RF Devices – Lecture 1

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NIU PHYS 790D Fall 2019

Essential Beam Physics Calculations

October 15th, 2019

First, Sources and Recommendations

- Recommended Literature:
  - RF Linear Accelerators – T. Wangler
    - RF and Beam Optics for Linear Machines
    - Extensive history with NC structures at Los Alamos
  - RF Superconductivity for Accelerators – H. Padamsee
    - Formative figure in Superconducting RF
    - Lots of SRF theory and application, including cavity processing and limitations
  - Microwave Electronics – J.C. Slater
    - Head of the MIT Rad Lab and Bell Laboratories (!)
    - Difficult to find, but outstandingly written and understandable
  - Microwave Engineering – D.M. Pozar
    - Common text for general microwave engineering classes

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RF Lectures Overview

- My goal is to give you a basic introduction to RF systems
- The class should introduce you to:
  – EM Theory (Analytical calculations, scaling, derivations)
  – Practical Concerns (Implementation, design, and limitations)
  – Maxwell Equations, Helmholtz Equations
  – Standard Transmission Line/Cavity Geometries
  – Figures of Merit and Optimization
- Some Real Design Considerations
  – Practical Concerns and Implications (fabrication, cleaning, etc.)
  – Many Examples of RF structures in use today

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Quick Introduction

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Rutherford's Desire

- Ernest Rutherford's discovery of the structure of the nucleus in 1909 opened new fields of experimentation
  - Nuclear Physics
  - Eventually, High Energy Physics, Light Sources, etc.
- These fields (and eventually others) require particle "accelerators"
  - Rutherford expressed a long-standing "ambition to have available for study a copious supply of atoms and electrons which have individual energy far transcending that of the alpha and beta particles" available from natural sources.
  - This desire has lead to a century of accelerator physics research

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High Energy Particles

- What does the science want?
  - More Energy!
    - Different science available
  - Controllable/Tunable Energy!
    - Dynamic behavior studies
    - Fine structure investigations like resonances
  - More Intensity!
    - Take data faster
    - Study rare processes
      - Rare isotopes
      - Neutrinos
      - Rare particle decays
  - Variety of Particle Beams!
    - The ability to create and use beams of any element/isotopes

- What do they REALLY want?

Wall power

Desired Beam

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What force to use?

- Strong
- Weak
- Electromagnetic
- Gravity

$$\frac{d\vec{p}}{dt} = \vec{F}$$

- Radioactive decay
  - Limited Natural Sources (some artificial sources, not much better)
  - Limited Intensity
  - Very specific energies
  - Limited set of available beams
- Extremely weak
  - Using the Sun's gravity well, you could get a proton up to ~22 MeV
  - No comments on the practicality
- Electric Fields!
  - Magnetic fields can't do work
  $\Delta E = F \cdot \Delta x = \left(q\vec{v} \times \vec{B} \right) \cdot \vec{v}\Delta t = 0$

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Most Simple Accelerator

- We don't want Magnetic Fields

  - Steering should be left to the magnets (most of the time), we're just talking about energy gain

  - If you assume B = 0, then we only get *static* electric fields

  - We want only longitudinal electric fields, strong in the desired direction

V = 0
Focusing Electrode

V = 10000V
Anode

Cathode

Diode Pierce Geometry

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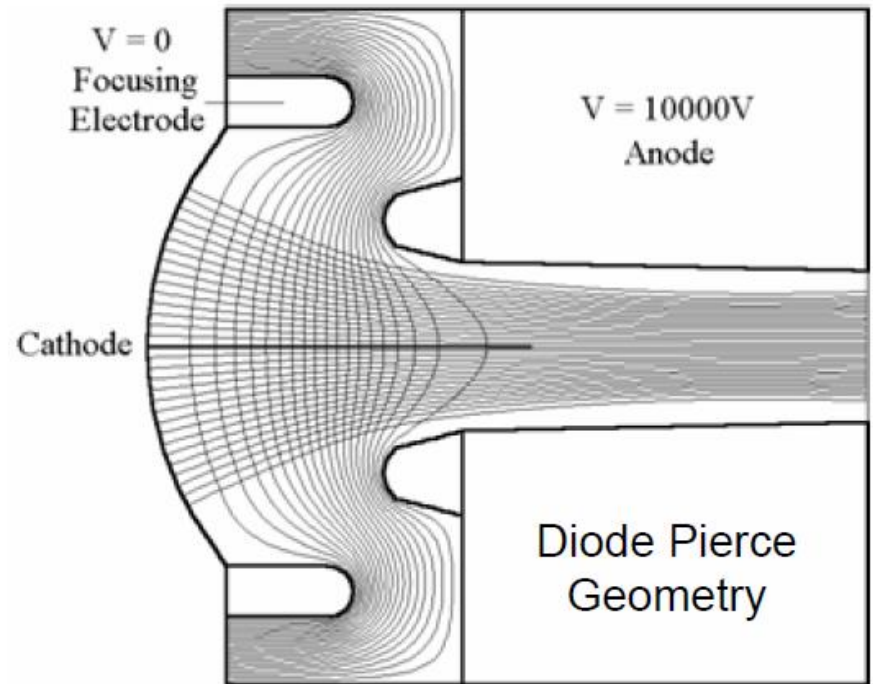
Static Electric Fields

- Electrostatic Accelerators
  - Limited energy gain (60 MeV/q)
  - Can accelerate DC beams (used often for particle sources)
- Tandem Accelerators
  - By changing the particle charge from negative to positive, twice the energy can be achieved (limited current)
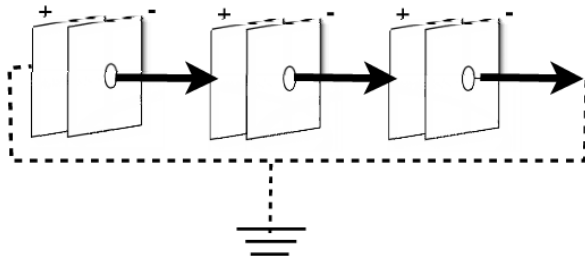
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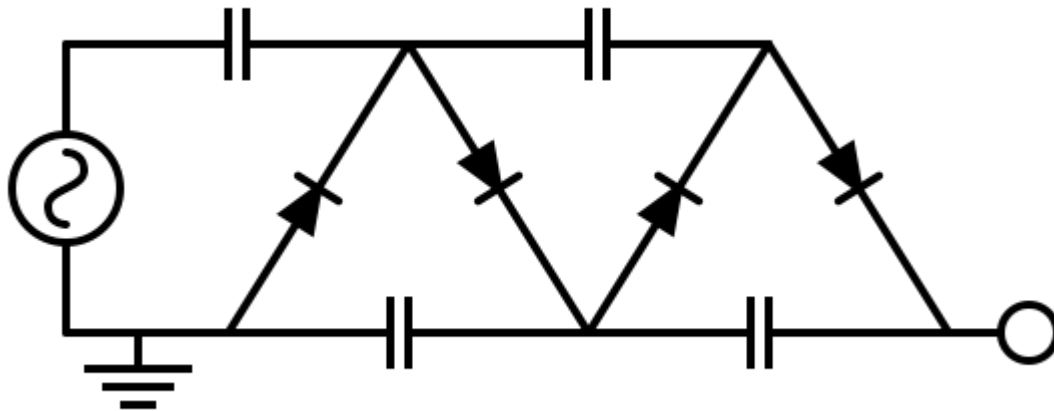
The Benefits of RF Fields
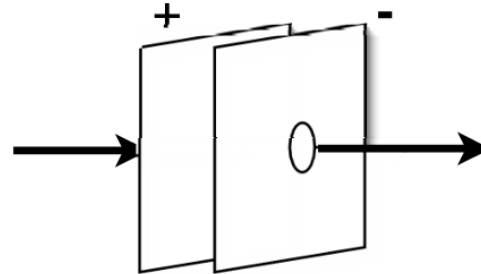
- **The Need for AC Systems**

Circular Accelerator

$V(t)$
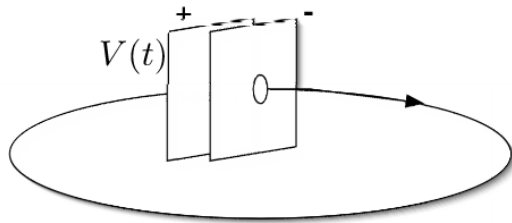
Linear Accelerator

$$\text{energy gain} = q \cdot V$$

\+ \-

DC systems limited to a few MV

$$\oint (q\vec{E}) \cdot d\vec{s} = work = \Delta(energy)$$

To gain energy, a time-varying field is required:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{A}$$

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Wideroe Drift Tube Linac

- First RF accelerator conceived and demonstrated by Wideroe in 1927 in Aachen, Germany

- RF voltage of 25 kV from 1 MHz oscillator was applied to single electrode between two ground planes

- Accelerated potassium ions to 50 keV, two gaps for twice the voltage

- Sloan and Lawrence built one of these style linacs with 30 electrodes, applying 42 kV to get mercury ions up to 1.36 MeV
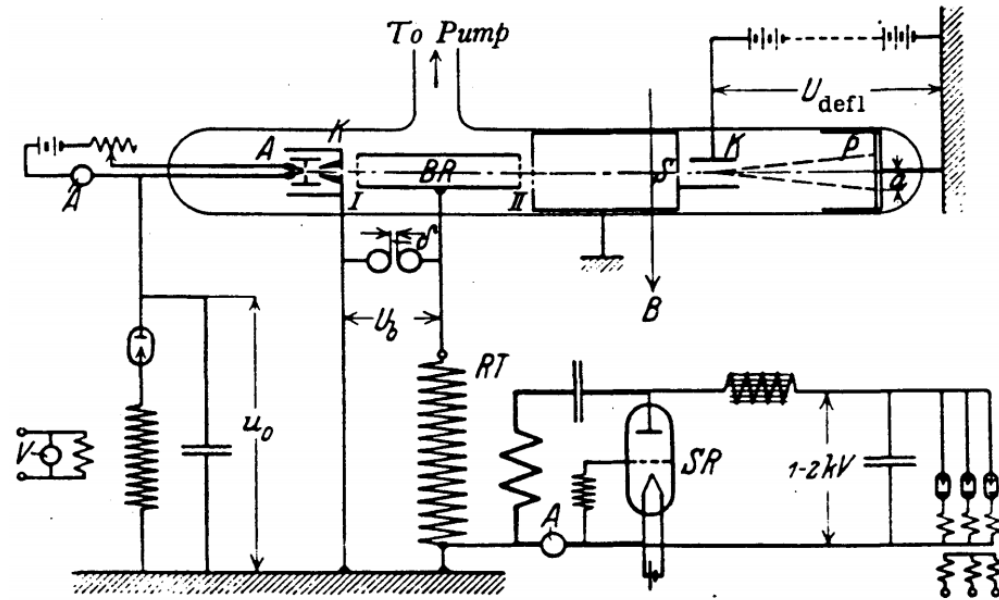
Wideroe's Device (from his thesis)

How do we create V(t)?

- What do we need?
  - Correct Frequency
  - Accelerating fields that are easy to access
  - "Clean" accelerating field distribution
  - Reasonable mechanical properties
  - Efficiency energy storage

- Coaxial Waveguide Modes!
- Cylindrical Waveguide Modes!
  - Everything else is just topological adjustments of these

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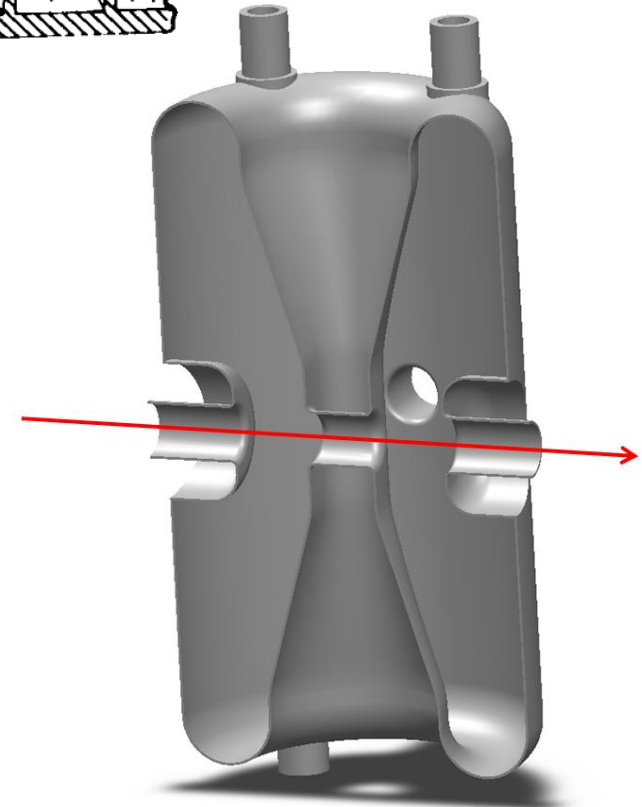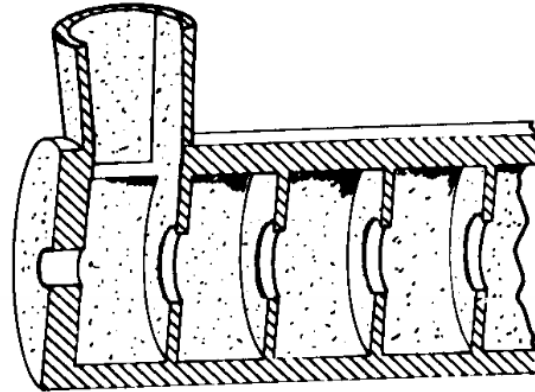
How are these cavities judged?

$$\Delta W = q \times V_0 \times TTF(\beta) \times \cos(\phi)$$

Charge

Maximum Accelerating Voltage

Transit Time Factor

Longitudinal Focusing Term

- R/Q
  - Measure efficiency of transferring energy to the beam

$$R/Q = \frac{V_{acc}^2}{\omega U}$$

- Geometry Factor
  - Measure of efficiency of energy storage in the cavity

$$G = \frac{\omega U}{P_d/R_s}$$

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Other Design Considerations

- Mechanical Issues
  - In operation, a cavity is exposed to many different pressures that deform their shape
  - How this deformation changes the cavity frequency and performance must be well understood
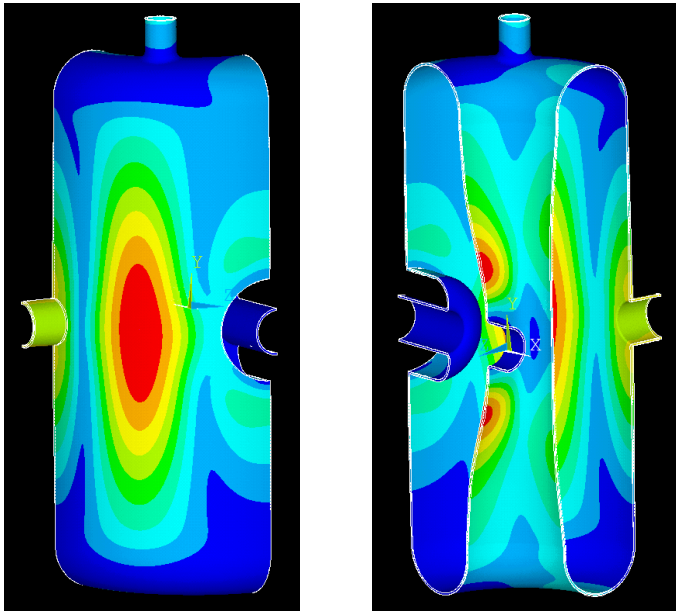
- Higher Order Modes
  - As the beam passes through the cavity, it can excite ALL cavity modes
  - Strong beam asymmetry or offset increases the strength of this coupling
  - These modes must be strongly damped or they can cause emittance growth or beam breakup… in some cases
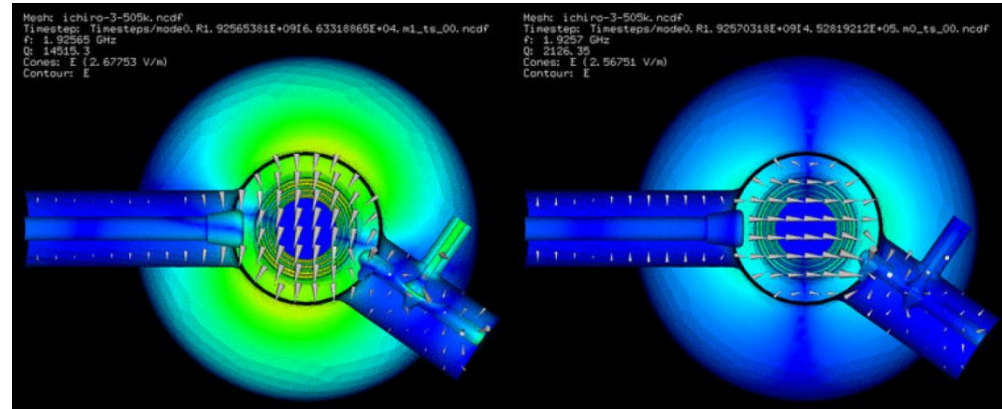
Cavity Design for Different Accelerators

- Synchrotrons (Ring Machines)
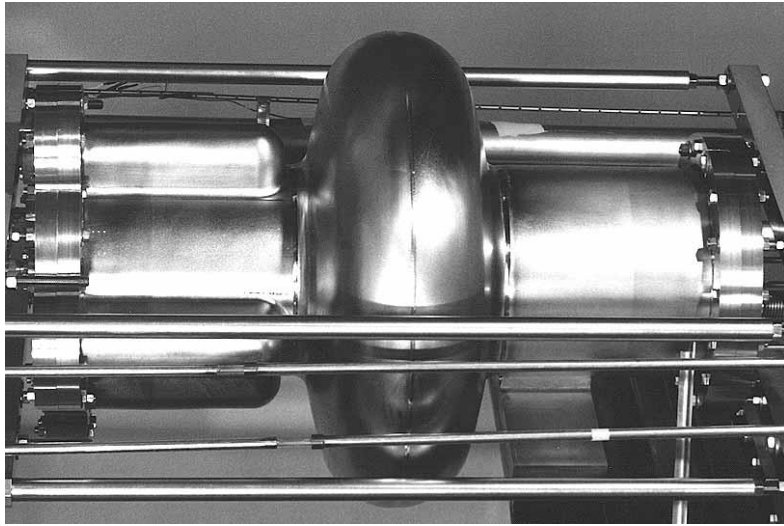 - The beam sees the cavity MANY times, low gradient is typical
  - Field must be very clean and stable
  - Very heavy higher order mode damping
  - Very large aperture
  - Acceleration and bunching

- Linacs (Linear Accelerators)
  - Single (or low #) pass machine
  - High Gradient is KEY (reduces # of cavities needed, therefore $$$)
  - Reliability and ease of fabrication is very important (many cavities)
  - Efficiency of operation also important

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Summary by Tom Wangler (RF Linear Accelerators)

- In DC accelerators the energy gain is limited by the maximum applied voltage, which is limited by electric breakdown.

- In RF accelerators (linacs, synchrotrons, cyclotrons) the final energy can exceed the maximum applied voltage, which is applied repeatedly to the beam. **The final energy is limited only by economics.**

- Synchrotrons are limited to low beam currents by beam instabilities associated with the repetitive cycling of the beam from turn to turn through unavoidable focusing lattice errors. (less true than it used to be)

- Cyclotrons are not pulsed but are limited to low beam currents by weak focusing and same inherent circular machine instabilities.

- Linacs can deliver high beam currents because they can provide strong focusing to confine the beam and are not subject to circular-machine repetitive instabilities.

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Maxwell to Helmholtz

Basic Equations

- Maxwell's Equations are very general, govern all classical electromagnetic interactions

- $\vec{\nabla} \cdot \vec{D} = \rho$

- $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

- $\vec{\nabla} \cdot \vec{B} = 0$

- $\vec{\nabla} \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

Where $\vec{D} = \epsilon_0 \vec{E}$ and $\vec{B} = \mu_0 \vec{H}$

- Lorentz Force
- Stored Energy density in EM fields
- Poynting Vector is useful conceptual tool (direction of energy flow in EM fields)

- $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

- $u = \dfrac{1}{2}\left(\epsilon_0 \vec{E}^2 + \dfrac{1}{\mu_0}\vec{B}^2\right)$ in vacuum

- $\vec{S} = \dfrac{1}{\mu_0}\vec{E} \times \vec{B}$ in vacuum

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Helmholtz Equation

Any solution to the wave equation with a time-independent form can be represented by the Helmholtz Equation

The solutions we want for RF applications are sinusoidal in time with frequency ω.

Particles will generally be accelerated in vacuum, so we can assume $\rho = \vec{J} = \epsilon_r = \mu_r = 0, c^2\mu_0\epsilon_0 = 1$

- Starting from $\left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\vec{E}(\vec{r}, t) = 0$

- Assume $\vec{E}(\vec{r}, t) = \vec{E}_s(\vec{r})T(t)$

- Gives $\frac{\nabla^2 \vec{E}_s}{\vec{E}_s} = \frac{1}{c^2 T}\frac{d^2 T}{dt^2}$

- Assume both sides equal to a constant $-k^2$

- Gives:

- $(\nabla^2 + k^2)\vec{E}_s = 0$

- $\left(\frac{d^2}{dt^2} + \omega^2\right)T = 0$

- We want: $T = e^{i\omega t}$ Good!

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Bring it all together - Free Space and Beyond

- Plugging this back into Maxwell's Equations, we get: $\omega = kc$

- Solving for $\vec{E} = \vec{E}_0 e^{-i\vec{k}\cdot\vec{r} - i\omega t}$

- Note that the magnetic field is specified: $\vec{B} = \dfrac{i}{\omega}\vec{k} \times \vec{E}$

- While we've solved for one frequency, we don't have to only work in one frequency

- If required we can solve for as many frequencies as required and add using Fourier Superposition to give the full, final field distribution

- Now, we have a powerful tool, just solve the Helmholtz Equation with whatever boundary conditions are appropriate to give us the RF fields in a structure of our choosing!

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Boundary Conditions

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Boundary Conditions

- For most of this class, we'll only be considering surface boundary conditions

- For magnet design, it's often important to simulate the device plus its surroundings because those fields (can) penetrate through materials, significantly complicating simulation

- For RF, metallic walls serves as an effectively perfect shield with very small (negligible) penetration depth

- Additionally, RF transmission will be bounded and well behaved, not free travel of plane waves

- Free propagation and interaction with EM fields and charges, like in a Free-Electron Laser are significantly more complex and will not be covered here (See G. Stupakov's class at USPAS)

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Perfect Conductor Boundary Conditions

- Gauss'/Faraday's Laws applied to a pillbox/loop on the metallic surface gives two conditions:

- $D_{n1} - D_{n2} = \rho_s$
- $E_{t1} - E_{t2} = 0$
- Any field in the metal (2) will drive currents, so the only stable configuration is with $D_{n2} = E_{t2} = 0$.

- Gauss'/Ampere's Laws applied to a pillbox/loop on the metallic surface gives two conditions:

- $B_{n1} - B_{n2} = 0$
- $H_{t1} - H_{t2} = J_s$
- Again, surface currents are fine, but not inside the material, so $B_{n2} = H_{t2} = 0$.

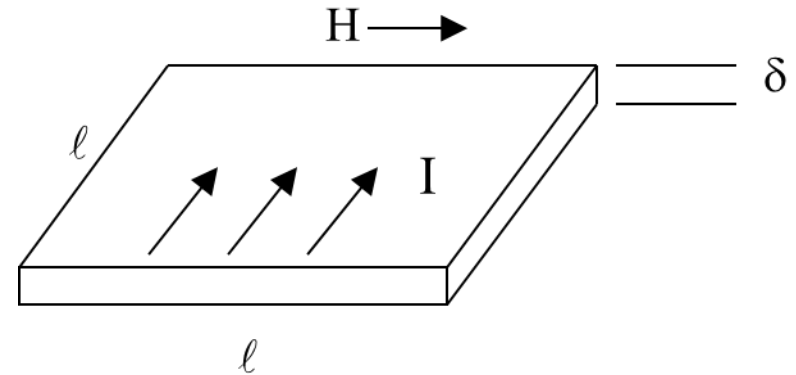Electric Metallic Boundary Condition:
$$E_{transverse} = 0$$

Magnetic Metallic Boundary Condition:
$$H_{normal} = 0$$

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Normal Conducting Materials – Skin Depth

- The charge/current density at the surface isn't infinite.

- Fields do penetrate into the material some amount.

- Fortunately, the generalized Helmholtz equation still applies, and gives

- $\delta^{-1} = \sqrt{\pi f \mu_0 \sigma}$

- We will get to losses later

For copper at 1 GHz, this gives a 2um skin depth. This perturbation on the very small for most applications. (Also, very simulation intensive)

Magnetic fields drive surface currents in a thin layer near the surface to oppose the fields penetrating the metal

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Additional Boundaries - In ComputerLand™

The boundary conditions given are for real materials and will always represent the physical boundaries of your device.

Additional types of boundaries are used to reduce the size of the simulation or represent an interface with the outside world.

I'll talk about losses later.

- Symmetry Boundaries
  - In 2D sims - rotational symmetry
  - In 3D sims - used to reduce size of the simulation and generally takes one of the following forms:
    - Electric – just like metallic wall
    - Magnetic - opposite conditions of electric B.C.
    - Periodic – Fields must be related at two planes
  - Ports to the outside – Places for power to flow into or out of your structure
    - More complex B.C. that are defined in software.

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Basic RF Design Process – So Far

1. Define closed boundary that has all the features you want to simulate.

2. Choose appropriate boundary conditions, including fixed frequency or eigenmode solver?

3. Use software to solve the Helmholtz Equation for the system

4. Repeat. Many times.

- So, now the question becomes what geometries should we use?

- In the next section, we'll cover the most common geometries, play with them a while, then start to torture them into the most unreasonable geometries we can think of and see what's useful.

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Circular Waveguide Modes

Assumptions

- Metallic circular tube with Radius: R

- Axis of tube points in $+\hat{z}$

- If we assume that the wave is propagating in $+\hat{z}$, we get:

- $\vec{E}(\vec{x}, t) = \vec{E}_0(\rho, \phi)e^{ikz - i\omega t}$

- $\vec{H}(\vec{x}, t) = \vec{H}_0(\rho, \phi)e^{ikz - i\omega t}$

- This is just the generalized plane wave solution to Helmholtz

- Now, we assume $B_z = 0$ (gotta pick something)

- Also, we need the fields to be azimuthally consistent, so let's assume they have a ϕ dependence of the form $e^{im\phi}$ where m is an integer.

- $E_z(\rho, \phi, z, t) = f(\rho)e^{ikz - i\omega t - im\phi}$

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Helmholtz Solutions

- $E_z(\rho, \phi, z, t) = f(\rho)e^{ikz - i\omega t - im\phi}$

- $\dfrac{1}{c^2}\dfrac{\partial^2 \mathcal{F}}{\partial t^2} = \nabla^2 \mathcal{F} \Rightarrow \dfrac{1}{\rho}\dfrac{d}{d\rho}\rho\dfrac{df}{d\rho} - \dfrac{m^2}{\rho^2}f + \left(\dfrac{\omega^2}{c^2} - k^2\right)f = 0$
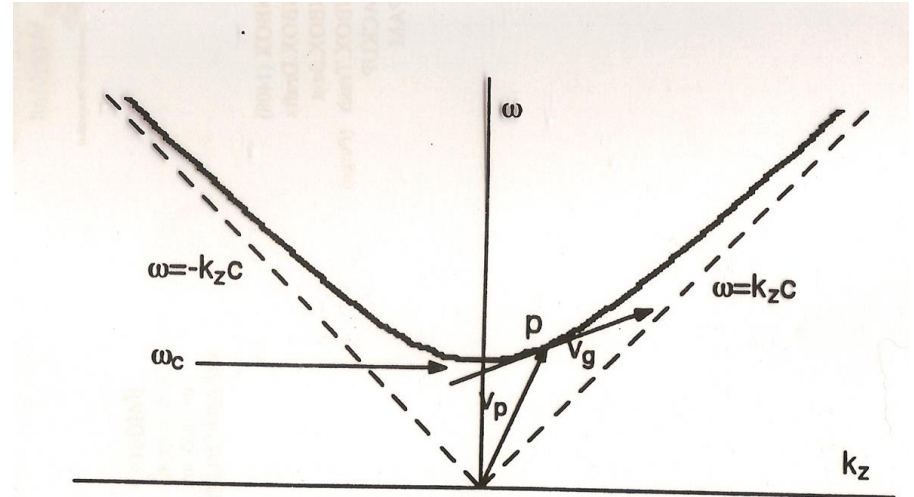
Math!

- $f(\rho) = E_0 J_m(k_\perp \rho)$ Bessel Functions! Wee.
- One last thing, we have to satisfy our electric field boundary condition: $\hat{n} \times \hat{E} = 0$
- Thus: $E_z(\rho = R) = 0$
- Gives the condition that $k_\perp R = j_{m,n}$ where $j_{m,n}$ is the n_{th} Bessel Zero of order m.

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Dispersion Curve

- The relationship between the wavenumber k and frequency ω is called the dispersion curve. For a circular waveguide, we get the following equation:

- $k_{m,n} = \pm \left(\dfrac{\omega^2}{c^2} - \dfrac{j_{m,n}}{R^2} \right)^{1/2}$

- Remember that $v_P = {}^{\omega}/_{k}$ and $v_G = {}^{d\omega}/_{dk}$

- Phase advances at v_p, energy advances at v_G

There is a frequency with no real solutions for k! Called Cutoff.

$$\omega_c = \frac{j_{m,n} c}{R_c}$$

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Practical Effects of Cutoff

- Cutoff is not a hard limit, i.e. frequencies above cutoff propagate freely and with no problems and frequencies below cutoff die immediately.

- From the dispersion curve, it's clear that the group velocity drops to zero at cutoff. Waves just above cutoff propagate very slowly, and this makes their transmission very sensitive to mechanical errors or perturbations (fractionally larger change in impedance close to cutoff). Very dangerous design situation.

- What does it mean to be below cutoff? Imaginary wave number!

- Imaginary k means that the wave propagates like e^{-kz}, exponentially decaying. Can be slowly so if k is small.

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Dispersion – Other Comments

- Every mode (different $j_{m,n}$) has a different cutoff. Higher order modes (higher m and n) will have a higher cutoff frequency. The most important implication of this is that there is a frequency range where only ONE mode is allowed. Which is nice.

- Circular Waveguide modes have a rotational degeneracy, which is a huge pain to work with. The lowest mode is a dipole mode, which with the slightest provocation, will rotate as it propagates. (Which is why you only use circular waveguides except in very specific applications)

Phase Velocity Synchronization

- To be useful for acceleration, the phase velocity must be synchronized with the particles ($v_P \leq c$)

- For Circular Waveguide modes:

- $$\frac{v_P}{c} = \frac{\omega}{c\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{j_{m,n}}{R}\right)^2}} = \frac{\omega}{\sqrt{\omega^2 - \left(\frac{j_{m,n}c}{R}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{j_{m,n}c}{\omega R}\right)^2}} > 1$$

- Not great. Why doesn't this break physics?

- This means that we can't synchronize no matter what.

- What can we do to slow down the phase velocity?

- Note:

  - $$\frac{v_G}{c} = \sqrt{1 - \left(\frac{j_{m,n}c}{\omega R}\right)^2}$$ which is always less than 1. Phew.

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Transverse Magnetic Modes (TM$_{m,n}$)

- Note: Use $\vec{\nabla} \times \vec{E} = i\omega\vec{B}$ to get the magnetic field.

- $E_z = E_0 J_m \left(j_{m,n} \frac{\rho}{R} \right) e^{ik_{m,n}z - i\omega t - im\phi}$

- $E_\rho = E_0 \frac{ik_{m,n}R}{j_{m,n}} J'_m \left(j_{m,n} \frac{\rho}{R} \right) e^{ik_{m,n}z - i\omega t - im\phi}$

- $E_\phi = -E_0 \frac{mk_{m,n}R^2}{\rho j_{m,n}^2} J_m \left(j_{m,n} \frac{\rho}{R} \right) e^{ik_{m,n}z - i\omega t - im\phi}$

- $B_\rho = E_0 \frac{m\omega R^2}{c^2 j_{m,n}} J_m \left(j_{m,n} \frac{\rho}{R} \right) e^{ik_{m,n}z - i\omega t - im\phi}$

- $B_\phi = E_0 \frac{i\omega R}{c^2 j_{m,n}} J'_m \left(j_{m,n} \frac{\rho}{R} \right) e^{ik_{m,n}z - i\omega t - im\phi}$

- Standard notation with m,n indicating the order.

- If you need TE modes, set $E_z = 0$ and go crazy.

Standing Wave Structures

- You can build an entirely different class of accelerating structures if you counter-propagate two traveling waves with $\pm \, k_{m,n}$

- Then you get a space terms:

- $\frac{1}{2}\left(e^{ik_{m,n}z} + e^{-ik_{m,n}z}\right) = \cos(k_{m,n}z)$

- $\frac{1}{2}\left(k_{m,n}e^{ik_{m,n}z} - k_{m,n}e^{-ik_{m,n}z}\right) = ik_{m,n}\sin(k_{m,n}z)$

- Rolling the factor of 2 into E_0 makes sense here.

- Assuming metal walls at 0 and L gives the requirement that $E_\phi(z = L) = E_\phi(z = 0) = E_\rho(z = L) = E_\rho(z = 0) = 0$

- Which leads to: $\sin(k_{m,n}L) = 0 \; \therefore \; k_{m,n}L = l\pi$

- Where l is the longitudinal order (TM$_{mnl}$ modes!)
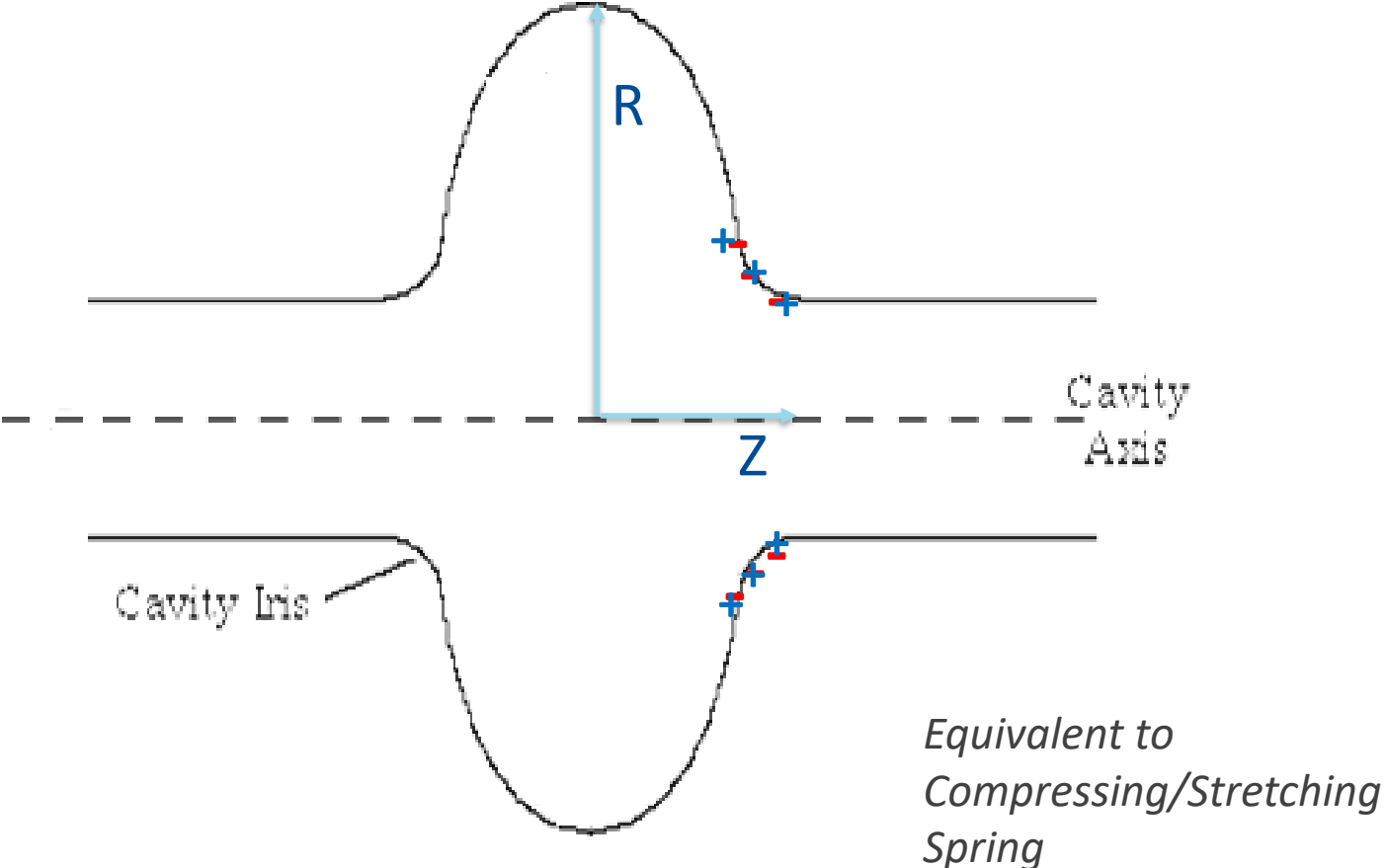
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Standing Waveguide Modes

- $E_z = E_0 J_m \left(j_{m,n} \frac{\rho}{R} \right) \cos \left(\frac{l\pi z}{L} \right) e^{-i\omega t - im\phi}$

- $E_\rho = -E_0 \frac{l\pi R}{j_{m,n} L} J'_m \left(j_{m,n} \frac{\rho}{R} \right) \sin \left(\frac{l\pi z}{L} \right) e^{-i\omega t - im\phi}$

- $E_\phi = -E_0 \frac{iml\pi R^2}{\rho j_{m,n}^2 L} J_m \left(j_{m,n} \frac{\rho}{R} \right) \sin \left(\frac{l\pi z}{L} \right) e^{-i\omega t - im\phi}$

- $B_\rho = E_0 \frac{m\omega R^2}{c^2 \rho j_{m,n}^2 L} J_m \left(j_{m,n} \frac{\rho}{R} \right) \cos \left(\frac{l\pi z}{L} \right) e^{-i\omega t - im\phi}$

- $B_\phi = E_0 \frac{i\omega R}{c^2 j_{m,n}} J'_m \left(j_{m,n} \frac{\rho}{R} \right) \cos \left(\frac{l\pi z}{L} \right) e^{-i\omega t - im\phi}$

- Note the change in the dispersion curve! No longer continuous with all frequencies allowed.

- $\omega_{m,n,l} = \sqrt{\left[\left(\frac{cl\pi}{L} \right)^2 + \left(\frac{c j_{m,n}}{R} \right)^2 \right]}$
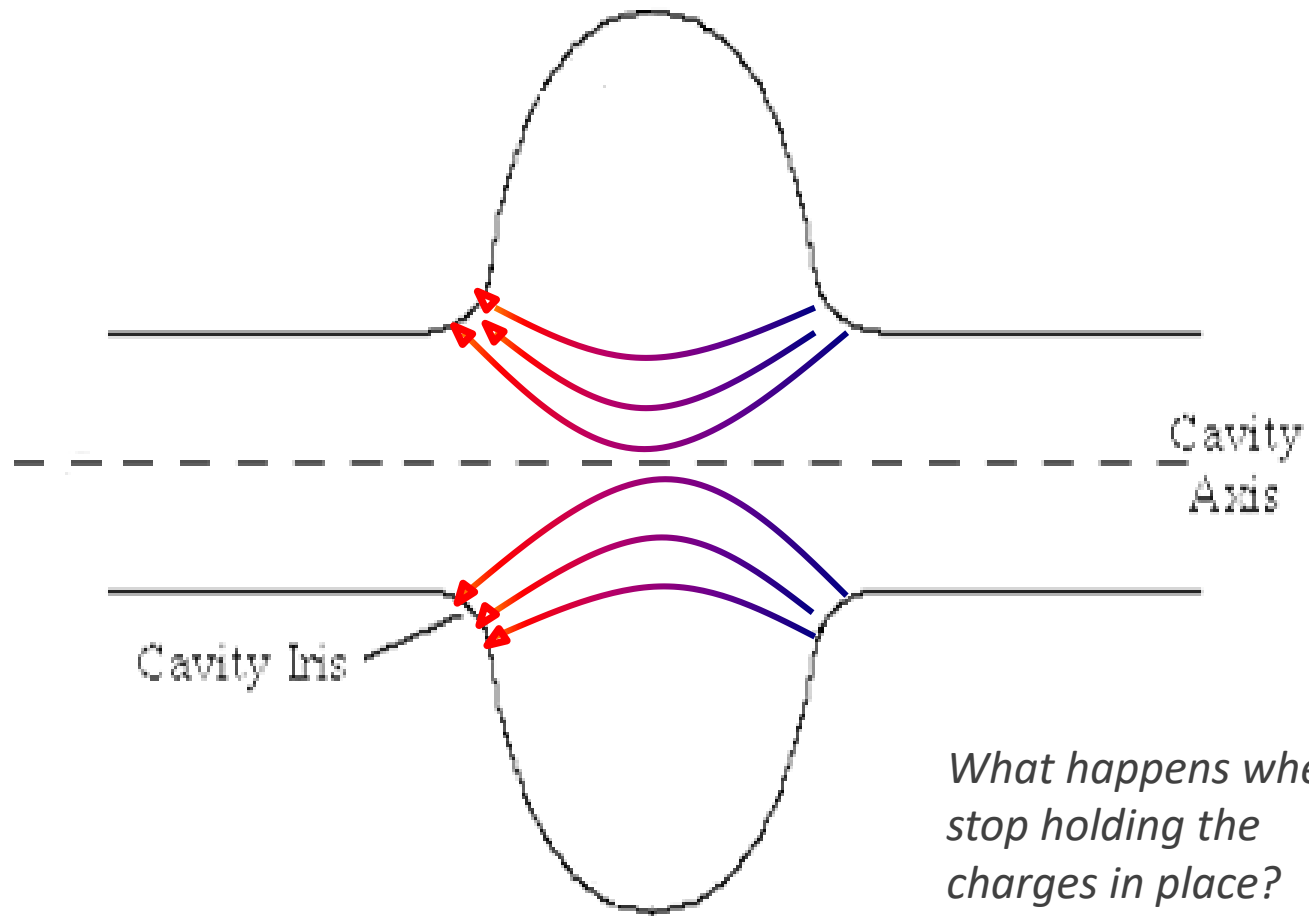
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Pillbox Cavity

- You can repeat all this for TE modes, but we want longitudinal electric fields for acceleration!

- Pick the lowest frequency, simplest mode: TM_{010}

- $B_\rho = E_\rho = E_\phi = 0$ and $j_{m,n} = 2.405$

- $E_z = E_0 J_0 \left(\frac{2.405 \rho}{R} \right) e^{-i\omega t}$

- $H_\phi = \frac{E_0}{\eta} J_1 \left(\frac{2.405 \rho}{R} \right) e^{-i\omega t} e^{\frac{i3\pi}{2}}$ with $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \cong 376.7 \; \Omega$ is the impedance of free space.

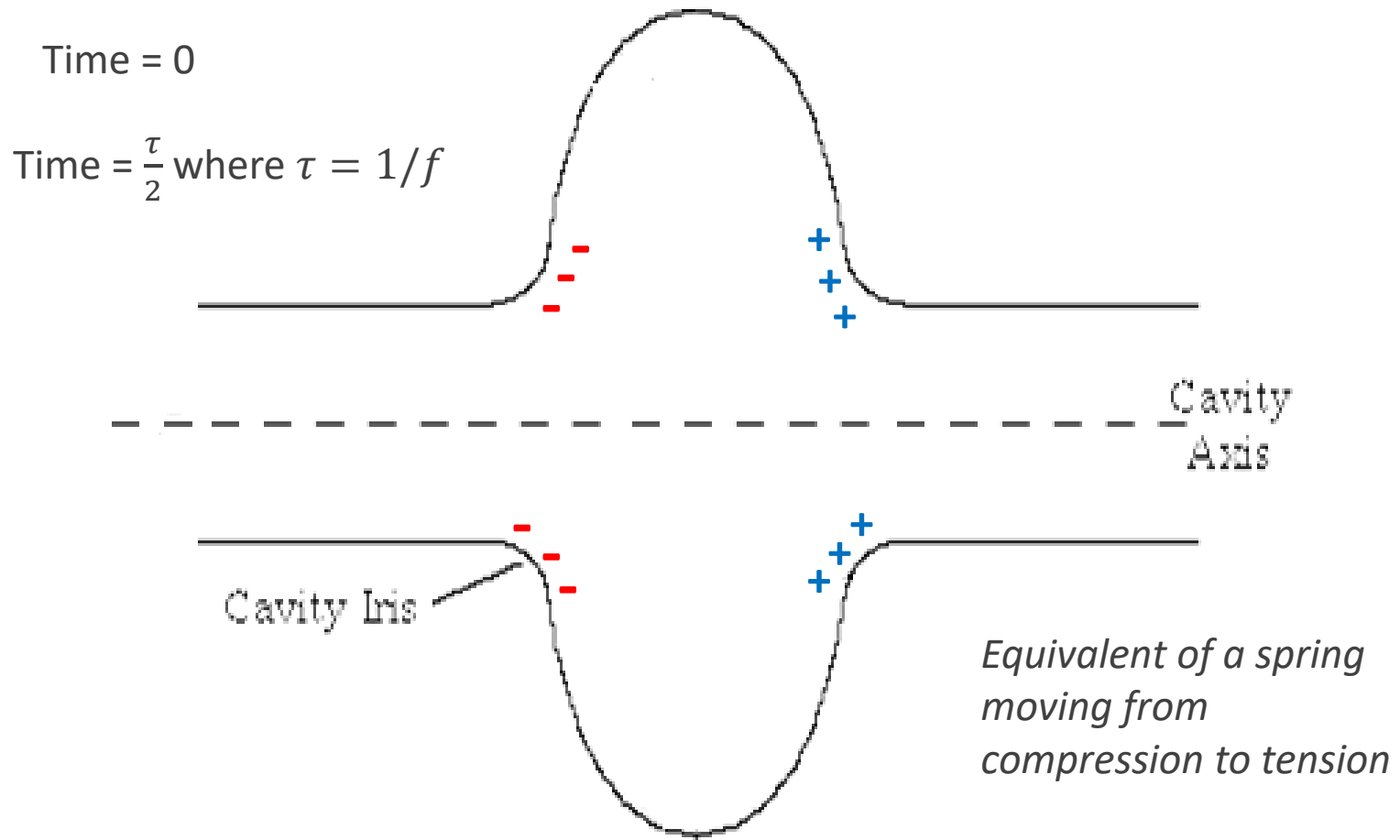- $\omega_{010} = \frac{2.405 c}{R}$ Note: only depends on radius, not length!

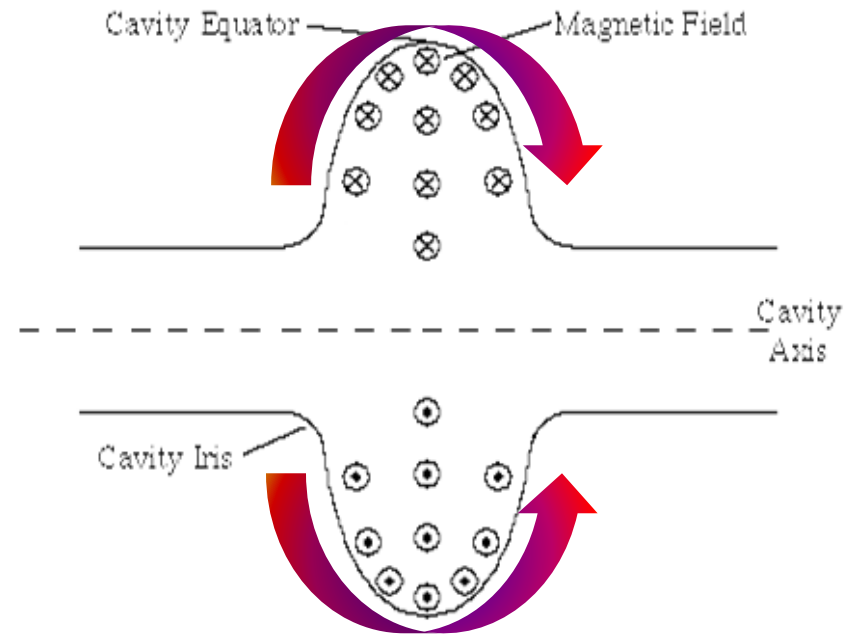🟦 **Fermilab**

Pillbox Monopole Mode – Current Model

R

Z

Cavity Axis

Cavity Iris

Equivalent to Compressing/Stretching Spring

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Resulting Electric Fields

Cavity Axis

Cavity Iris

What happens when we stop holding the charges in place?

Releasing the Spring

Time = 0

Time = $\frac{\tau}{2}$ where $\tau = 1/f$

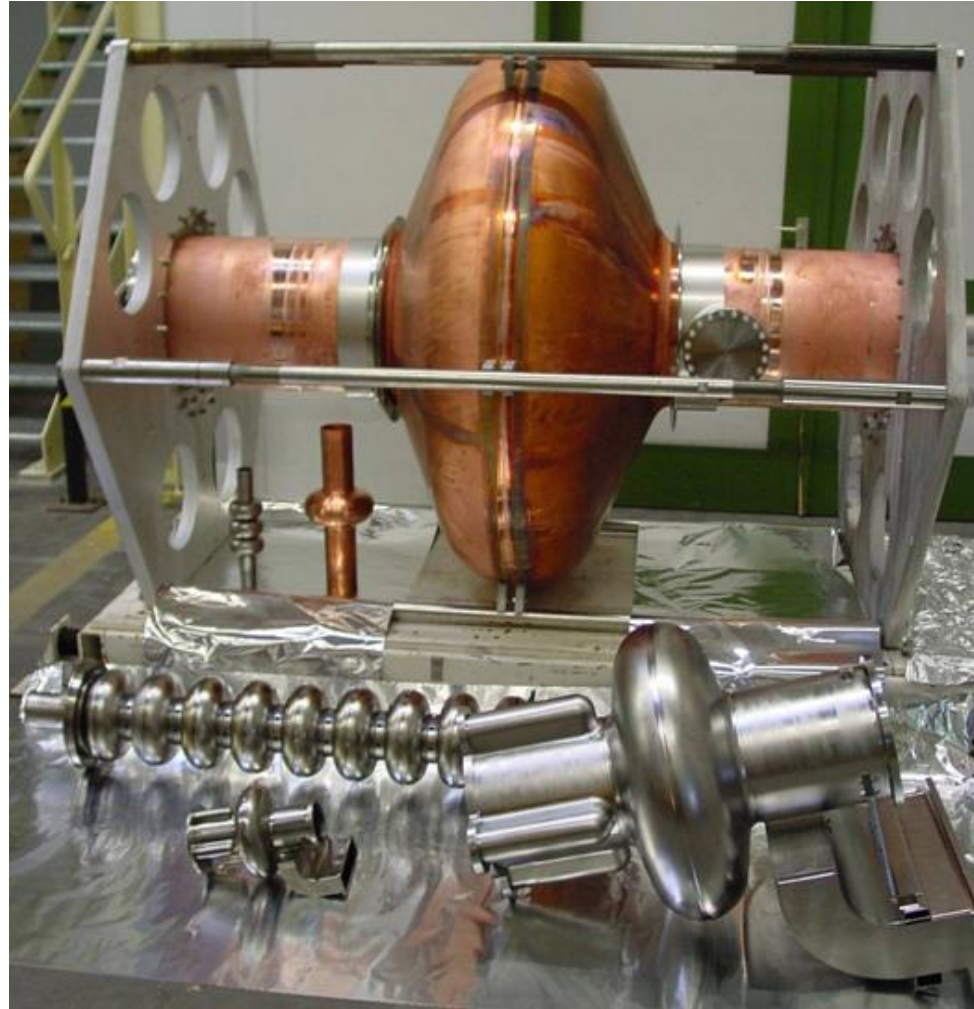*Equivalent of a spring moving from compression to tension*

Magnetic Fields

- At t = τ/4, all electric fields are gone, replaced by magnetic fields

- The moving charges act as currents, creating the magnetic fields around the cavity equator
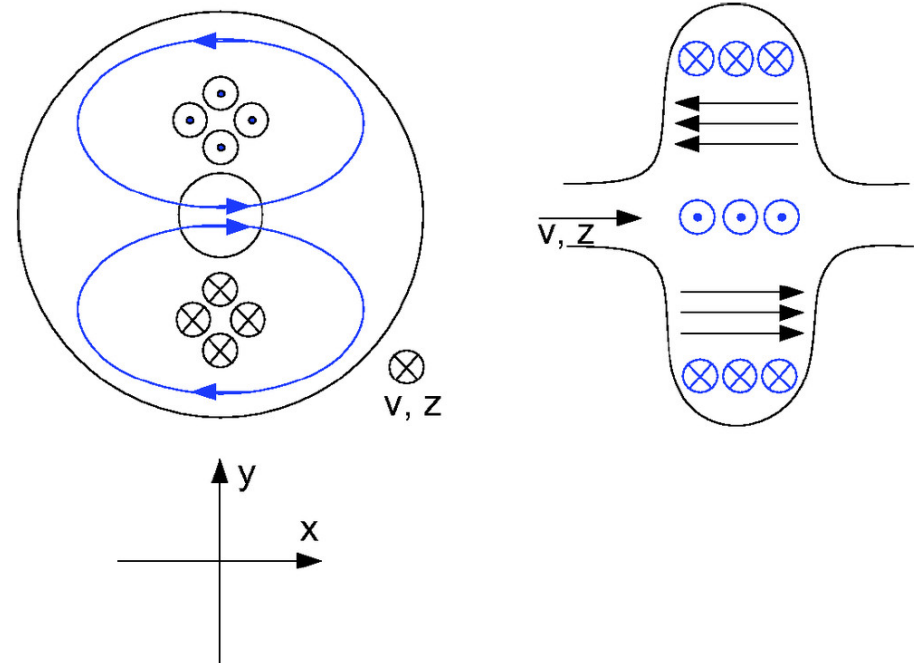
Examples of Monopole-Mode (Accelerating) Cavities

SPX Dipole-Mode Cavity

- Dipole-Mode: Two high electric field regions
- A repetition of the process we just used for the monopole mode shows:
  - Shape of Magnetic field
  - τ will be smaller (higher frequency)
- **Strong, Transverse Magnetic Field on Axis**
  - **Produces desired deflection**
- Degenerate modes must be split

- SPX Deflecting Mode

Now we…. Wellll…… First thing's first. RF Losses!

- Now it's unavoidable, how is power dissipated in a metallic surface?

- We showed that the skin depth was related to the conductivity and frequency: $\delta^{-1} = \sqrt{\pi f \mu_0 \sigma}$

- This came from solving for the fields in a metallic layer as it screened the imposed fields, and we did it with the Electric Field: $E_z = E_0 e^{-\tau_n x}$ where $\tau_n = \sqrt{i \omega \sigma \mu_0}$ (the real part of this gives the skin depth)

- We want the surface resistance, which is the real part of the surface impedance.

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Surface Resistance – Normal Conducting Materials

- First, need the total current: $I = \int_0^\infty j_z(x)dx =$
  $\int_0^\infty j_0 \, e^{-\tau_n x}dx = j_0/\tau_n$

- So, Impedance $Z_s = \dfrac{E_0}{I} = \dfrac{\tau_n}{\sigma} = \dfrac{\sqrt{i\omega\mu_0\sigma}}{\sigma} = R_s + iX_s$

- Turn the crank: $R_s = \sqrt{\dfrac{\pi\mu_0 f}{\sigma}} = \dfrac{1}{\sigma\delta}$

- Two things to note:
  - Highly conducting materials, low R_s ($\sim m\Omega$), good!
  - $R_s \propto f^{\frac{1}{2}}$ Increases with frequency, but not quickly.

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Surface Resistance – Superconducting Materials!

- Some materials, when cooled below a certain 'transition' temperature lose their DC resistance.

- Technically they are even better than a 'perfect conductor' because upon transition, they expel magnetic field instead of trap it.

- Most common superconducting material for cavities (but not only!) is niobium (9.2 K)

- However, no free lunch. While DC resistance is zero, RF resistance is merely very, very small (electrons still have mass, after all)

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Surface Resistance – Superconductivity!

- The physics of this is very different than normal metals:
  - Surface resistance is now determined by a far more complex physical process, modeled by BCS theory:
  - $R_{BCS} = \dfrac{2^{-4} C_{RRR}}{T} \left(\dfrac{f}{1.5}\right)^2 e^{-\frac{17.67}{T}}$
    - f is in GHz
    - T is in Kelvin
    - C_{RRR} varies from 1 to 1.5 depending on material purity
  - Even worse! High magnetic fields (the thing we'll be applying to the cavity) break the superconducting state.
  - If the superconductivity is broken in one place, it reverts to a normal conducting metal, and the dissipated power there will almost certainly rapidly heat the rest of the cavity above the transition temperature.

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Superconducting Practicalities

- Runaway is called a quench, and it's a bad thing.

- Peak surface magnetic field matters quite a bit for superconducting applications, often totally dominating design

- The real surface resistance, what's achievable, is actually a combination of effects:

- $R_S = R_{BCS} + R_{res}$ where R_{res} is a combination of many factors
  - Impurities on the cavity surface
  - Adsorbed gasses
  - Ambient magnetic field trapped during cooldown
  - Many more

- Modern processing techniques can achieve $R_s = 10n\Omega$ reliably in most applications, and sometimes $< 1n\Omega$ in certain circumstances (real cavities, though!).

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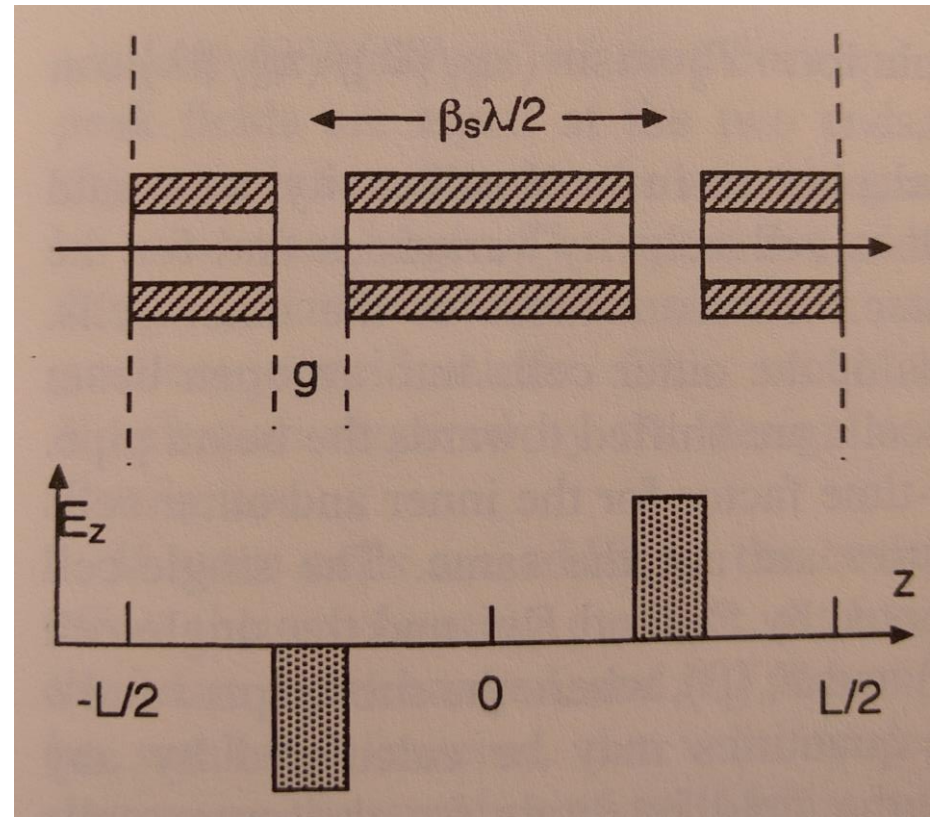# More superconducting…

- Last take away points:
    - $R_{S,SRF} \propto f^2$ Pushes applications to lower frequency
    - Complex dependence on temperature, but lower is almost universally better (from a performance point of view, not cost!)
    - Achieving the best performance is very labor/infrastructure/cost intensive. Just ask LCLS-II! Or ILC! Or XFEL! Or CEBAF!
- I'll spare you the math, but the equivalent skin depth for this application is about 350Å.
- Also, remember your Carnot: $\eta_c = \frac{T_C}{T_H - T_C}$, and operating at $4K$, we get $\eta_c = 0.013$. We save six orders of magnitude on R_s but lose three because of the temperature. We gain efficiency, but pay for it in complexity.

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Pillbox, for real this time.

- What quantities do we care about?
  - Accelerating Voltage
  - Stored Energy
  - Peak Surface Fields
  - Efficiency of storing energy
  - Efficiency of transferring energy to the beam
- Peak Fields are obviously defined.
- Let's tackle the others in detail.

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Accelerating Voltage – Transit Time Factor

- We must now account for the finite speed of the particles

- $dV_{acc} = E_{acc}(z)e^{i\omega t + \phi}dz$

- $dV_0 = |E_{acc}(z)|dz$

- $T = V_{acc}/V_0$

- Integrating across all synchronized gaps gives a Transit Time Factor less than 1

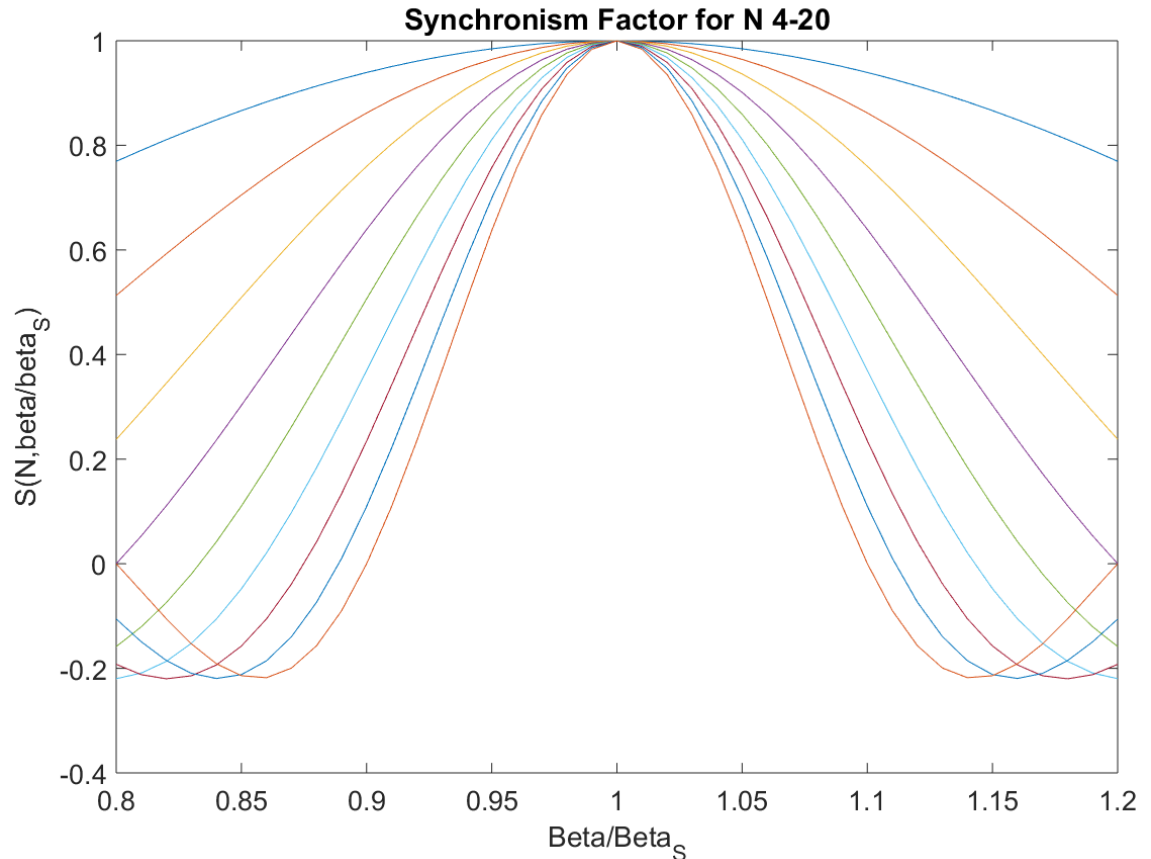- Model as $T = T_g S\left(N, \dfrac{\beta_s}{\beta}\right)$

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Gap Synchronism

Plotted is the synchronism factor for 20% error in β for gaps ranging from 4 to 20.

Larger number of gaps have smaller velocity acceptance.

Machine parameters drive design here, heavy ion v electrons, for instance.

For wide range of β, multiple cavity types may be needed.

FRIB is accelerating anything from carbon to uranium, so the acceptance has to be huge, SLC was only electrons

Synchronism Factor for N 4-20

PIP-II Cavity Choices

- Optimization of cavity styles, number, etc. is a very complex process
- Ultimately it comes down to complexity and cost
- This also shows the limited usefulness of geometric beta
- Look at the difference made by having one module of HWRs!
- Largest cost savings comes from reducing number of cavity types
- Electron machines don't worry about this

Table 3.6: Accelerating cavities in the PIP-II Linac and their operating ranges in the Linac. ($\beta_g = \beta_G$ for the HWR, SSR1 and SSR2 cavities, β_g for the elliptic cavities is defined as the ratio of regular cell length to half-wavelength. Fitting to Eq. 3.3 for the elliptic cavities yields: $\beta_G = 0.64$ for LB650 and $\beta_G = 0.947$ for HB650.)

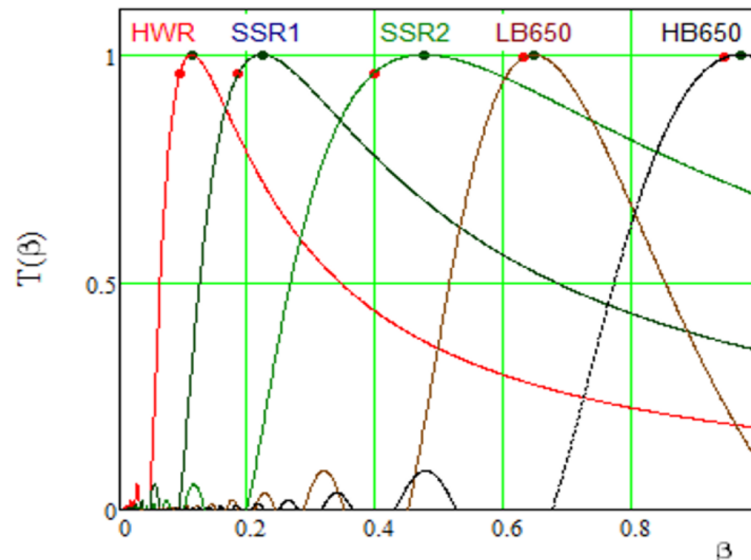| Cavity name	β_g	β_{opt}	Freq. (MHz)	Cavity type	Energy gain at β_{opt} per cavity (MeV)	Energy range (MeV)
HWR	-	0.112	162.5	Half wave resonator	2	2.1 - 10.3
SSR1	-	0.222	325	Single-spoke resonator	2.05	10.3 - 35
SSR2	-	0.475	325	Single-spoke resonator	5	35 - 185
LB650	0.61	0.65	650	Elliptic 5-cell cavity	11.9	185 - 500
HB650	0.92	0.971	650	Elliptic 5-cell cavity	19.9	500 - 800

Figure 3.18: Variation in the transit time factor with beam velocity for the PIP-II cavities. Red dots mark the position of β_G, and blue dots the position of β_{opt}.

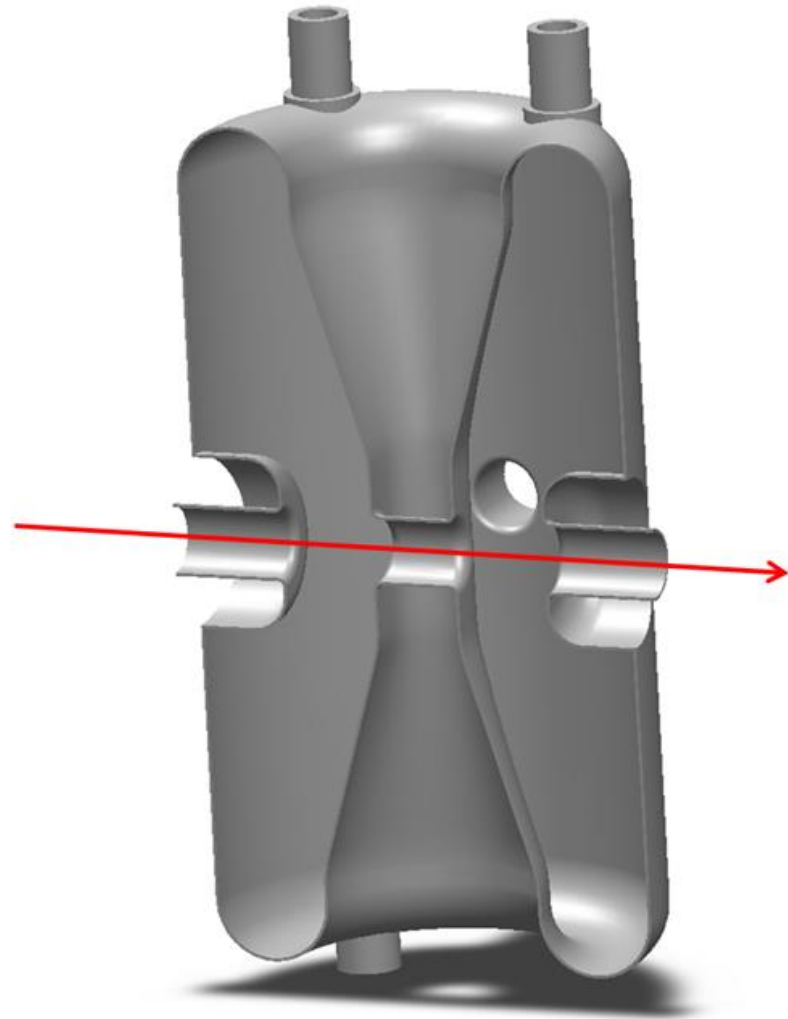🟦 **Fermilab**

Effective Length – A Warning

One Last Comment:

An often quoted figure of merit is the Accelerating Electric Field: $E_{acc} = \frac{V_{acc}}{L}$

While pillbox-style cavities are relative easy to determine the length, more complex geometries are more open to interpretation.

V_{acc} is unambiguous.

Pillbox: $E_{acc} = \frac{V_{acc}}{L} = \frac{2E_0}{\pi}$

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Stored Energy

- We stated earlier: $u = \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right)$

- So it follows that $U = \int_V \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right)dV$

- While this is generally true, we can chose a time where this calculation is easier. Choose time such that the electric fields are zero and magnetic fields are maximized.

- So, $U = \int_V \frac{1}{2}\left(\frac{1}{\mu_0}\vec{B}^2\right)dV$

- Generally, this is done for you in simulation. For a pillbox, this can be done analytically.

- $U = E_0^2 \pi L \epsilon_0 \int_0^R \rho J_1^2\left(\frac{2.405\rho}{R}\right)d\rho = \frac{\pi \epsilon_0 E_0^2}{2}J_1^2(2.405)LR^2$

Peak Surface Fields

- We want to calculate the peak surface fields.

- $E_{pk} = E_0$ is easy.

- Maximizing magnetic field on the end wall:

- $B_{pk} = \frac{E_0}{c} J_1(1.84) = \frac{E_0}{c} 0.583$ or where $\rho = 0.77R$

- But what we also want are normalized quantities.

- $\frac{B_{pk}}{\sqrt{U}}, \frac{E_{pk}}{\sqrt{U}}$ and, by extension, $\frac{V_{acc}}{\sqrt{U}}$

- These quantities can be scaled nicely, and are less prone to change during optimization of unrelated features.

- Speaking of, that last one seems quite useful…

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Shunt Impedance

- Remember, we want a quantity that can be used to judge the efficiency of transferring the stored energy to the beam.

- The (effective) shunt impedance is defined as:

- $\frac{R}{Q} \overset{\text{def}}{=} \frac{V_{acc}^2}{\omega U}$ which is the ratio of the accelerating voltage squared and the reactive power in the cavity (in the equivalent circuit).

- This is a purely geometric factor that is very useful in describing the accelerating efficiency of a cavity geometry.

- Other definitions of this may not include the TTF, or may have a factor of two for historical reasons, so watch out.

- Note that this does not scale with frequency. You can directly scale a geometry to a different frequency, and this will stay the same. Very useful.

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Shunt Impedance 2

- Smashing together the equations we know for a pillbox:

- $\frac{R}{Q} \cong 150 \, [\Omega] \, \frac{L}{R} \cong 196\beta \, [\Omega]$

- Linear with optimum particle velocity! Higher frequencies are better.

- Makes sense, U scales like L, but so does V_{acc}.

- Note: Reactive Power in circuit theory is the power flow IN the resonator, in this case equivalent to ωU

- Think of this as the full stored energy in the cavity flowing through a plane ω times a second

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Quality Factor

- A standard metric for how efficiently a resonator stores energy is the quality factor.

- This is a quantity related to the number of cycles it would take to dissipate a given amount of stored energy.

- $Q_0 = \frac{\omega U}{P_d}$ But this means that we need a definition of P_d

- Fortunately, we've done the ground work:

- $P_d = \frac{1}{2} R_s \int_S \left| \vec{H} \right|^2 dA$ Integrated over the cavity walls

- Note the implicit assumption, that surface resistance is uniform over the entire cavity! Probably not the greatest assumption for superconductors, but not much else you can do without significant effort.

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Temperature Mapping

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Geometry Factor

- R_s is quite variable, especially for superconducting cavities.
- The quality factor that doesn't depend on R_s would be of great usefulness.
- The R_s dependence comes from the dissipated power.
- $Q_0 = \frac{\omega U}{P_d} = \frac{\omega U}{\frac{P_d}{R_s} R_s}, G = R_s Q_0 = \frac{\omega U}{\frac{P_d}{R_s}}$
- This, while adding dimensions to the quality, depends strictly on geometry and not material.
- Again, doesn't scale with frequency (make sure to gather all the scaling of U and P_d)

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Pillbox Quality Factor

- $P_d = \frac{R_s E_0^2}{\eta^2}\left\{2\pi \int_0^R \rho J_1^2\left(\frac{2.405\rho}{R}\right) d\rho + \pi R L J_1^2(2.405)\right\}$

- Outer wall + end wall

- $P_d = \frac{\pi R_s E_0^2}{\eta^2} J_1^2(2.405)R(R+L)$

- Giving:

- $G = \frac{\omega_0 \mu_0 L R^2}{2(R^2+RL)} = \eta\,\frac{2.405L}{2(R+L)} = \frac{453\frac{L}{R}}{1+\frac{L}{R}}\,[\Omega]$ With an optimum L ...

- $\frac{L}{R} = \frac{\beta\pi}{2.405}\,,\qquad G = 257\beta\,[\Omega]$

- A highly useful result, indicating that pillbox cavities are more efficient at higher optimum particle velocities.

Cryogenic Efficiency

- A quantity that is often used to compare efficiency of superconducting cavities is $\frac{R}{Q} * G = \frac{V_{acc}^2}{\frac{P_d}{R_S}}$

- Calculates directly cost of voltage to dissipated power.

- Cryogenic refrigeration is at a premium, so this can be an excellent comparison between very different cavity geometries.

Pillbox Scaling

Clearly better at high beta, best at $\beta = 1$.
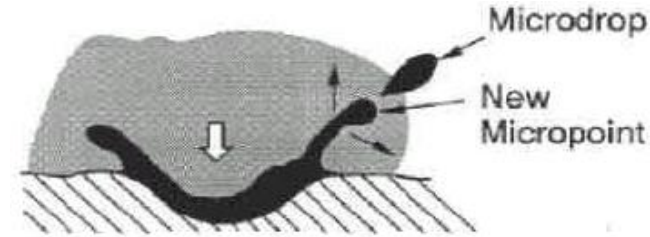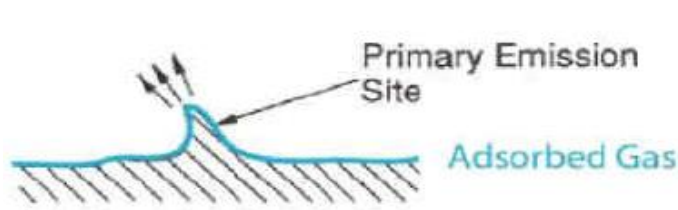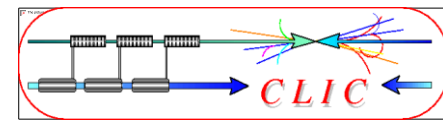
Mechanical concerns also come into play:

Aspect ratio:
$$\frac{L}{R} = \frac{\beta\pi}{2.405}$$

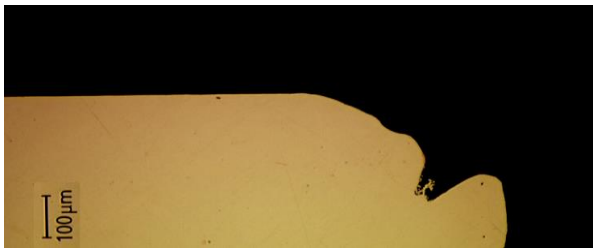This gets pretty sub-optimal at low beta, thin pancake cavities have poor mechanical properties.

- $G = 257\beta\,[\Omega]$

- $\dfrac{R}{Q} = 196\beta\,[\Omega]$

- $E_{pk} = E_0$

- $cB_{pk} = 0.583E_0$

- $U = \dfrac{\pi\epsilon_0 E_0^2}{2}J_1^2(2.405)LR^2$

- $P_d = \dfrac{\pi R_s E_0^2}{\eta^2}J_1^2(2.405)R(R+L)$

- $TTF = \dfrac{2}{\pi}$

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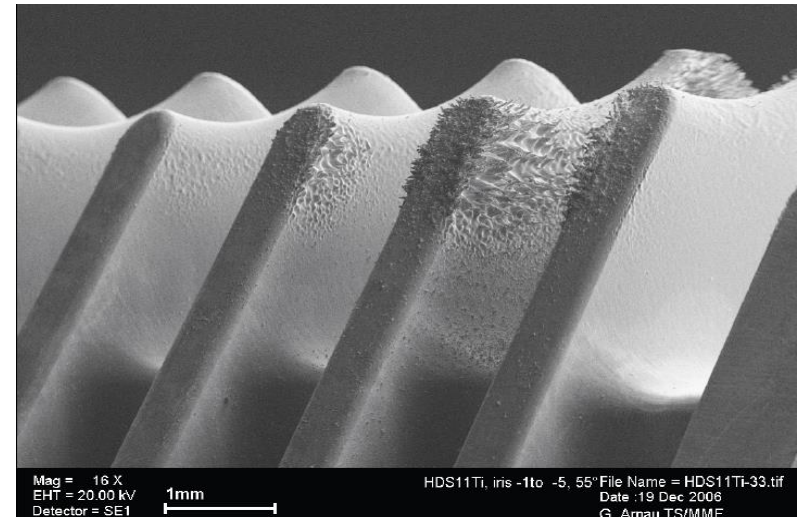
Material Comparison

- Superconducting Cavity
  - Peak Surface Fields dominate design
  - ~220 mT is theoretical max, 120 mT is doing very well in practice
  - Pushes for high Q
  - Technologically Challenging
  - Processing requirements put significant constraints on complex cavity geometries
  - $R_s \propto f^2, P_d \propto f, Q \propto f^{-2}$

- Normal Conducting Cavity
  - Limited by dissipated power
  - Limits duty cycle or gradient
  - Pushes for highest $\frac{R}{Q}$
  - Local power density also a concern (local heating), maxes at ~20 W/cm^2
  - Electrical breakdown limited peak electric fields
  - Cheaper material (copper!)
  - Cooling design can be quite complex (non-uniform)
  - $R_s \propto f^{\frac{1}{2}}, P_d \propto f^{-\frac{1}{2}}, Q \propto f^{-\frac{1}{2}}$

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- More energy: electrons generate plasma and melt surface

- Molten surface splatters and generates new field emission points!
  $\Rightarrow$ limits the achievable field

- Excessive fields can also damage the structures

- Design structures with low E_{surf}/E_{acc}

- Study new materials (Mo, W)

Damaged CLIC structure iris
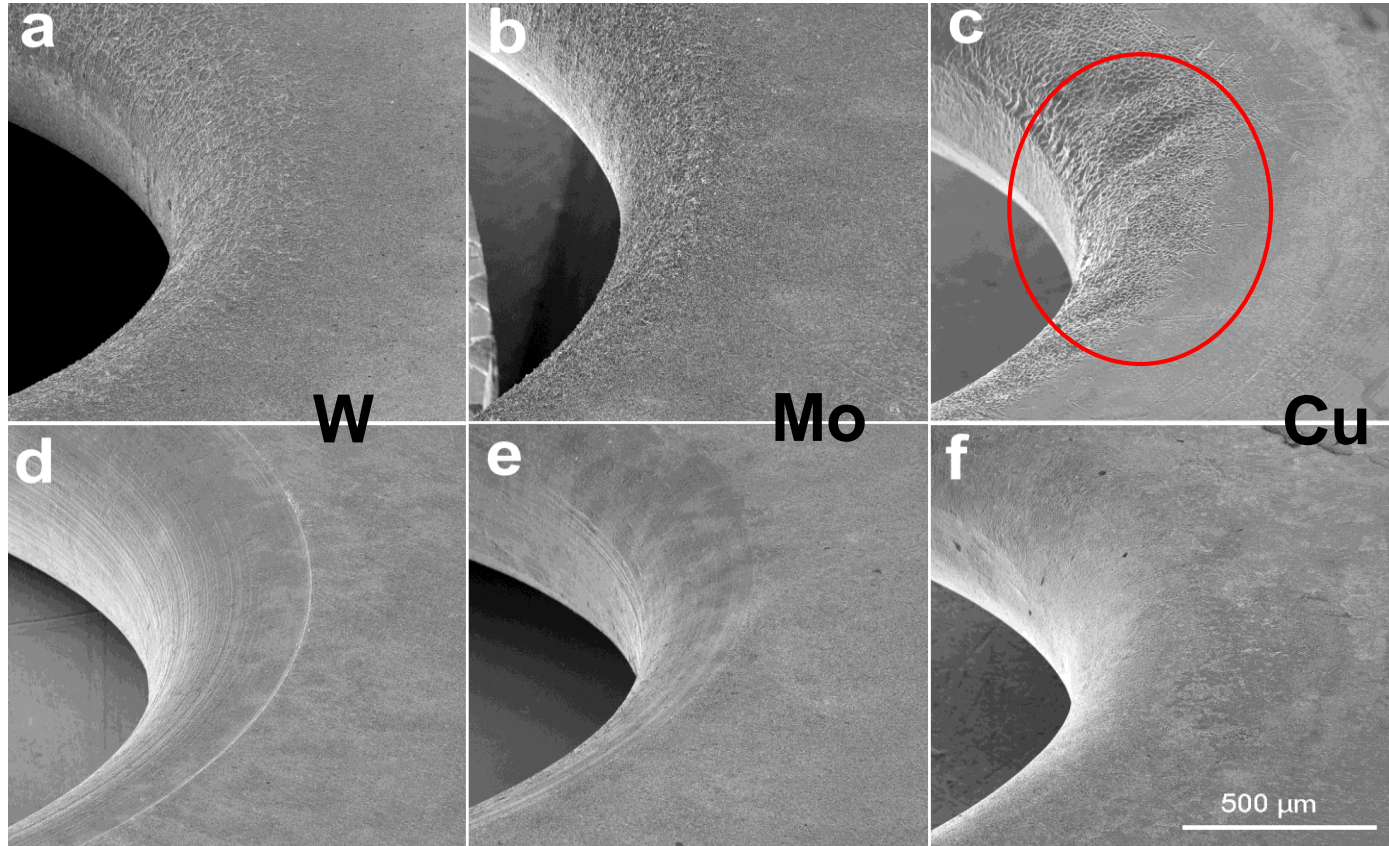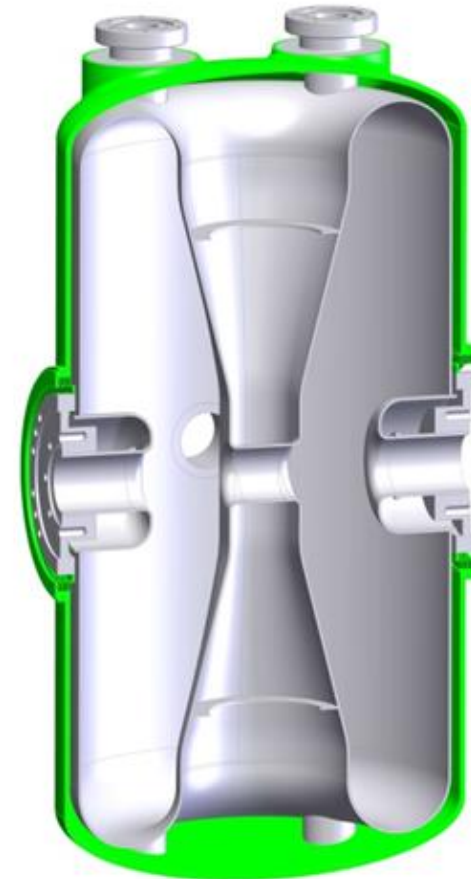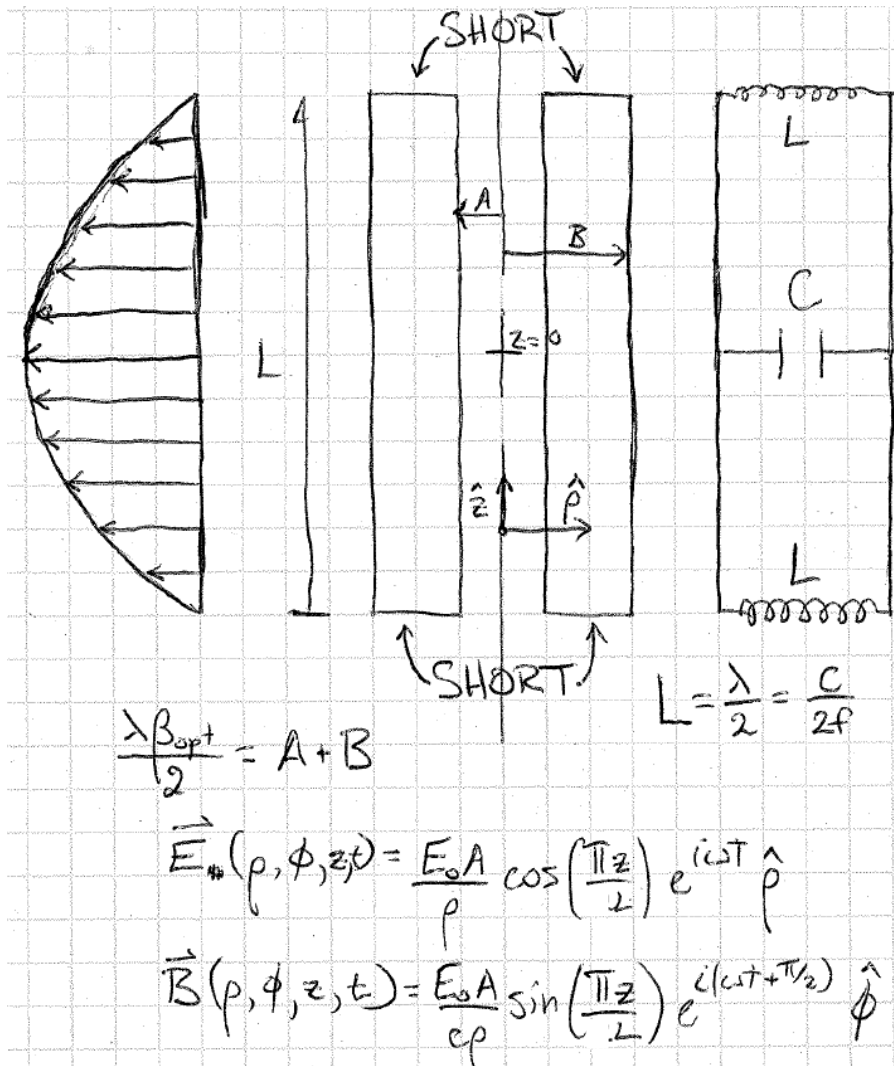
First
iris

downstream
iris

Damage on iris after runs of the 30-cell clamped structures tested in CTFII.
First (a, b and c) and generic irises (d, e and f) of W ,Mo and Cu structures respectively.

Coaxial Resonators

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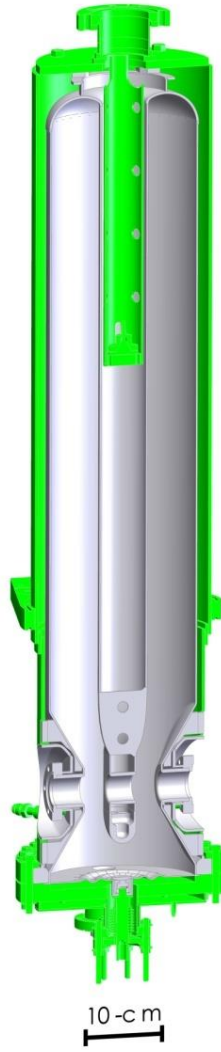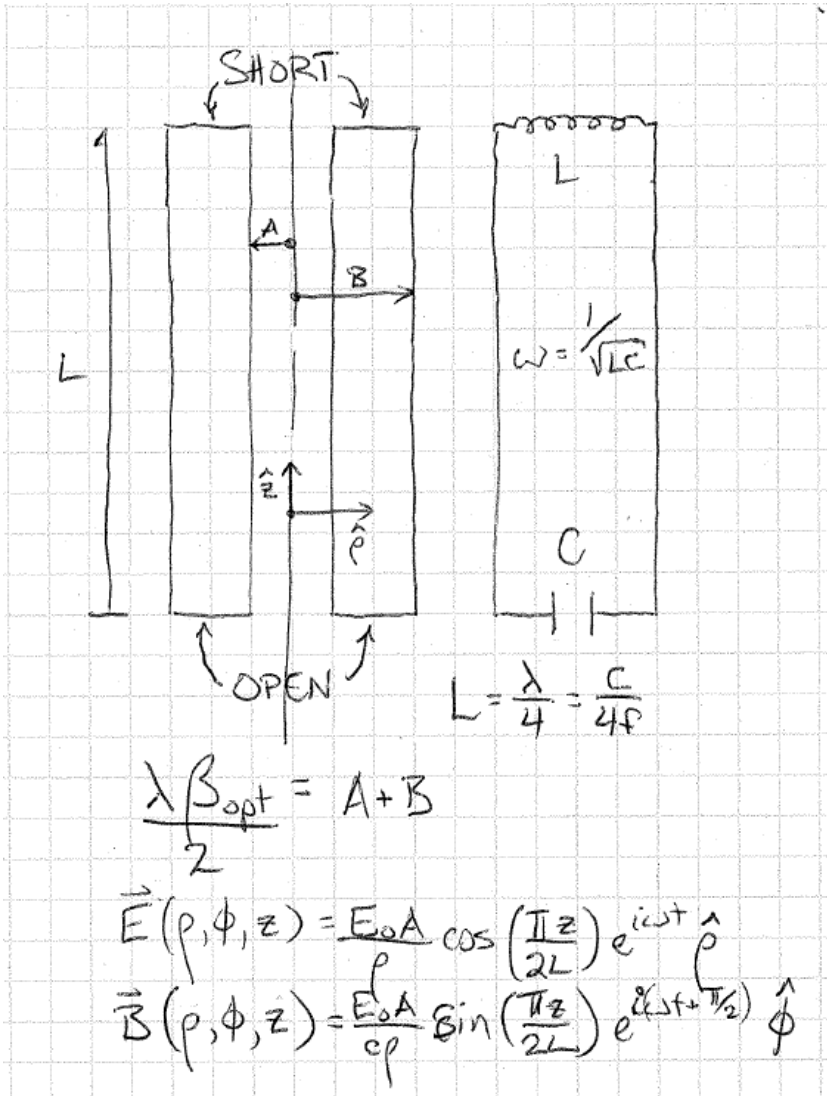
Coaxial Waveguide

- A fundamentally different transmission line is coaxial geometry

- In contrast to circular/rectangular waveguide, there is a second conducting surface that's disconnected (in a waveguide) from the outer conductor.

- Assume that we have a cylindrical outer conductor, radius b and co-radial inner conductor, radius a. Both are aligned on the $\hat{z}$ axis.

- Solving the Helmholtz Equation and putting shorting plates at $\pm \frac{L}{2}$ we get similar solutions:

- $E_\rho = \frac{E_0 a}{\rho} \cos\left(\frac{p\pi z}{2L}\right) e^{i\omega t}, \; B_\phi = -i \frac{E_0 a}{\rho c} \sin\left(\frac{p\pi z}{2L}\right) e^{i\omega t}$

- $\omega = pc\pi/2L$

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Half Wave Resonator

$$\frac{\lambda \beta_{opt}}{2} = A + B$$

$$\vec{E}_{\#}(\rho, \phi, z, t) = \frac{E_0 A}{\rho} \cos\left(\frac{\pi z}{L}\right) e^{i\omega T} \hat{\rho}$$

$$\vec{B}(\rho, \phi, z, t) = \frac{E_0 A}{c\rho} \sin\left(\frac{\pi z}{L}\right) e^{i(\omega t + \pi/2)} \hat{\phi}$$

$$L = \frac{\lambda}{2} = \frac{C}{2f}$$

10-cm

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Quarter-Wave Resonator

Short

"Open"

10-cm

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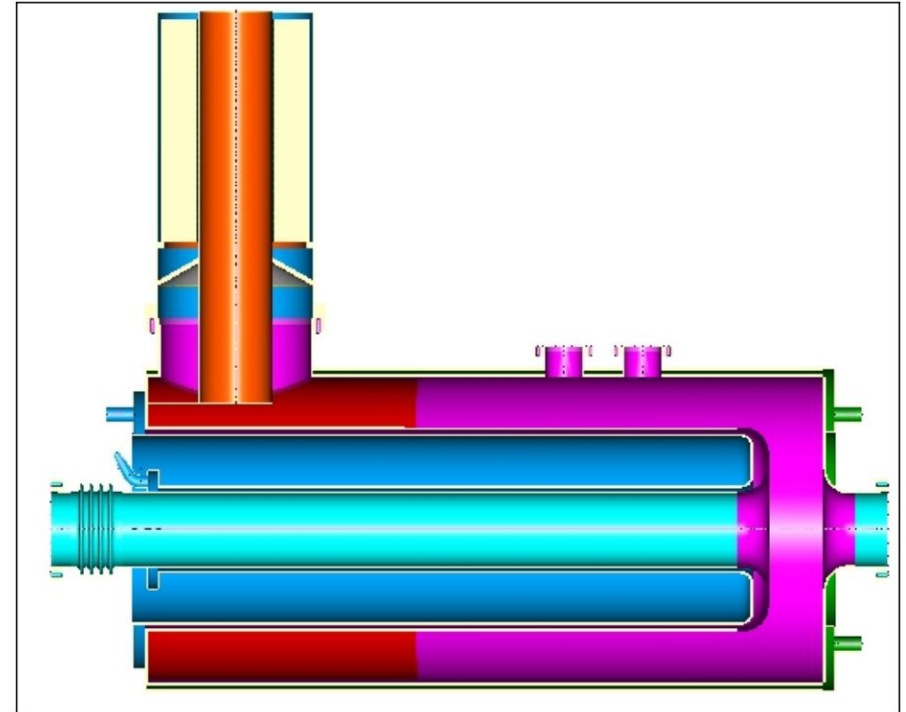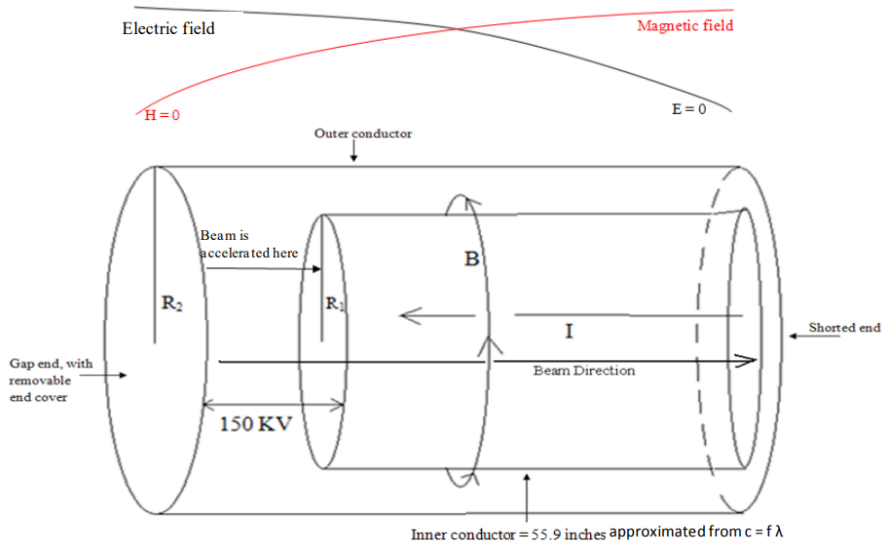
Coaxial Cavity Discussion

- Decouples beam line/accelerating gap size/geometry from the transverse dimension.

- Allows very low frequency resonators with small gaps in a mechanically robust geometry, very low beta resonators.

- Complicated fabrication and processing

- Quarter Wave Resonators are significantly different from ideal because the 'open' boundary condition isn't physical.

- Lack of rotational symmetry can lead to transverse accelerating fields, especially with QWRs.

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Technical Point of Order (Quarter Wave OR Pillbox?)

FNAL Recycler Upgrade RF Cavity

https://indico.fnal.gov/event/2665/contribution/10/material/paper/1.pdf

FNAL Main Injector RF Cavity

http://lss.fnal.gov/archive/2005/conf/fermilab-conf-05-102-ad.pdf

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Summary

- We went through an overview of how bound RF waves could be used to build accelerators, and we saw a bit of the process of optimization that's used to arrive at a final design.

- We got to a pillbox cavity, the most simple work-horse cavity topology that's used today.

- For homework, you'll explore simple cavity geometries and fields.

- Tomorrow is a deep dive into how we characterize and optimize these cavities and into the zoo of topologies actually used in the real world (read: lots of pictures!).

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