing magnetic field on the end wall:
- $B_{pk} = \frac{E_0}{c} J_1(1.84) = \frac{E_0}{c} 0.583$ or where $\rho = 0.77R$
- But what we also want are normalized quantities.

•
$$\frac{B_{pk}}{\sqrt{U}}$$
, $\frac{E_{pk}}{\sqrt{U}}$ and, by extension, $\frac{V_{acc}}{\sqrt{U}}$

- These quantities can be scaled nicely, and are less prone to change during optimization of unrelated features.
- Speaking of, that last one seems quite useful...



Shunt Impedance

- Remember, we want a quantity that can be used to judge the efficiency of transferring the stored energy to the beam.
- The (effective) shunt impedance is defined as:
- $\frac{R}{Q} \stackrel{\text{def}}{=} \frac{V_{acc}^2}{\omega U}$ which is the ratio of the accelerating voltage squared and the reactive power in the cavity (in the equivalent circuit).
- This is a purely geometric factor that is very useful in describing the accelerating efficiency of a cavity geometry.
- Other definitions of this may not include the TTF, or may have a factor of two for historical reasons, so watch out.
- Note that this does not scale with frequency. You can directly scale a geometry to a different frequency, and this will stay the same. Very useful.



Shunt Impedance 2

- Smashing together the equations we know for a pillbox:
- $\frac{R}{Q} \cong 150 \ [\Omega] \frac{L}{R} \cong 196 \beta [\Omega]$
- Linear with optimum particle velocity! Higher frequencies are better.
- Makes sense, U scales like L, but so does V_{acc} .
- Note: Reactive Power in circuit theory is the power flow IN the resonator, in this case equivalent to ωU
- Think of this as the full stored energy in the cavity flowing through a plane ω times a second



Quality Factor

- A standard metric for how efficiently a resonator stores energy is the quality factor.
- This is a quantity related to the number of cycles it would take to dissipate a given amount of stored energy.
- $Q_0 = \frac{\omega U}{P_d}$ But this means that we need a definition of P_d
- Fortunately, we've done the ground work:
- $P_d = \frac{1}{2} R_s \int_S |\vec{H}|^2 dA$ Integrated over the cavity walls
- Note the implicit assumption, that surface resistance is uniform over the entire cavity! Probably not the greatest assumption for superconductors, but not much else you can do without significant effort.

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Geometry Factor

- R_s is quite variable, especially for superconducting cavities.
- The quality factor that doesn't depend on R_s would be of great usefulness.
- The R_s dependence comes from the dissipated power.

•
$$Q_0 = \frac{\omega U}{P_d} = \frac{\omega U}{\frac{P_d}{R_s}R_s}$$
, $G = R_s Q_0 = \frac{\omega U}{\frac{P_d}{R_s}}$

- This, while adding dimensions to the quality, depends strictly on geometry and not material.
- Again, doesn't scale with frequency (make sure to gather all the scaling of *U* and *P_d*)



Pillbox Quality Factor

•
$$P_d = \frac{R_s E_0^2}{\eta^2} \left\{ 2\pi \int_0^R \rho J_1^2 \left(\frac{2.405\rho}{R} \right) d\rho + \pi R L J_1^2 (2.405) \right\}$$

• Outer wall + end wall

•
$$P_d = \frac{\pi R_s E_0^2}{\eta^2} J_1^2 (2.405) R(R+L)$$

• Giving:

•
$$G = \frac{\omega_0 \mu_0 L R^2}{2(R^2 + RL)} = \eta \frac{2.405L}{2(R+L)} = \frac{453\frac{L}{R}}{1 + \frac{L}{R}} [\Omega]$$
 With an optimum L ...
• $\frac{L}{R} = \frac{\beta \pi}{2.405}$, $G = 257\beta [\Omega]$

• A highly useful result, indicating that pillbox cavities are more efficient at higher optimum particle velocities.

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Pillbox Scaling

Clearly better at high beta, best at $\beta = 1$.

Mechanical concerns also come into play:

Aspect ratio:

$$\frac{L}{R} = \frac{\beta\pi}{2.405}$$

This gets pretty suboptimal at low beta, thin pancake cavities have poor mechanical properties. • $G = 257\beta[\Omega]$ • $\frac{R}{Q} = 196\beta[\Omega]$

• $E_{pk} = E_0$

•
$$cB_{pk} = 0.583E_0$$

•
$$U = \frac{\pi\epsilon_0 E_0^2}{2} J_1^2 (2.405) LR^2$$

$$P_d = \frac{\pi R_s E_0^2}{\eta^2} J_1^2 (2.405) R(R+L)$$

TTE - $\frac{2}{\pi^2}$

$$TTF = -\frac{\pi}{\pi}$$



Material Comparison

- Superconducting Cavity
 - Peak Surface Fields dominate design
 - ~220 mT is theoretical max, 120 mT is doing very well in practice
 - Pushes for high Q
 - Technologically Challenging
 - Processing requirements put significant constraints on complex cavity geometries

$$-R_s \propto f^2, P_d \propto f, Q \propto f^{-2}$$

- Normal Conducting Cavity
 - Limited by dissipated power
 - Limits duty cycle or gradient
 - Pushes for highest $\frac{R}{Q}$
 - Local power density also a concern (local heating), maxes at ~20 W/cm²
 - Electrical breakdown limited peak electric fields
 - Cheaper material (copper!)
 - Cooling design can be quite complex (non-uniform)

$$- R_s \propto f^{\frac{1}{2}}, P_d \propto f^{-\frac{1}{2}}, Q \propto f^{-\frac{1}{2}}$$

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Conditioning limits





- More energy: electrons generate plasma and melt surface
- Molten surface splatters and generates new field emission points!
 ⇒ limits the achievable field
- Excessive fields can also damage the structures
- Design structures with low E_{surf}/E_{acc}
- Study new materials (Mo, W)



Damaged CLIC structure iris



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First iris

iris



Damage on iris after runs of the 30-cell clamped structures tested in CTFII. First (a, b and c) and generic irises (d, e and f) of W ,Mo and Cu structures respectively.



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Technical Point of Order (Quarter Wave OR Pillbox?)





FNAL Recycler Upgrade RF Cavity https://indico.fnal.gov/event/2665/con tribution/10/material/paper/1.pdf

FNAL Main Injector RF Cavity http://lss.fnal.gov/archive/2005/conf/fe rmilab-conf-05-102-ad.pdf



Topological Morphing of Modes





Actual Pillbox Cavity, same as we derived, but with beam pipes added and outer conductor modified to improve quality factor and other factors.

BNL Photo-Injector "Quarter-Wave" Cavity Not really. Modified for lower frequency in a compact shape.



Wideröe Linac – Sloan/Lawrence Structure

Dipole mode in a cylindrical waveguide cavity has opposite voltages at opposite edges of the cavity.

Putting loading elements bring this voltage to the beam axis, and oppositely loaded elements on the beam axis give a pi-mode like structure.

The spacing between the gaps has to be tuned to keep synchronization.



Wideröe (3)

As the tank gets long, compared to the wavelength, it becomes better to drive the cavity in a quarter wave resonator mode.

In principle, this is the same, although the voltage on the central electrode goes like a QWR, highest at the end.

Electrodes can be made longer, keeping synchronism, to add focusing elements.





Alvarez Drift-Tube Linac



Using a pillbox cavity mode, but with drift tubes to shield the particle from deceleration fields.

Fields can be tuned to be uniform, giving uniform field per gap.

Each drift tube shields the fields, giving field-free regions. Synchronous acceleration requires $\beta\lambda$ gap separation.



Linac 2

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- 3 RF tanks, 750 keV to 50 MeV, 34 m long.
- Frequency 200 MHz, about 30 MW.



The drift tubes of Linac2 Er







Linac4 DTL (3 ... 50 MeV)

- Three tanks 39/42/30 cells
- Permanent magnetic quadrupoles
- Drift tube alignment relies on machining tolerances and not on alignment mechanism
- All PMQ centres aligned within ±0.1 mm!
- Conditioning time per tank: 1-2 weeks
- December 2015: Fully commissioned if Linac2 fails, we have an emergency plan.





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DTL Tuning

Large stubs are machined to tune inductance of tank.

Each cell has a stub tuner $(\lambda/4)$ that can be trimmed to modify the capacitance of each drift tube.

Also! Nominally, each 'cell' is independent. In reality, this isn't true (errors, etc)

These tuning rods also perturb the fields, Coupling each 'cell' together.





Side-Coupled Linacs in Practice



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Superconducting Cavities

Superconducting cavities can be made with large apertures because the design focus is peak surface fields and Q rather than R/Q.

This means high coupling, and lower sensitivity to errors.

Also, you essentially never see superconducting cavities longer than 9-cells because of processing effects.

Still need to tune SRF cavities, though.





Bead Pull and Correction

Cavities are made field-flat by pulling a ceramic bead through the cavity.

This perturbs the resonant frequency of the cavity proportional to the field where the bead is.

This profile can be used to calculate the cell-by-cell errors, and each cell is tuned individually.

This process is repeated until the cavity gradient is even between all the cells.





Quarter Wave Resonators

HORI

2

apt

- Coaxial Resonator
 - Effective open and short termination
- Low Frequency Structure
 - Allows for efficient acceleration of low beta beams
- Accelerating Field
 - Two gap structure (Pi-Mode like)
- Steering
 - Asymmetric design leads to slight beam steering
- Open end for access/processing
 - Open end for cavity processing and inspection





Half Wave Resonators

- Coaxial Resonator
 - Two effective short terminations
- Higher Frequency Structure than QWR
- Accelerating Field
 - Two gap structure (Pi-Mode like)
- HWR v. QWR
 - Higher optimum beta
 - No beam steering
 - Double the losses
 - No easy access





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Single Spoke Resonator

Topologically identical to the Half Wave Resonator

Mechanically weaker in the beam axis.





Multi-Spoke Resonator

Essentially a multi-cell version of the single spoke resonator/half wave resonator.

More compact than equivalent medium-beta geometries, but significantly more mechanically complex.





Couplers

We can generate power in a variety of ways, but we have get it from the source to the cavity.

Waveguides/Coax Transmission Lines

J

$$f_c = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For rectangular waveguide, but no cutoff for coaxial lines.

Higher power = Larger Coax





High-Power Couplers





http://arxiv.org/ftp/arxiv/papers/1501/1501.07129.pdf

Design Topics for Superconducting RF Cavities and

H. Padamsee



Equivalent Circuit for Driving a Cavity



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Many models are simplification, but all the relevant parts are there: Generator, Transmission Line, Coupler, Cavity, Beam



Cavity Mode-Coupler Mode Interface

Power flowing in the coupler must be 'coupled' to the cavity mode.

How easily power flows into the cavity mode is related to the convolution of the cavity mode structure and the coupler field structure.

Coupler interface is geometry based.





Coupler Definitions

- We will now have to make the distinction between different quality factors.
- $Q_0 = \frac{\omega U}{P_d}$ where P_d is the dissipated power in the cavity walls
- Note that this depends on the geometry of the cavity, but also on the cavity material properties. When quality factor is quoted, this is often the number that people mean.
- $P_{tot} = P_d + P_e + P_t$
- Total power lost from the cavity is the sum of the losses in the walls and the power flowing out of both of the cavity couplers.

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- Generally, cavities are tested with an input and probe couplers.
- Input is meant to supply power, probe is a small field tap.

Loaded Q

- A real cavity has these three kinds of losses, and we need to first, treat them together, and then figure out how to solve for them separately.
- Define a combined Q_L called the Loaded Q.

•
$$Q_L \equiv \frac{\omega U}{P_{tot}}$$

• With no driving term, power will flow out of the cavity in relationship to the stored energy:

•
$$\frac{dU}{dt} = -P_{tot} = -\frac{\omega U}{Q_L}$$
, giving $U = U_0 e^{-\frac{\omega t}{Q_L}}$

• The cavity stored energy decays, with no drive, with a time constant $\tau_L = \frac{Q_L}{\omega}$. Note, this is power, the voltage will decay twice as fast.

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External Q

•
$$\frac{P_{tot}}{\omega U} = \frac{P_c + P_e + P_t}{\omega U}$$
, $\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e} + \frac{1}{Q_t}$

• With the definition of the Q-External as:

•
$$Q_e = \frac{\omega U}{P_e}$$
, $Q_t = \frac{\omega U}{P_t}$

 Note that these are definitions are for power flowing out of the cavity, which are effective losses, but the energy still exists somewhere as RF, not as heat.

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• We can define some very useful quantities:

•
$$\beta_e \equiv \frac{Q_0}{Q_e} = \frac{P_e}{P_d}$$
, $\beta_t \equiv \frac{Q_0}{Q_t} = \frac{P_t}{P_d}$

•
$$Q_0 = Q_L(1 + \beta_e + \beta_t)$$

Measurements

- The probe is generally approximated to be very weakly coupled ($\beta_t \ll 1$) because we desire it to be a small diagnostic signal (< 1 mW).
- So, let's assume that we're driving the cavity with one coupler only for now.

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- What we're looking for:
- Cavity response to a driving signal.
- $P_f, P_r, P_t, Q_0, Q_e, Q_t$ (we'll deal with probe signals later)
- Going through the circuit analysis:

•
$$\Gamma(\omega) = \frac{\beta_e - 1 - iQ_0\delta}{\beta_e + 1 + iQ_0\delta}, \delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

• On Resonance: $\Gamma = \frac{\beta_e - 1}{\beta_e + 1}$

Steady State Behavior

- So, we're driving a cavity with a fixed forward power on resonance.
- There are 4 Powers we care about:
 - P_f : Forward Power, coming from the generator to the cavity.
 - P_r : Reverse Power, coming back from the cavity.
 - P_e : Emitted power, from the cavity stored energy, leaking out through the coupler.
 - P_{ref} : Reflected power, incident power directly reflecting off of the cavity coupler boundary.
 - Reverse power is the vector sum of those two field components.
 - We'll worry about transmitted power later.

Steady State Behavior

• $U_0 = \frac{4\beta P_f}{(\beta+1)^2} \frac{Q_0}{\omega}$

•
$$\beta_e = \frac{\left(1 \pm \sqrt{\frac{P_r}{P_f}}\right)}{\left(1 \mp \sqrt{\frac{P_r}{P_f}}\right)}$$

- Note, that there is an ambiguity here. There are three states possible:
- Undercoupled: Weak coupling, most power reflected
- Overcoupled: Strong coupling, large emitted power
- Matched: Equal emitted and reflected 180° out of phase

 $-P_f = P_d!$



Dynamic Measurements

- Must break the ambiguity.
- Turning the RF drive on and off gives us this information.
- Overcoupled will be dominated by emitted power, undercoupled is dominated by reflected power.
- Turning off the drive power removes the reflected power component!

• On:
$$E(t) = E_0 \left[1 - e^{-\frac{t}{2\tau_L}} \right]; P_r = \left\{ 1 - \frac{2\beta}{1+\beta} \left[1 - e^{-\frac{t}{2\tau_L}} \right] \right\}^2 P_f$$

• Off:
$$E(t) = E_0 e^{-\frac{t}{2\tau_L}}; P_r = \left\{\frac{2\beta}{1+\beta}e^{-\frac{t}{2\tau_L}}\right\}^2 P_f$$



Square Wave Response



Calibrations

- Combining low field static and dynamic measurements characterizes the cavity.
- This gives us a measure of the stored energy, *U*, and can be used with a simultaneous measurement of the probe power to calculate $Q_t = \frac{\omega U}{P_t}$.
- Once we know Q_t, all we need is a static measurement to directly measure the stored energy in the cavity (thus gradient!).
- Also, an energy balance tells us that $P_d = P_f P_r P_t$, so we've also measured P_d , thus Q_0 .

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•
$$Q_0 = \frac{Q_t P_t}{P_f - P_r - P_t}$$

Cavity Bandwidth





Production Testing of SRF Cavities

 Qualification of SRF cavities starts with matched, or nearly matched cavity testing.

•
$$P_d = \frac{\omega U}{Q_0} \approx \frac{(2*\pi*1.3E9[Hz]*3.7[J])}{3E10} = 1[W]$$

- A 1 [W] amplifier can get $\sqrt{1012 * 2\pi * 1.3E9 * 3.7} = 5.5 MV!$
- Full gradient is $\sim 35 MV/m$, so that's pretty good!
- For a copper cavity, this would be half a kilowatt or more!
- Keep in mind that CW copper cavities exist!
 - Advanced Photon Source Cavities require 200 kW at 352 MHz, although most of that goes into the beam (1 MV, QL ~21e3, 100 mA of electrons), but it's a multi-MW class RF system! 20% of power going to the cavity. Compare to 0.05% for LCLS-II.

The Cost of Detuning

- When driven on-resonance, a cavity takes a minimum of RF power to maintain gradient
- However, for operation, it is required that cavities be tightly maintained at amplitude, phase, and frequency
- Cavity frequency will drive/shift based on thermal and mechanical effects, and RF power must be paid to continue operation

•
$$P_{Amp} = \frac{V^2 (1+\beta)^2}{4\beta Q_0 \left(\frac{r}{Q}\right)} \left[\left(1 + \frac{I_{beam} \left(\frac{r}{Q}\right) Q_0}{V(1+\beta)} \right)^2 + \left(\frac{Q_0}{1+\beta} \frac{2\delta f}{f}\right)^2 \right]$$

• Three places for the energy to go, cavity losses, giving energy to the beam, and compensating for detuning

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Cavity Tuning - Mechanical





Thin-wall SRF cavities can be mechanically tuned, changing length to change frequency (~500 kHz vs 1.3 GHz), think $\Delta L/L$ Fast and slow tuning possible, although fast tuning is quite challenging

Water Cooled - Tuning

Copper cavities must be water cooled, and while you can get mechanical tuners, they are often very stiff

Water cooling can be temperature adjusted to change the size of the cavities slightly, changing the frequency

Compare thermal expansion coefficients at 300 K to $\Delta L/L$ Copper: ~17e-6 per C





Linac4 DTL (3 ... 50 MeV)

- Three tanks 39/42/30 cells
- Permanent magnetic quadrupoles
- Drift tube alignment relies on machining tolerances and not on alignment mechanism
- All PMQ centres aligned within ±0.1 mm!
- Conditioning time per tank: 1-2 weeks
- December 2015: Fully commissioned if Linac2 fails, we have an emergency plan.





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RF Summary

- RF resonators are used to give energy to your beam in a way that DC accelerators cannot
- Storing and concentrating the EM energy allows compact and efficient acceleration, and RF resonators are designed for
 - R/Q: Efficient transfer of stored energy to the beam
 - Quality Factor: Efficient storage of the energy to the beam
- Further design considerations are:
 - Mechanical/thermal performance
 - Highest gradient/stored energy
 - Beam-induced Higher Order Modes
 - Overall power requirements

