

Beam analysis for secondary beams (Part I)

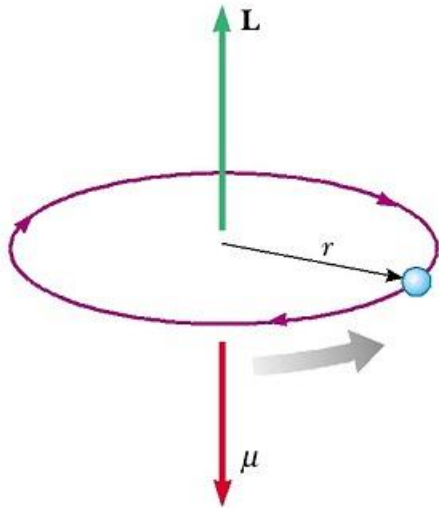
Diktys Stratakis
Fermi National Accelerator Laboratory

PHYS 790D, NIU, DeKalb, IL (Fall 2019)
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Outline

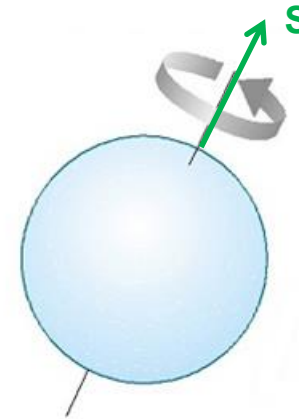
- Secondary beams and their applications
- Production of secondary beams
- Fermilab Muon Campus: a testbed for secondary beams
- Challenges on lattice design with secondaries:
 - Particle-matter interactions
 - Particle contamination and cleaning
 - Particle decay
 - Polarization tracking
- Introduction to G4beamline
- Examples of using G4beamline for modeling secondaries

g-factor or gyromagnetic ratio



Magnetic moment (μ) of a classical muon (mass m_μ) with charge e and angular momentum L :

$$\vec{\mu} = -\frac{e}{2m_\mu} \vec{L}$$



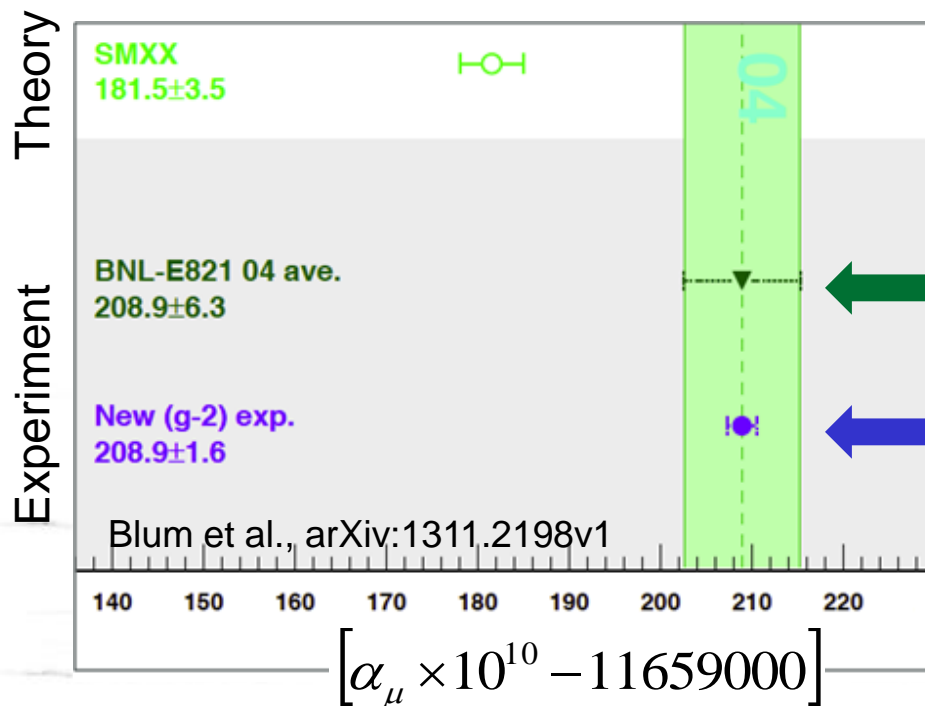
Magnetic moment of a muon with intrinsic spin angular momentum S :

$$\vec{\mu} = -g \frac{e}{2m_\mu} \vec{S}$$

- g-factor dictates the relationship between momentum & spin
- Its exact value is still an open question

A hint of new physics?

- Standard model: $g_{\text{theory}} = 2.00233183630(99)$
- Last measured : $g_{\text{meas}} = 2.00233184178(126)$
- What other physics must be added to make $g_{\text{theory}} = g_{\text{meas}}$?



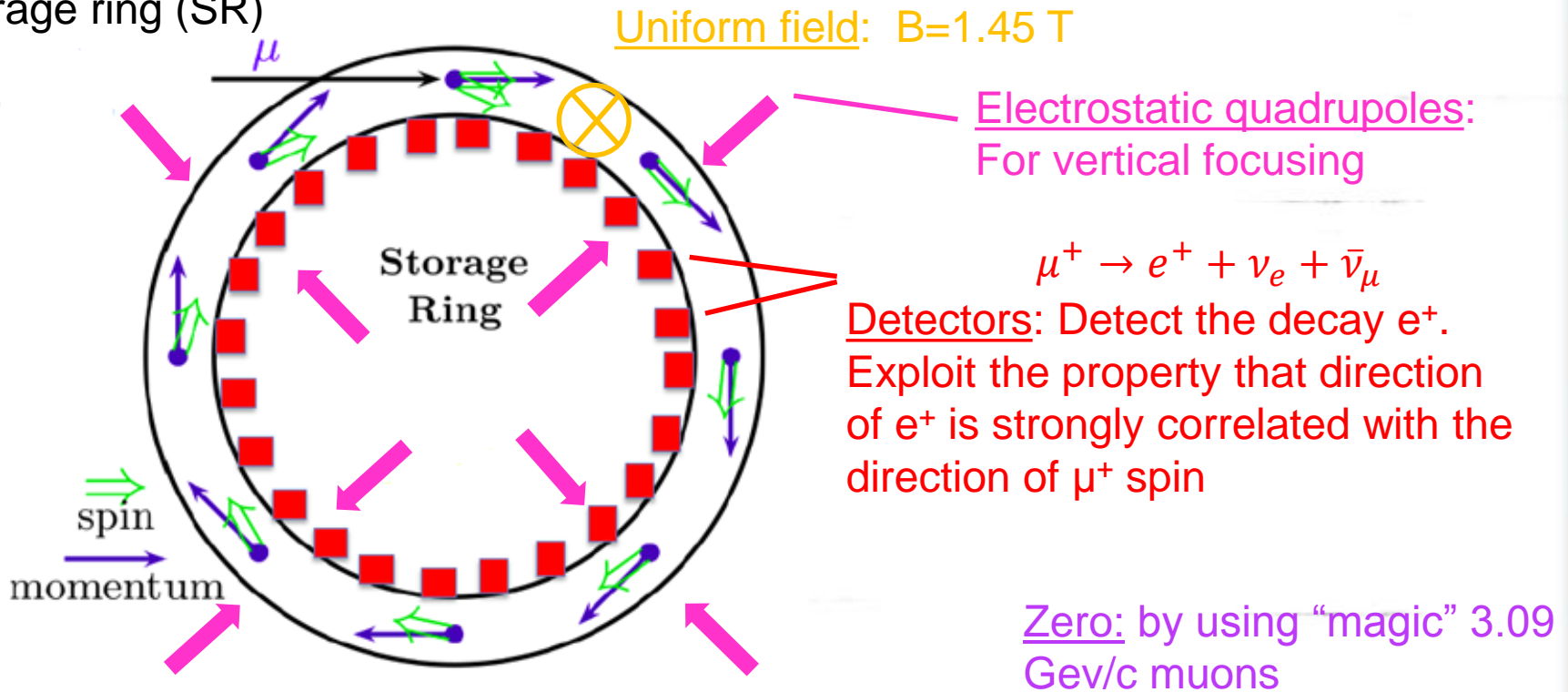
$$\alpha_{\mu} = \frac{g - 2}{2}$$

Current discrepancy between theory/experiment

Goal of the Fermilab Muon g-2 Experiment

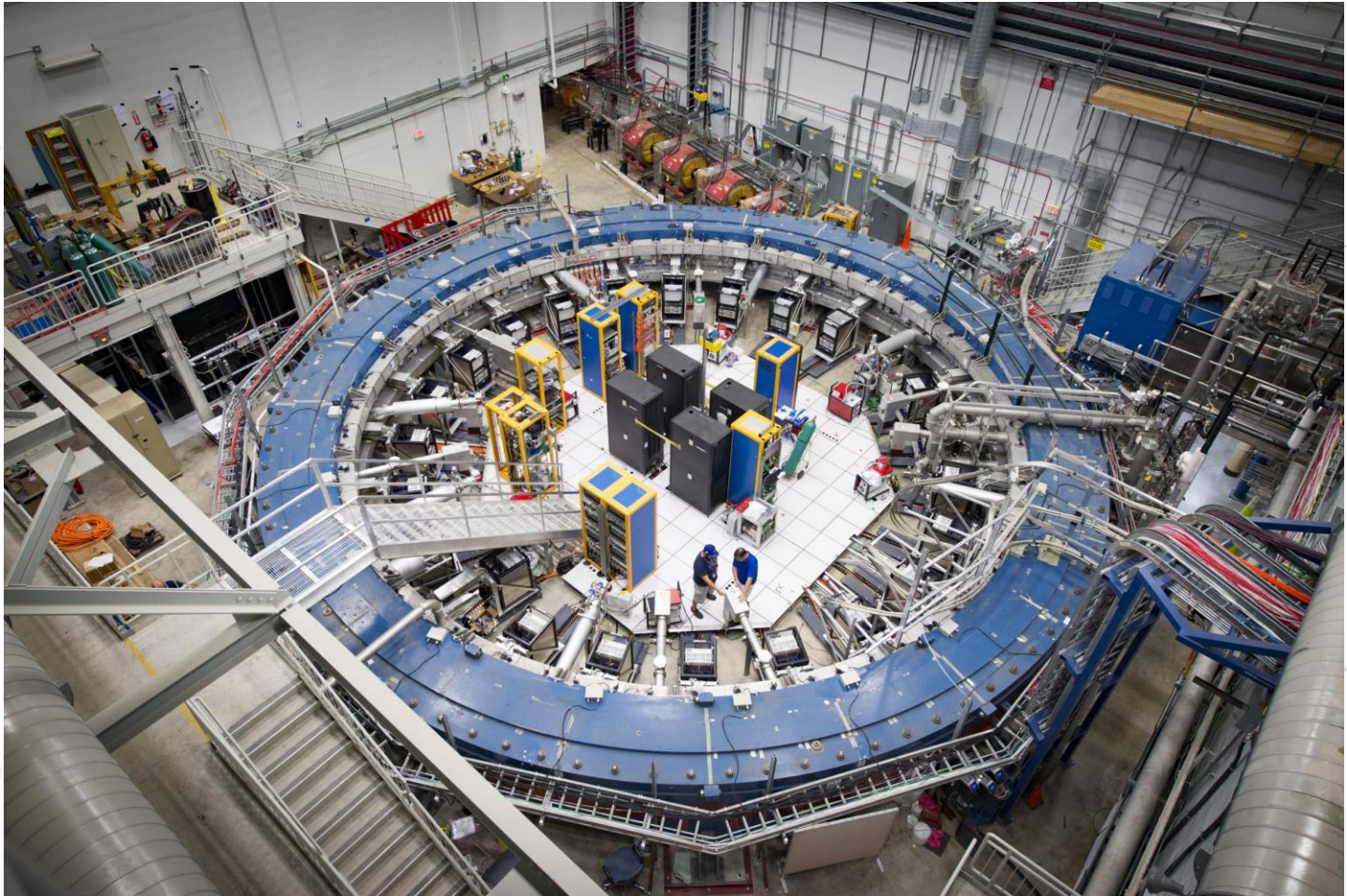
The Fermilab Muon g-2 Experiment (1)

Inject polarized μ^+ into a storage ring (SR)



Precession frequency:
$$\vec{\omega}_a = \frac{e}{mc} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]$$

The Fermilab Muon g-2 Experiment (2)



The Fermilab Mu2e experiment (1)

- Muons are stopped in a Al target and captured into an atomic orbital state of an Al nucleus. Most likely processes:

- Decay in orbit:

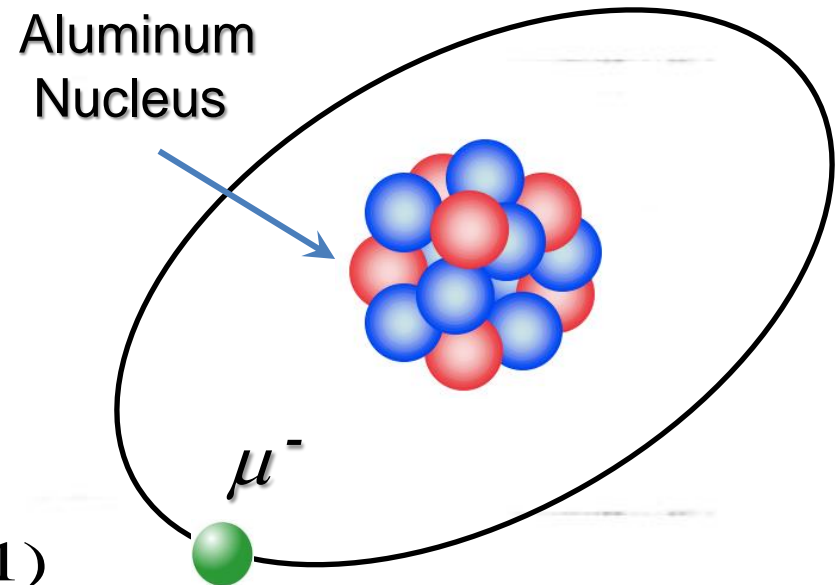
$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

L_e	0	=	1	-1	0
L_μ	1	=	0	0	1

- Muon capture:

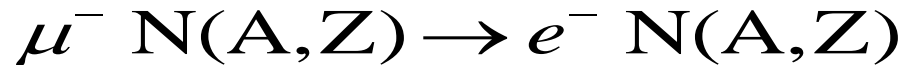
$$\mu^- N(A, Z) \rightarrow \nu_\mu N(A, Z - 1)$$

L_μ	1	=	1
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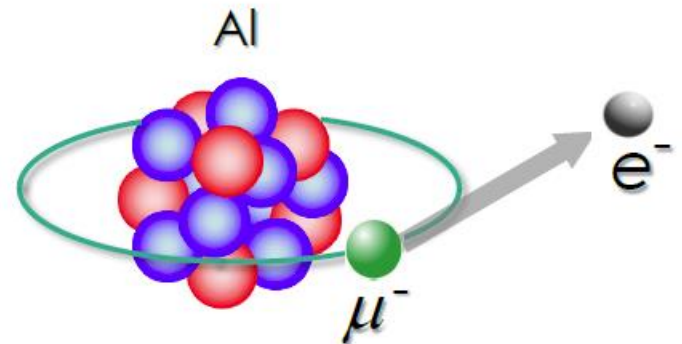


The Fermilab Mu2e experiment (2)

- Mu2e will look for a neutrinoless muon to electron conversion:

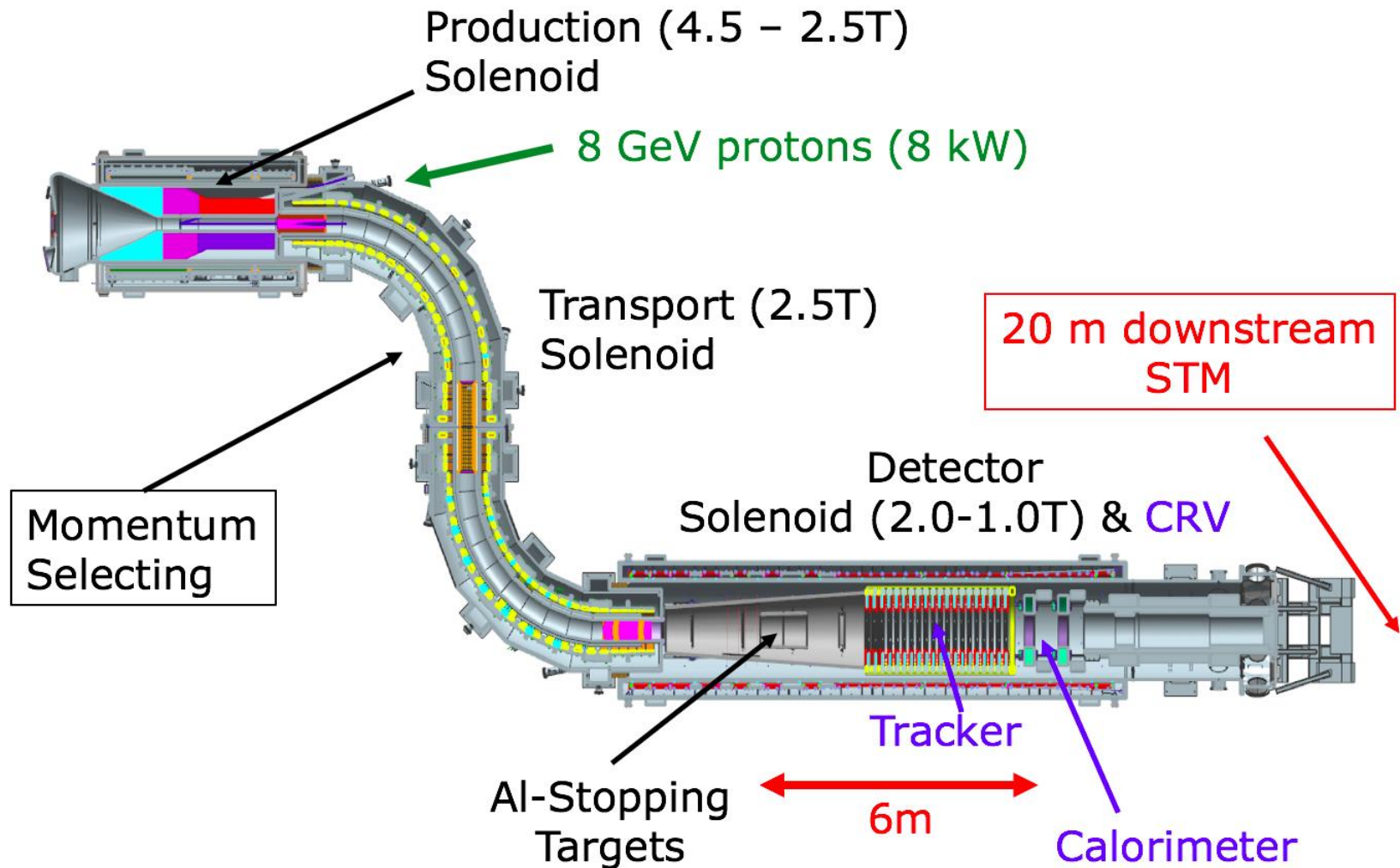


L_e	0	\neq	1
L_μ	1	\neq	0

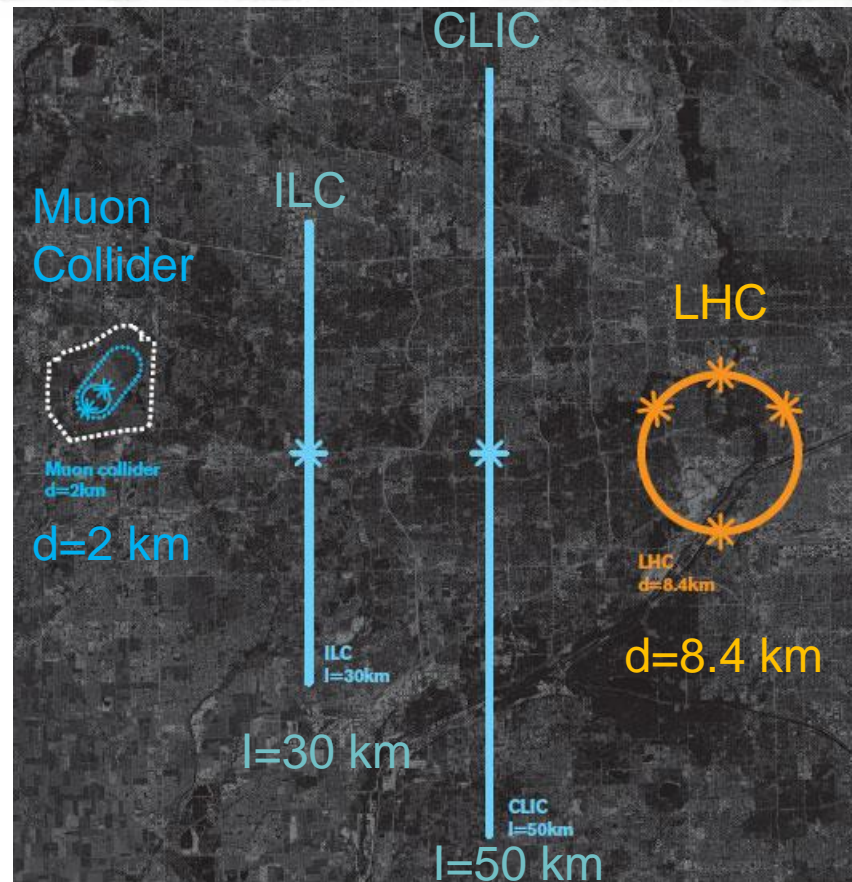


- The Mu2e experiment endeavors to detect Charged Lepton Flavor Violation

The Fermilab Mu2e experiment (3)



Towards a Muon Collider (far future)



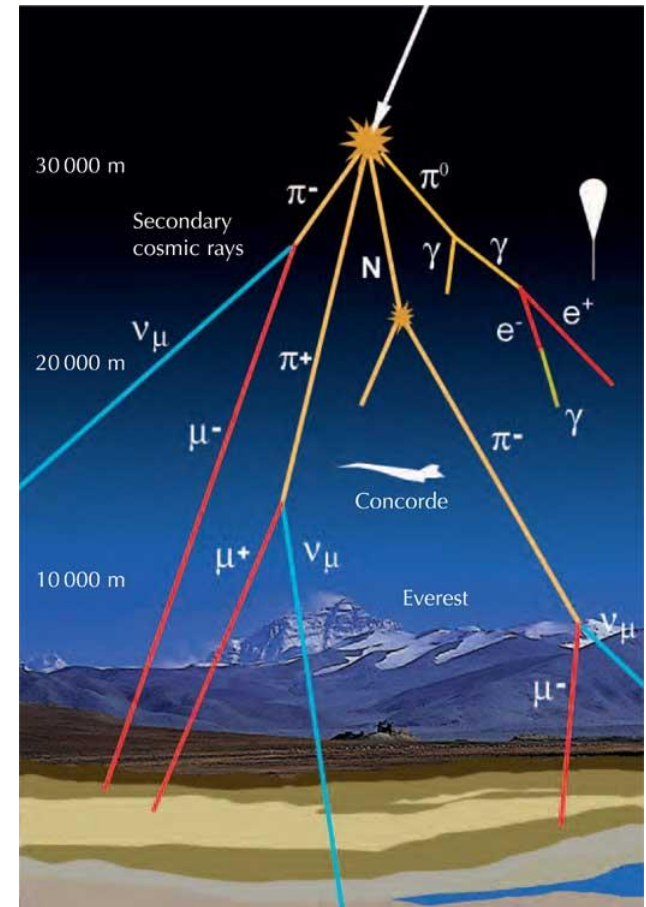
- As with an e^+e^- collider, a $\mu^+\mu^-$ collider would offer a precision probe of fundamental interactions

Why muons?

- Carry the same electrical charge as electrons
- Like electrons, muons are elementary particles and thus can produce “clean collisions”
- Muons are ~ 200 times heavier than electrons making it more sensitive to the discovery of new physics
- The large muon mass suppresses synchrotron radiation and thus can be accelerated in circular channels at much higher energy than electrons

Muon production

- Atmospheric muon beam
 - High energy protons strike atmosphere
 - Pions and kaons are produced
 - Pions decay before they interact
 - Muons are born
 - Arrive at sea level with a flux of ~ 1 muon per square cm per minute
- Unfortunately, these muons are not enough for executing the aforementioned experiments

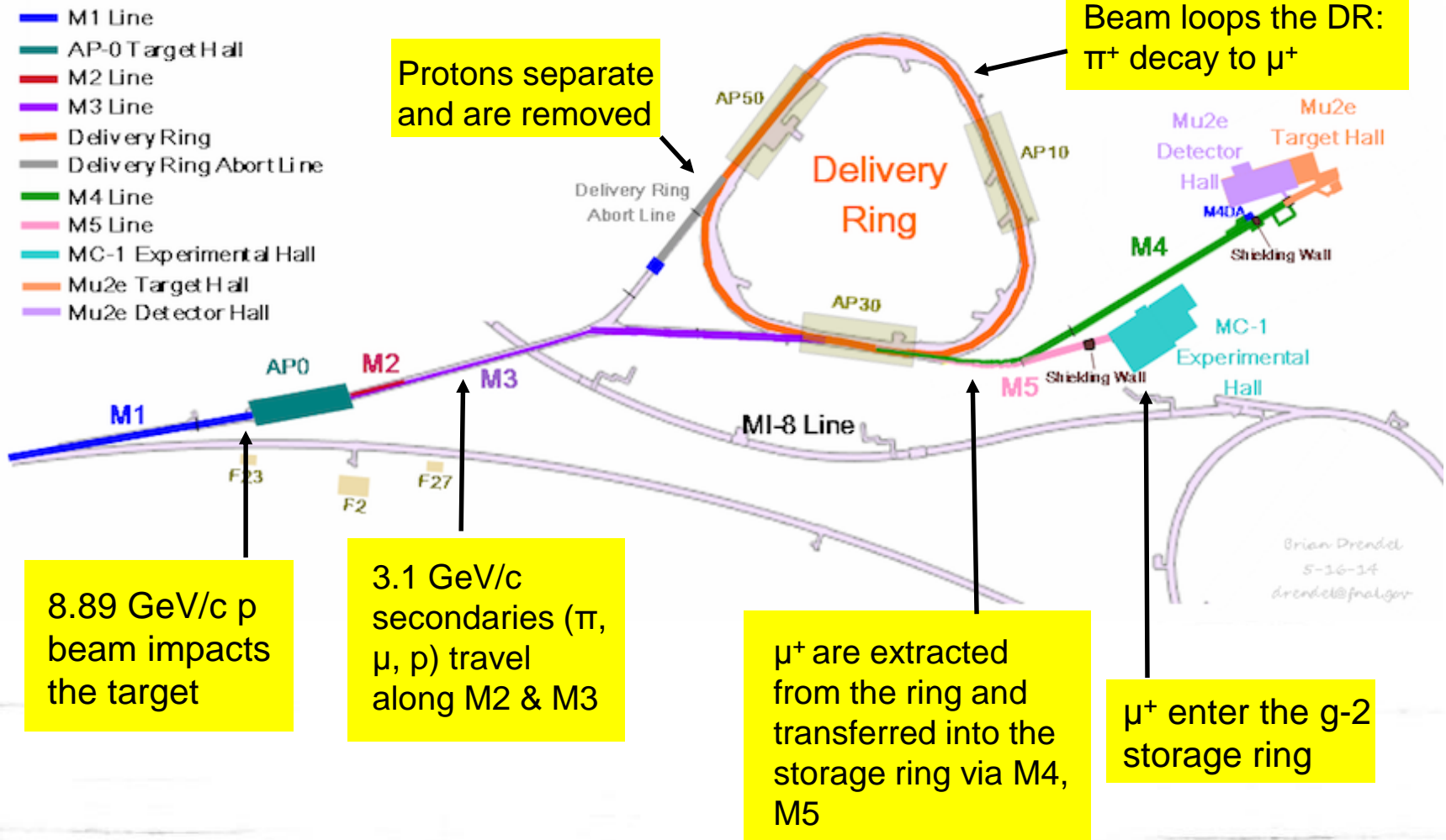


Creating a “human-made” muon beam

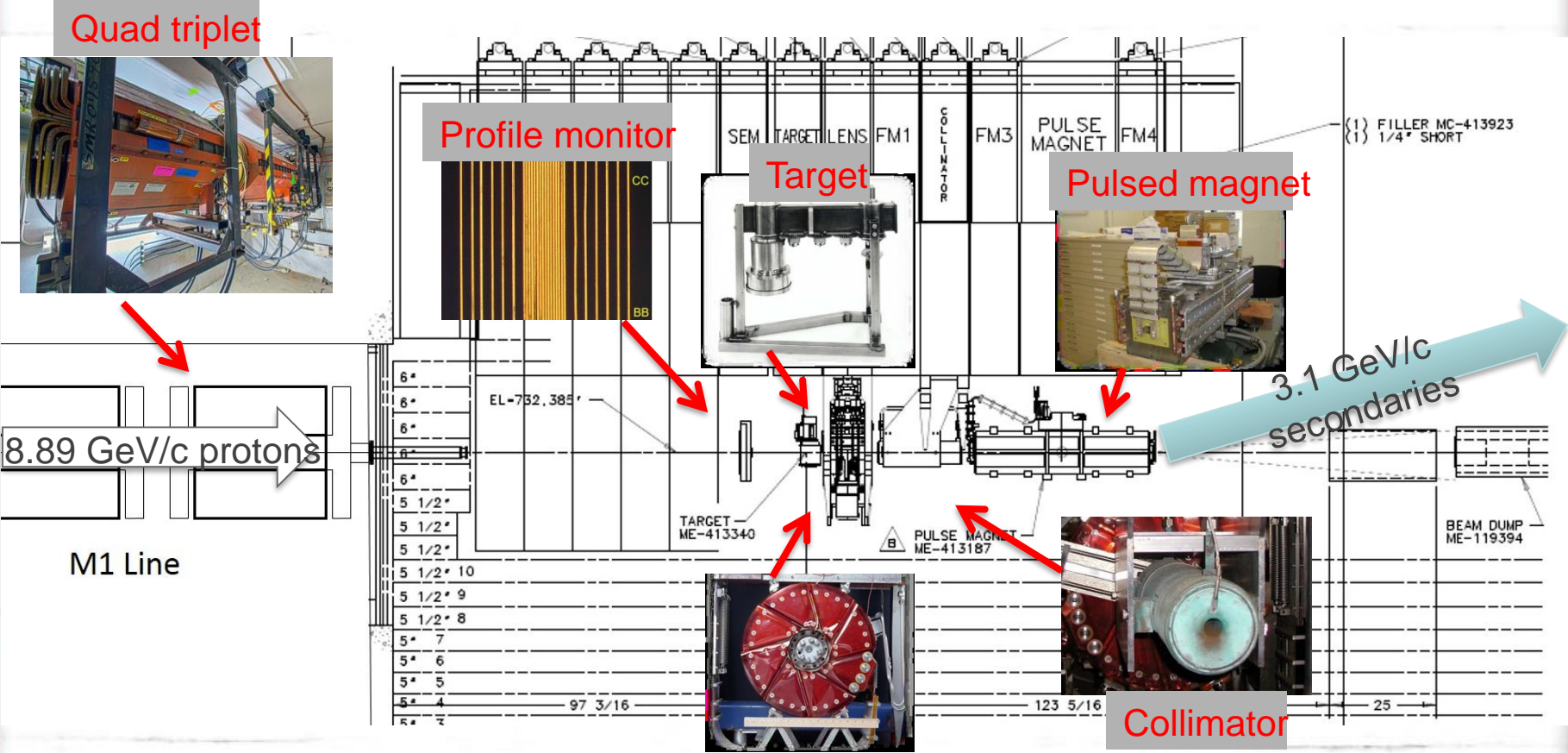
- High intensity muon beams are possible using high-energy accelerators
- Major production components:
 - Proton beam transport & tight focusing area
 - Pion production target
 - Focusing and energy selection system
 - Decay and muon transport region



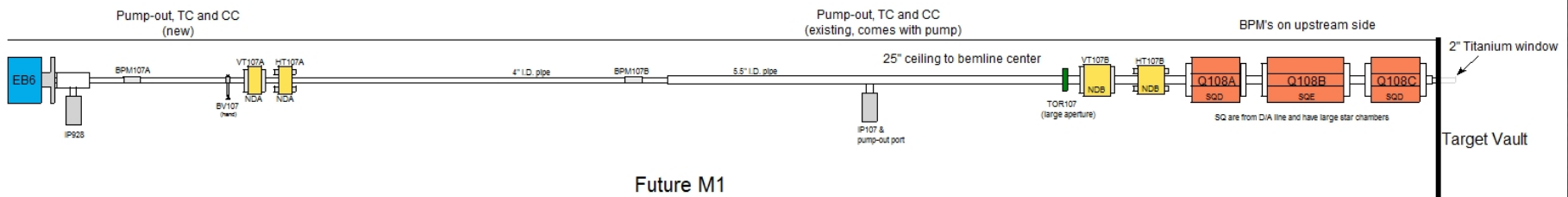
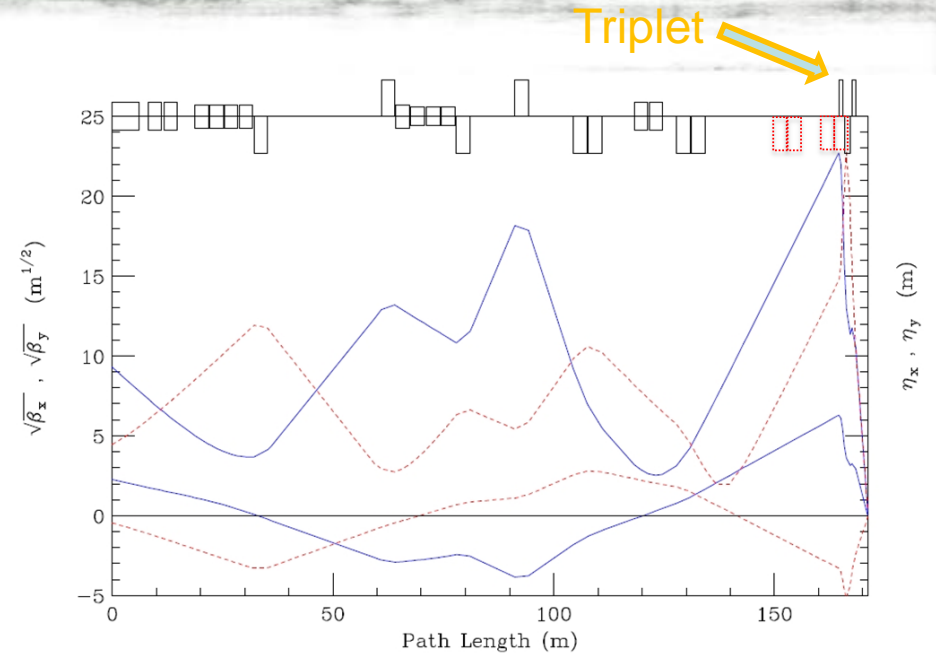
Fermilab Muon Campus accelerator



Secondary beam production scheme

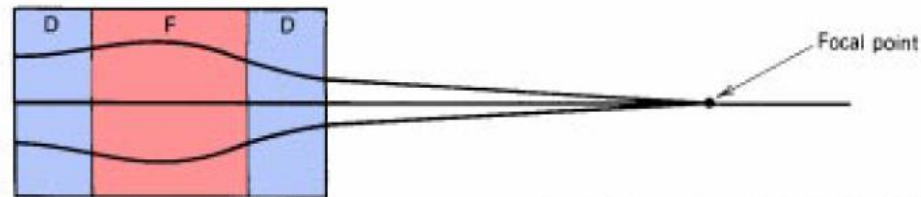
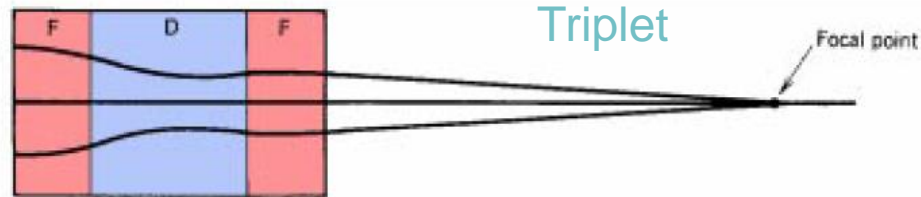
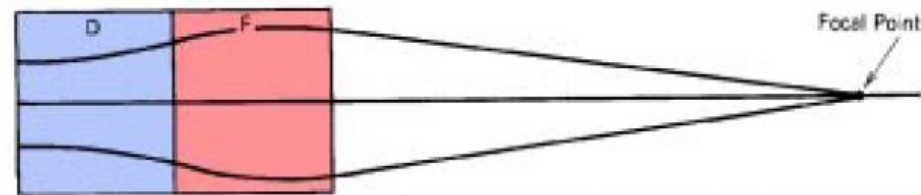
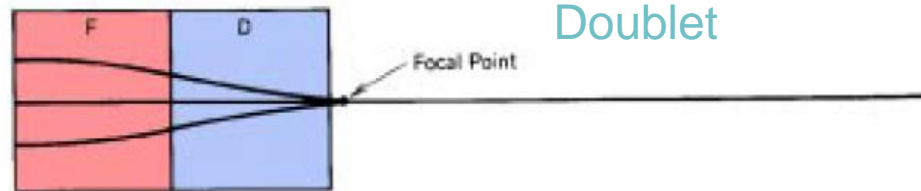


Quadrupole triplet for tight focusing



Future M1

Final focus with a quadrupole triplet



- Combination of equal D and F quads leads to net focusing
- BUT focusing is different in x and y directions

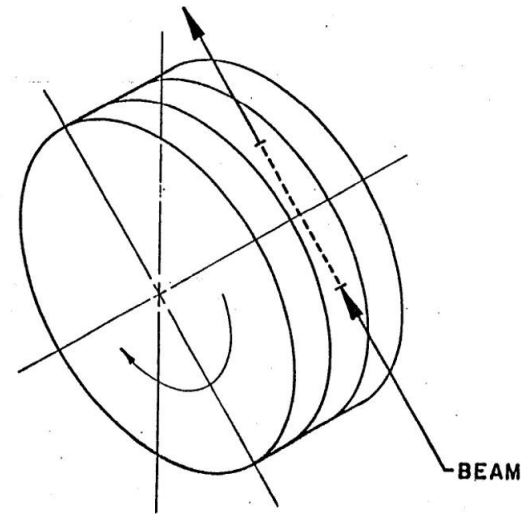
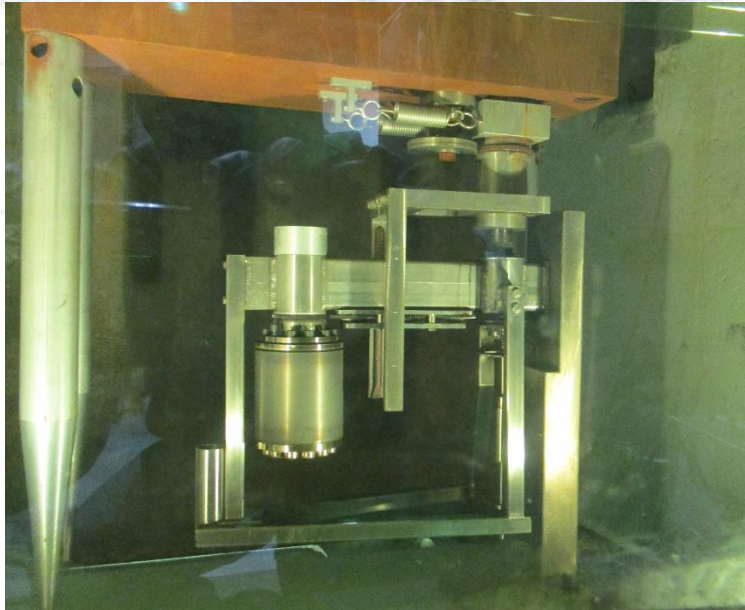
- A quad triplet focuses equally in both directions and thus focus to a point
- Allows stronger focusing
- Ideal for small spot sizes

Target considerations

- Production target should produce high yield of pions and muons
- Pion production rates are approximately independent of atomic number, although production of other particles (neutrons, gammas) increases with Z . Low Z materials minimize scattering
- Particle interactions should generate little heat and targets should dissipate heat easily
- Monolithic targets are not necessarily the best design – surface to volume ratio needs to be maximized
- For g-2, we rely on a Inconel 600 based target:

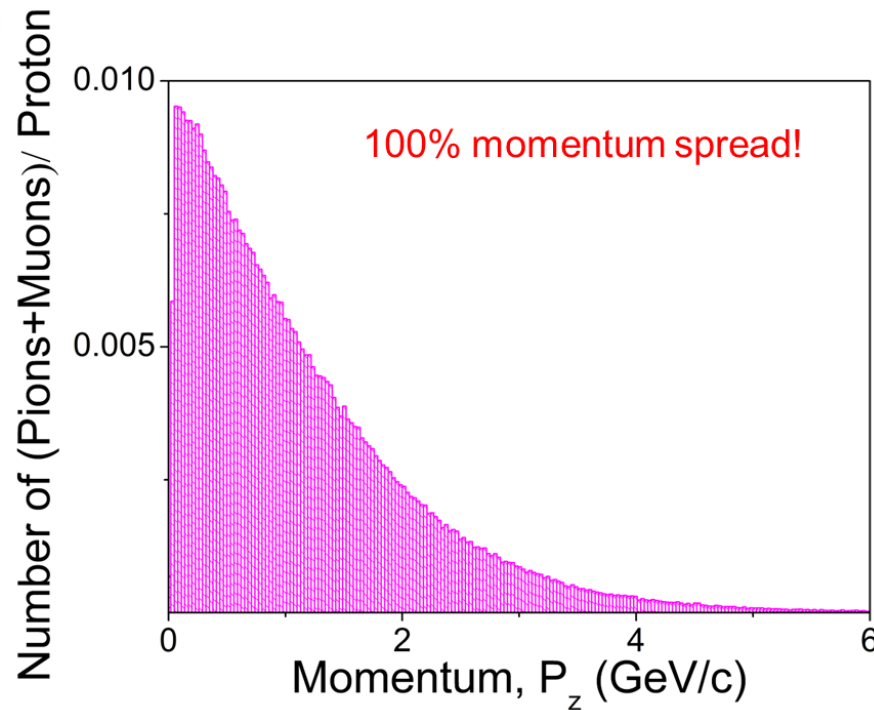
material Inconel600 Ni,0.72 Cr,0.15835 Fe,0.10 Mn,0.01 Cu,0.005 Si,0.005 C,0.0015 S,0.00015 density=8.47

Muon g-2 Target



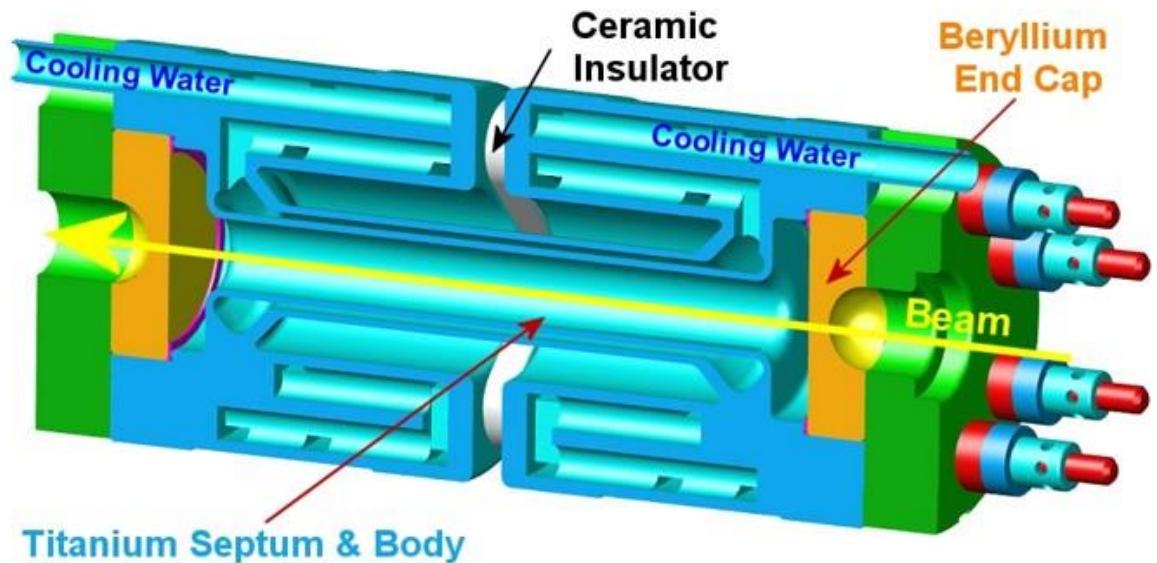
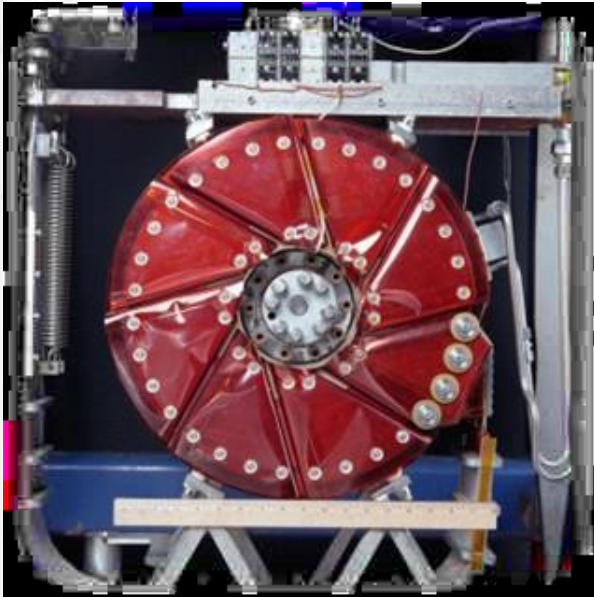
- Target has an outer Be cover to prevent target material from being sputtered onto nearby elements
- Is rotated one turn per 45 s & is moved vertically by 1 mm after each 2×10^{16} protons to spread the depletion uniformly

Beam distribution



- Beam distribution out of the target has an enormous energy spread and occupies a large phase-space distribution
- Strong focusing is necessary

Lithium lens



- The lens is a short (16 cm) cylindrical column of lithium metal with a constant current density around its axis giving an azimuthal B-field which focuses particles transversely (both planes)
- Lithium is chosen due to its large interaction length.

Considerations for muon capture

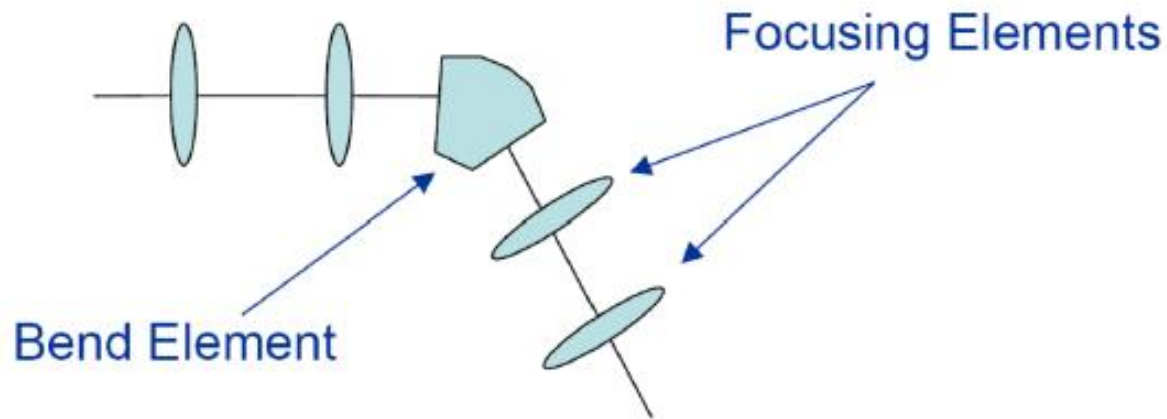
- Muons are coming from pion decay. The lifetime of a pions is 26 ns in their own frame. Therefore, a considerable long muon capture line is necessary
- MC capture lines are 280 m long. Exponential decay law predicts:

$$N = N_0 e^{-(t_{M2M3})/\gamma\tau_\pi} = 0.3N_0 \rightarrow 70\% \text{ of } \pi^+ \text{ decay}$$

- Daughter muons have equal or lower momentum and even larger momentum spread. They do not come from a single spot.
- The muon capture channel requires a high density of magnets so that to properly focus and transport the beam

Muon capture & transport line optics

- The capture and transport of secondary beam is done with magnets
- Mainly two types of magnets: bending magnets (dipoles) and focusing magnets (quadrupoles)



Dipole magnets

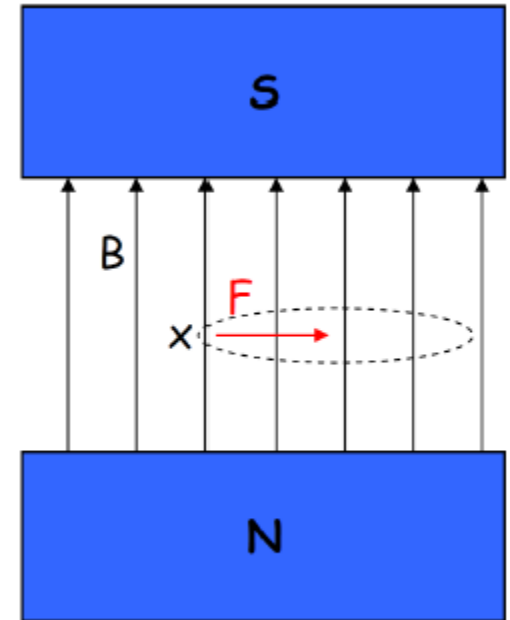
- Recall that the Lorentz force on a particle:

$$F = ma = e(E + v \times B) = \frac{mu^2}{r}$$

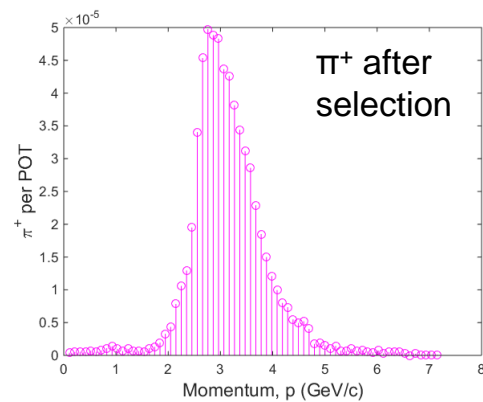
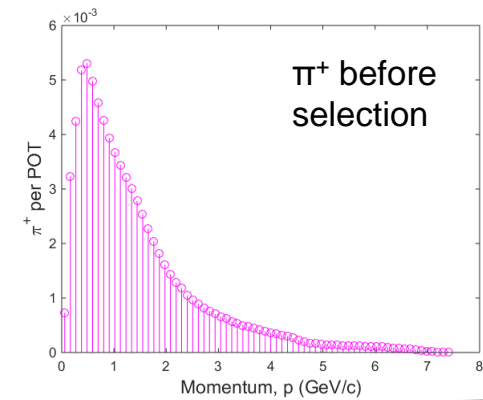
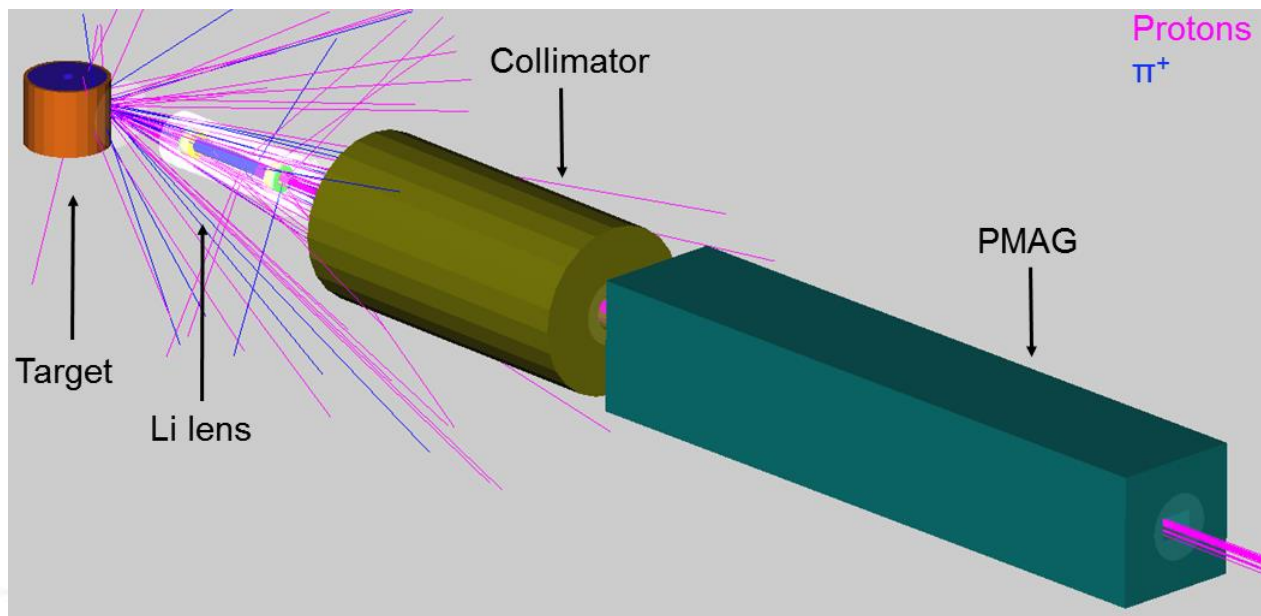
- In the absence of an E-field and assuming that B and v are perpendicular:

$$\frac{1}{r} = \frac{eB}{p}$$

- In an accelerator, dipoles are used to bend the beam trajectory. By using the appropriate field, one can tune the system so that particles of certain momentum can be transported only



Target station dipole



Quadrupole magnets

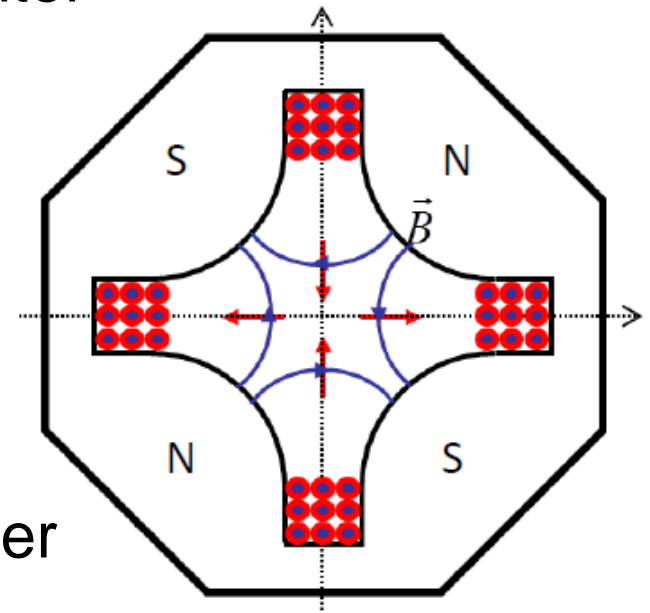
- Quad magnet has four poles and imparts a force proportional to distance from center
- Magnetic Field:

$$B_x = -Gy \text{ and } B_y = Gx$$

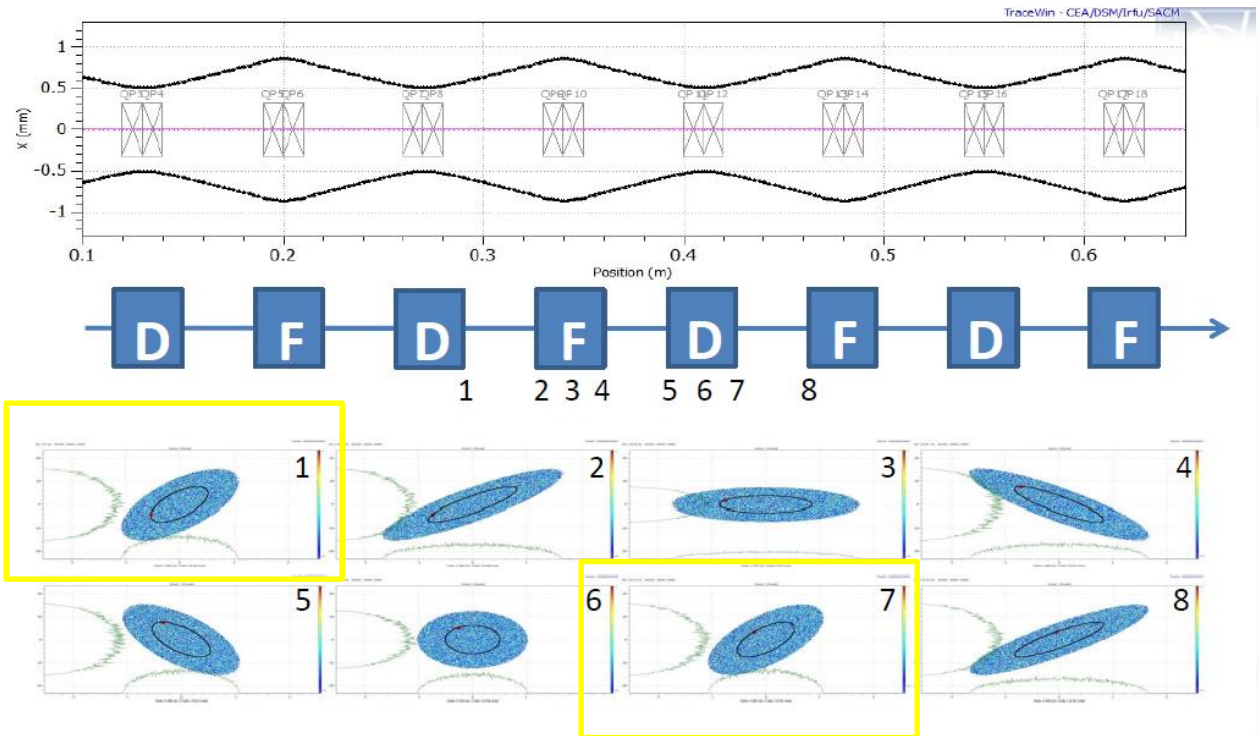
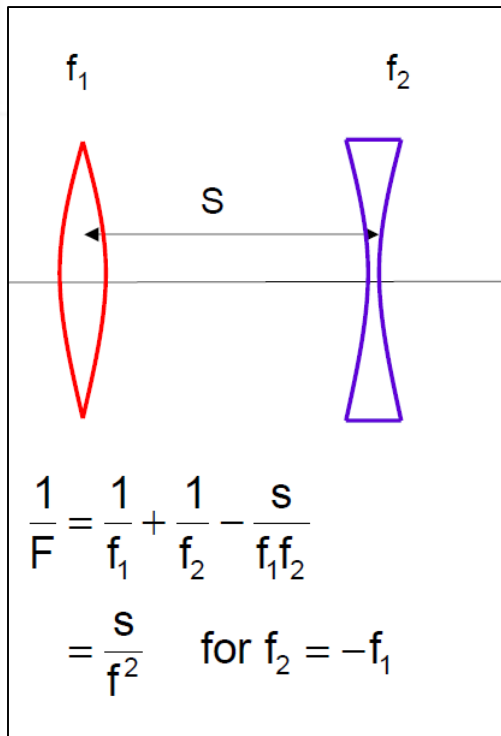
- Magnetic Force:

$$F_x = -qvGx \text{ and } F_y = qvGy$$

- Focus in one plane, defocus in the other
- Accelerators consist of a sequence of identical “FODO” cells which combine a focusing & defocusing quad, separated by a drift



Focusing Defocusing (FODO) lines



- The beam is matched if after every period the Twiss parameters are identical

Muon Campus beam lines

Muon campus M3 line



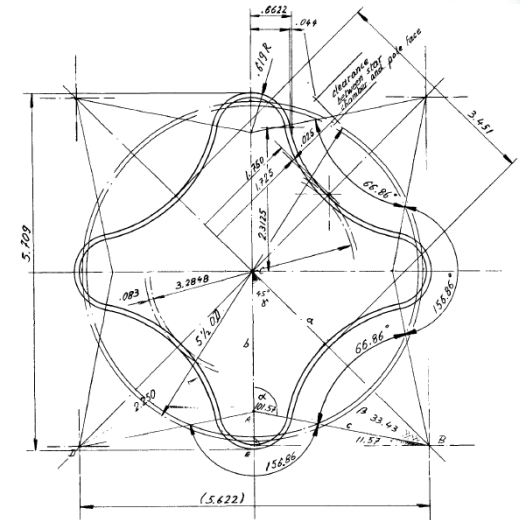
Quadrupole magnet

Muon campus Delivery Ring



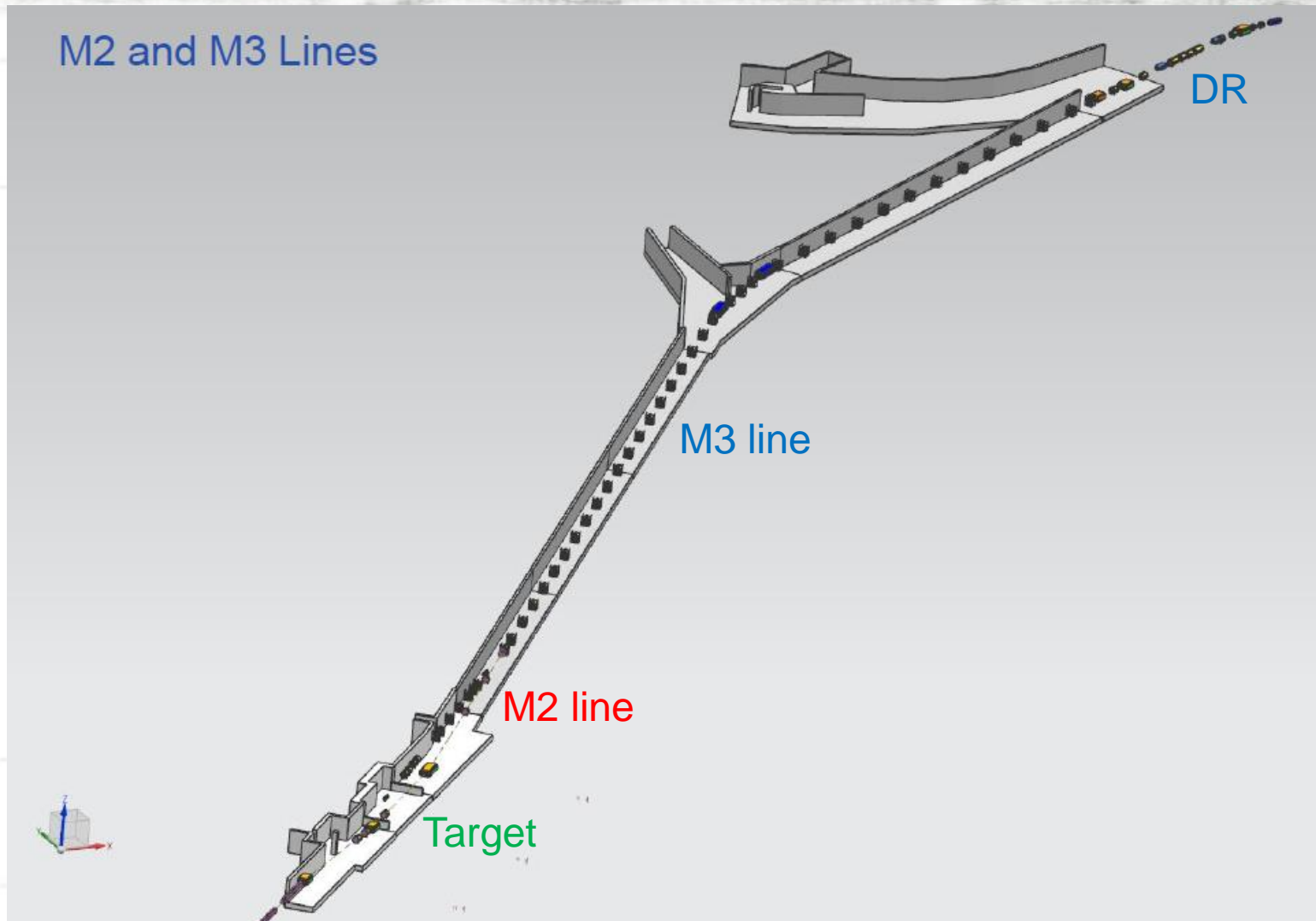
Dipole magnet

Muon Campus quad magnets



- Most Muon Campus quads have special vacuum chambers that conform to the poles in order to extend the aperture and therefore maximize capture

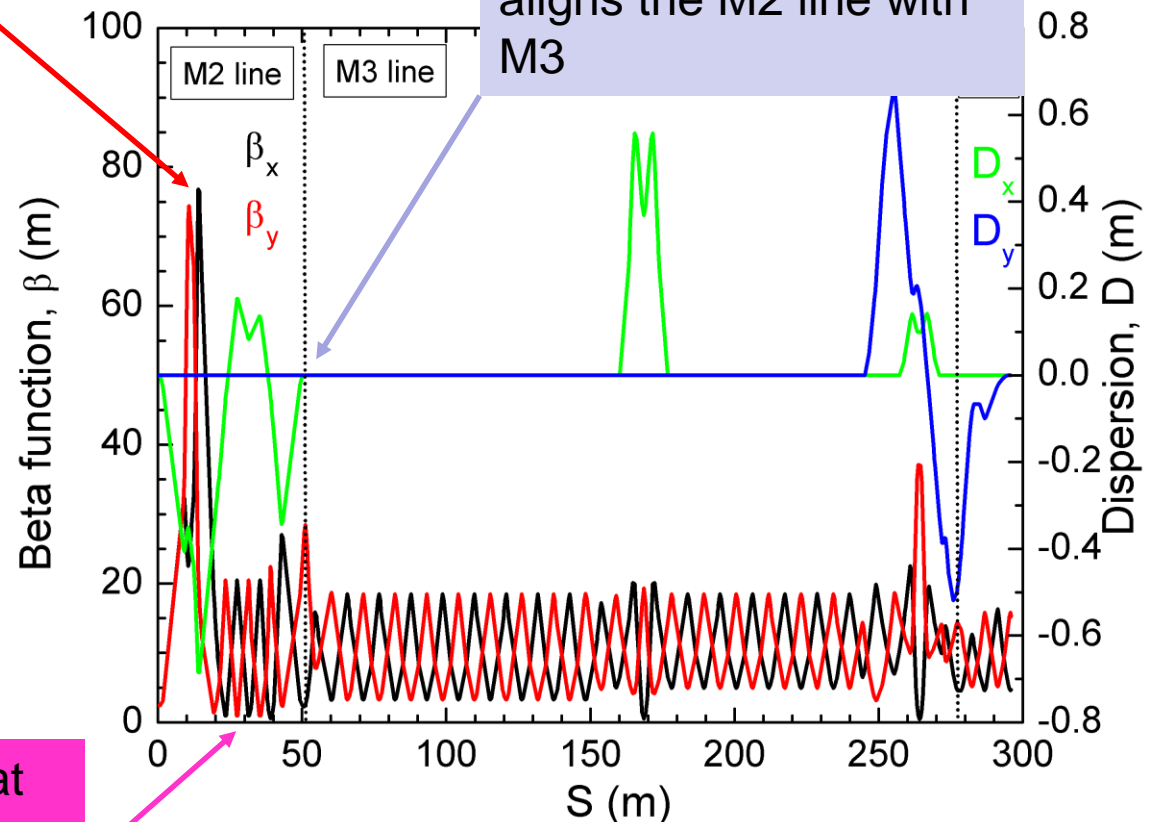
Layout of the Muon Capture line



Muon capture & transport line (M2)

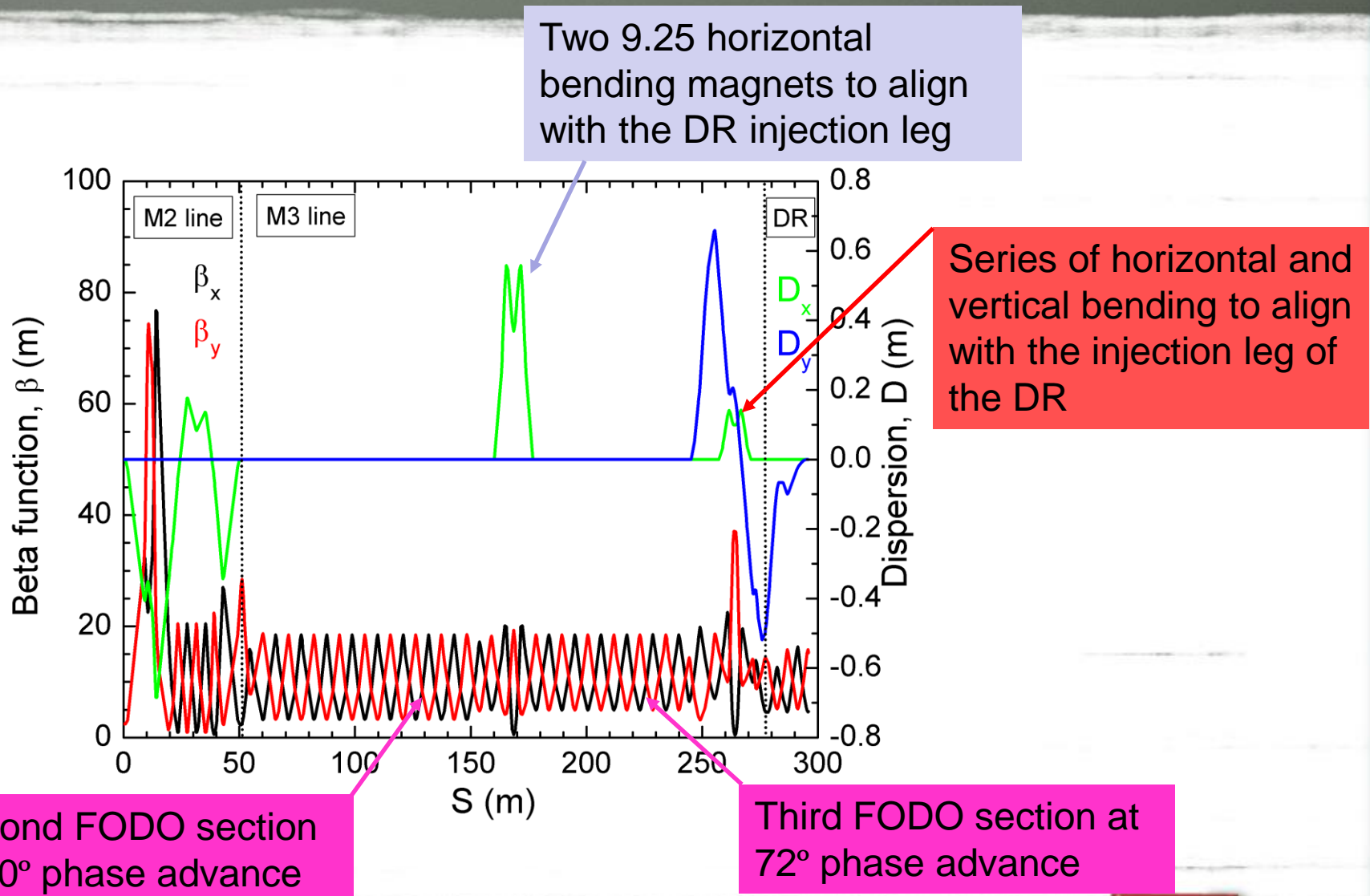
4 quads to match the target generated beam distribution

Second 3° bend that cancels dispersion and aligns the M2 line with M3



First FODO section at 120° phase advance

Muon capture & transport line (M3)

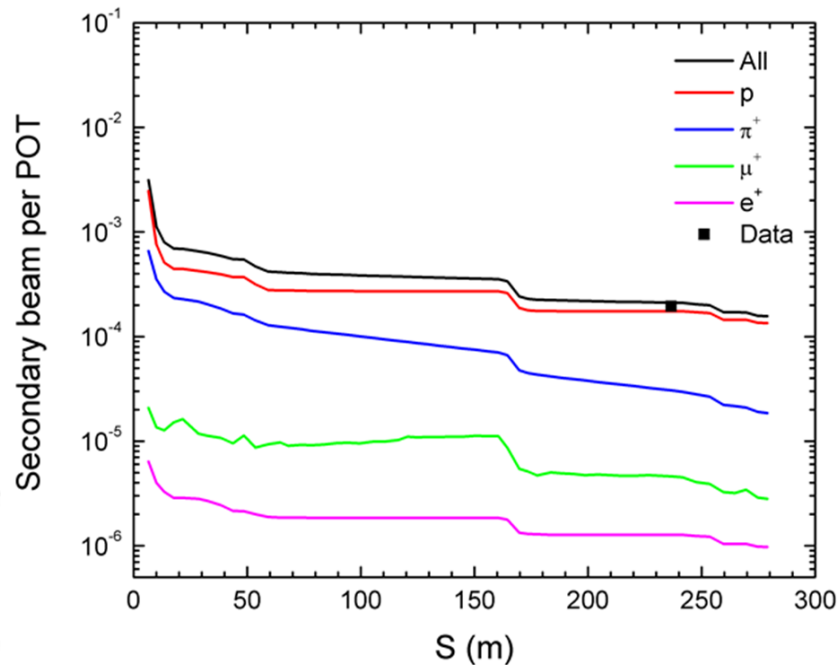


Simulation challenge #1

- Generation and transport of secondary beams involves particle-matter interactions
 - A code that can handle physical processes such as straggling, scattering and ionization is necessary
- Perfect examples are the target and the Li-lens

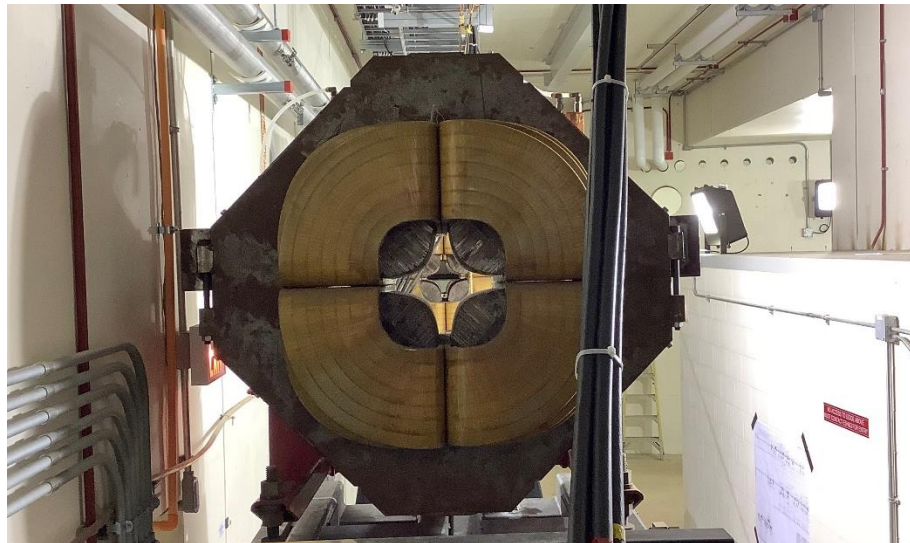
Simulation challenge #2

- Beam is contaminated with several species
 - Pions, muons, kaons, positrons
- Some are not stable particles and their evolution over distance needs to be estimated



Challenge #3

- Beamline is composed into several magnetic elements to properly focus the beam
 - Aperture restrictions
- Most Muon Campus quads have special vacuum chambers that conform to the poles in order to extend the aperture and therefore maximize capture



Challenge #4

- Energies of newborn muons is not the same
- In the pion rest frame:

$$p^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 30 \text{ MeV}/c$$

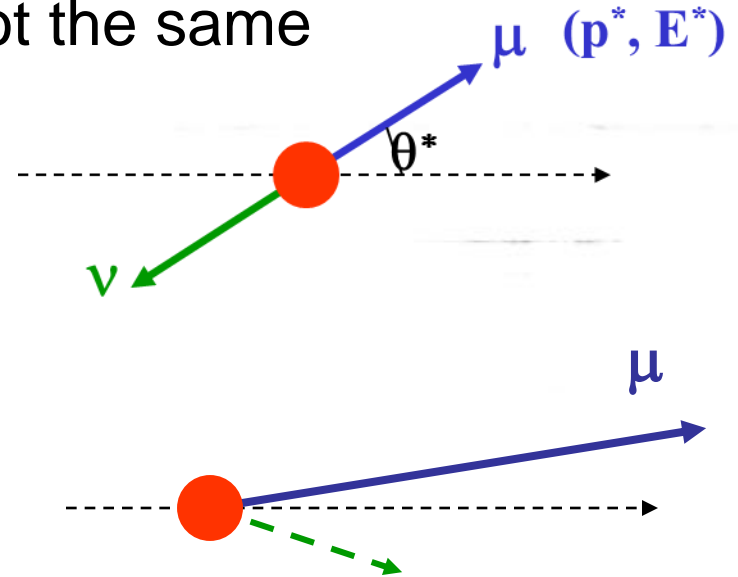
$$E^* = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 110 \text{ MeV}$$

- Boost to laboratory frame:

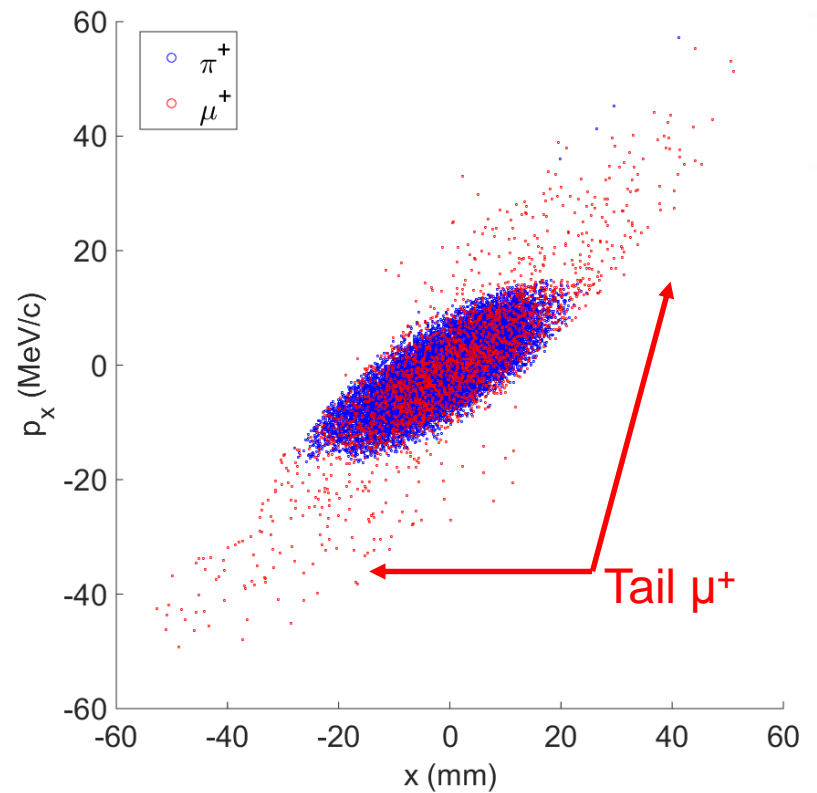
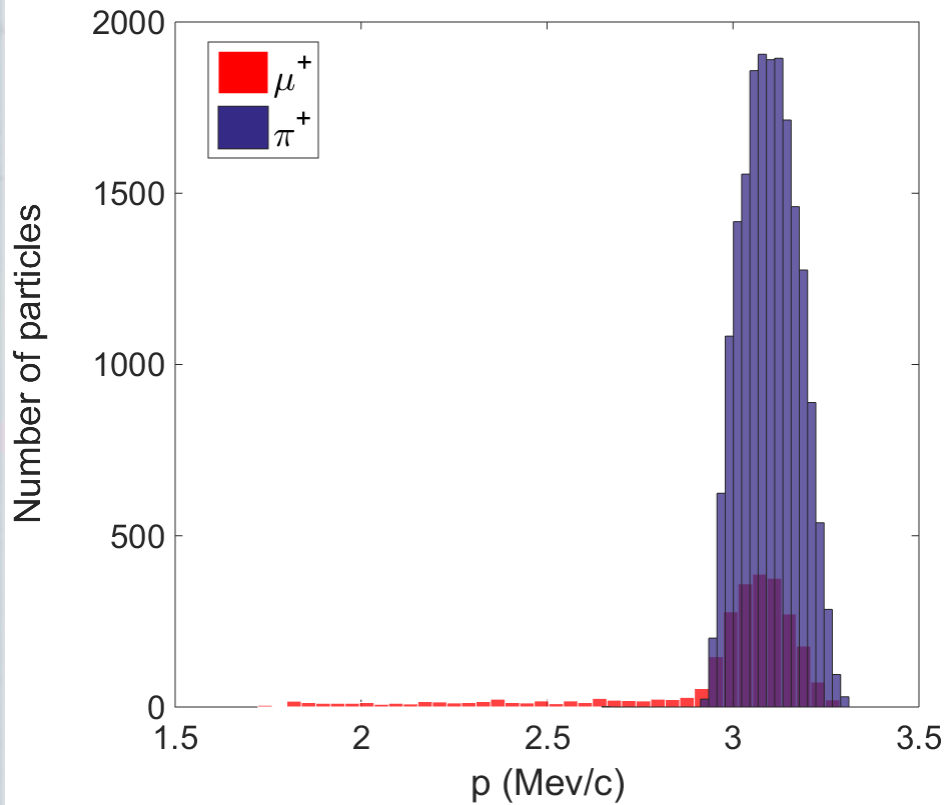
$$E_\mu = \gamma_\pi (E^* + \beta_\pi p^* \cos\theta^*)$$

- Limiting cases:

- $\cos\theta = +1 \rightarrow E_{max} = 1.00 \times E_\pi$ (forward decays)
- $\cos\theta = -1 \rightarrow E_{min} = 0.57 \times E_\pi$ (backward decays)

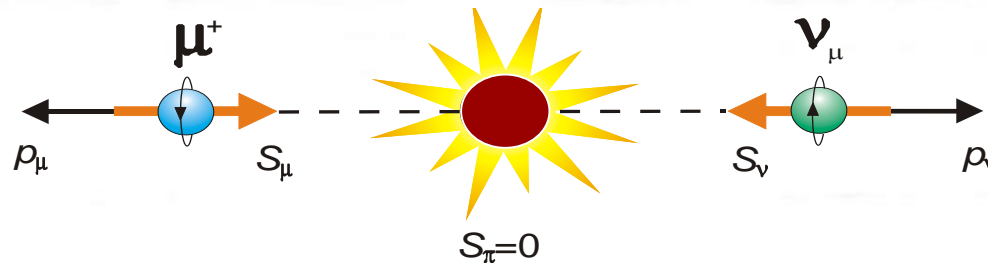


Muons at the end of M3



- Distribution of μ^+ has a long low-momentum tail

Challenge #5



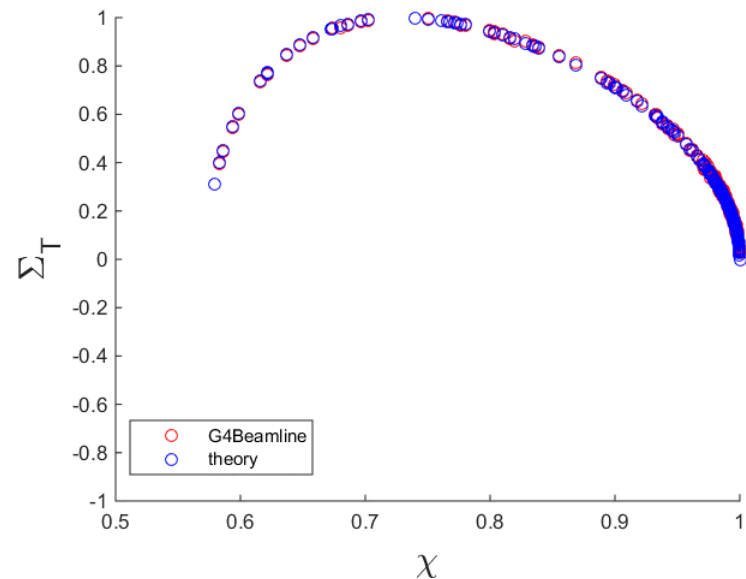
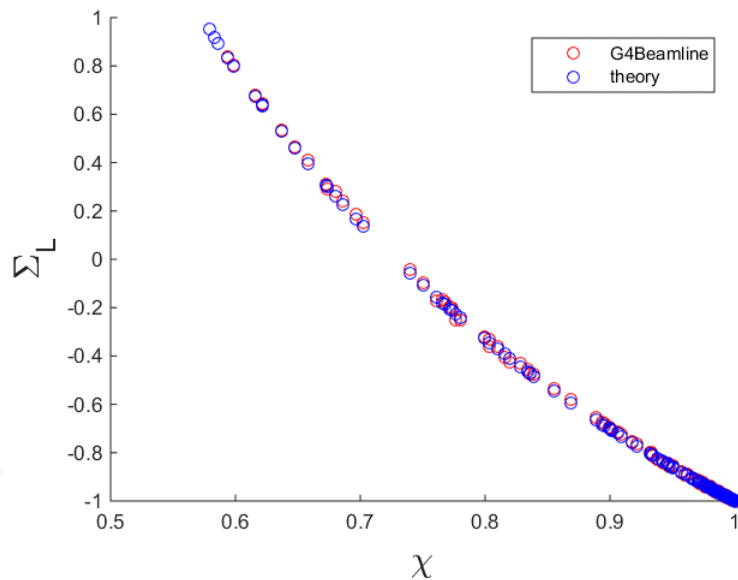
- Muons from pion decay are naturally polarized. Their polarization is highly depended on the momentum ratio between the new born muon and its parent pion, $x = p_\mu/p_\pi$
- Transverse polarization is given by:

$$P_T = \frac{2b}{x(1-b^2)} [(1-x)(x-b^2)]^{1/2}, \quad b = m_\mu/m_\pi$$

- Longitudinal polarization is given by: $P_L = \frac{x(1+b^2)-2b^2}{x(1-b^2)}$

Polarization in the Muon Campus (1)

- The rms momentum spread of the Muon Campus is $\sim 2\%$
- Muons from forward decays are surviving and the muon polarization is expected to be $>90\%$



Quick guide to G4beamline (1)

- G4beamline is a particle-tracking simulation program based on the Geant4 toolkit [<http://geant4.cern.ch>].
- All of the Geant4 physics lists are available, modeling most of what is known about particle interactions with matter.
- The program is optimized to model and evaluate the performance of beam lines.
 - It has a rich range of beam-line elements.
 - It has general-purpose geometrical solids and fields so you can construct custom elements (e.g. an electrostatic septum, multi-function magnets, complex absorbers).
 - It lets you easily lay out elements along the beam centerline

Quick guide to G4beamline (2)

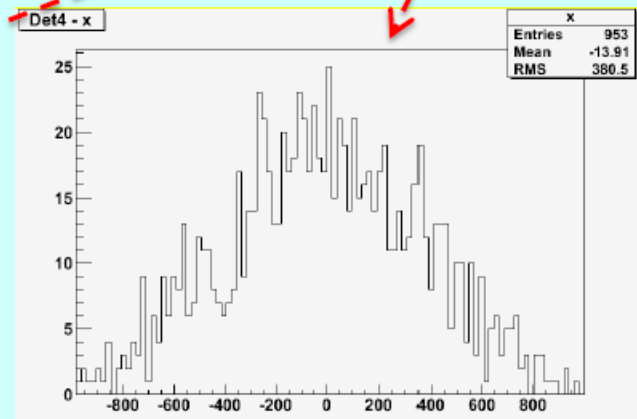
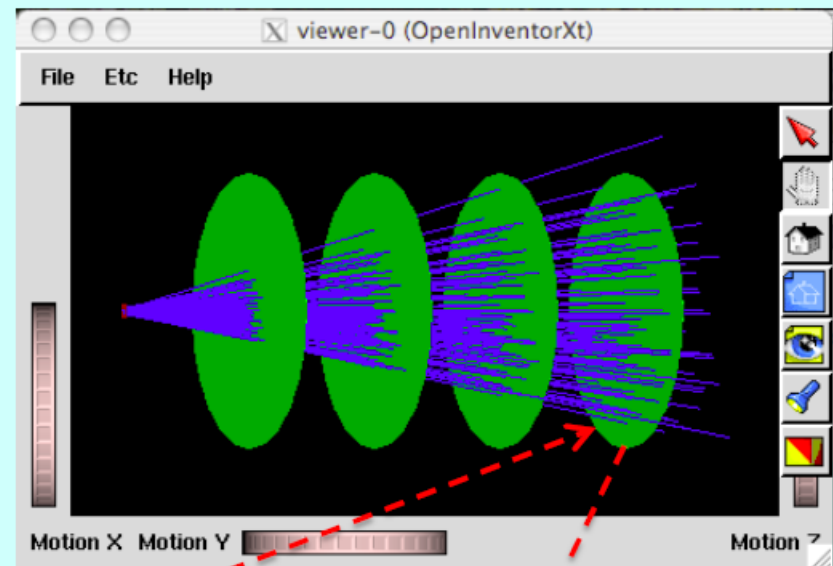
- The basic idea is to define each beamline element, and then place each one into the beamline at the appropriate place.
- All aspects of the simulation are specified in a single ASCII:
 - Geometry
 - Input Beam
 - Physics processes
 - Program control parameters
 - Generation of output NTuples
- The input file consists of a sequence of commands with named arguments. Each command has its own list of arguments
- Command and argument names are spelled out, so the input file becomes a record of the simulation that is readable by others

Quick guide to G4beamline (3)

- The system is described in a simple ASCII file:

```
# example1.in
physics QGSP_BERT
beam gaussian particle=mu+ nEvents=1000 \
  meanMomentum=200 \
  sigmaX=10.0 sigmaY=10.0 \
  sigmaXp=0.100 sigmaYp=0.100
# BeamVis just shows where the beam starts
box BeamVis width=100.0 height=100.0 \
  length=0.1 material=Vacuum color=1,0,0
place BeamVis z=0
virtualdetector Det radius=1000.0 color=0,1,0
place Det z=1000.0 rename=Det1
place Det z=2000.0 rename=Det2
place Det z=3000.0 rename=Det3
place Det z=4000.0 rename=Det4
```

- Visualization is included out-of-the-box
- Includes a user-friendly histogram tool: HistoRoot.



Quick guide to G4beamline (4)

G4beamline

Input file: Browse Output file: g4bl.out

Parameters: HistoRoot Events

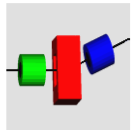
Viewer: none best Other: events/run: # runs:



G4beamline

3.02

<http://g4beamline.muonsinc.com>



- [Introduction](#)
- [Using the Program](#)
- [Visualizing the System](#)
- [HistoRoot - program to Create Histograms](#)
- [References](#)
- [Appendix: Using the OpenInventor Viewer](#)
- [Appendix: Using the Wired Viewer](#)
- [Appendix: G4beamline Commands](#)
- [Appendix: List of Particles \(QGSP\)](#)

NOTE: Before G4beamline can run, it requires that various Geant4 data-sets be downloaded and unpacked; which specific data-sets are required depends on the physics list used in the input file. When first run, G4beamline prompts the user to download them. If you find you need to download more data-sets, click [here](#).

Introduction

G4beamline is a particle-at-a-time simulation program based on Geant4 [\[1\]](#) and optimized for the simulation of beamlines. See the G4beamline User's Guide at the above URL for a description of the program (it is also installed with this distribution).

This brief description only discusses how to use the program, it does not describe how to construct a simulation's input file; for that see the G4beamline User's Guide.

Using the Program

Select the input file using the "Browse" button above, or via the File/Open menu. The directory containing it will become the current directory, and the output file will be "g4bl.out", as will this TextArea. Any additional Parameters used by the input file can be entered above. Pushing the "Run" button above will run the program, replacing this text with its output.

If you wish, you can enter a simple directory name into the "Directory" field; that directory will be created and the output files placed into it (including any NTuple output files). No '/' or '\' can be used.

While the program is running, the "Run" button above will change to "Abort" to permit you to stop the program immediately. Closing this window will also abort g4beamline.

After running the simulation (with viewer=none), the simulation usually creates one or more Root files containing the NTuples. You can use the [HistoRoot](#) program to generate histograms from these files.

IMPORTANT NOTE: With a viewer selected, NTuple output files are not written. This includes both .root files and ASCII files from virtualdetector-s, zntuple-s, etc.

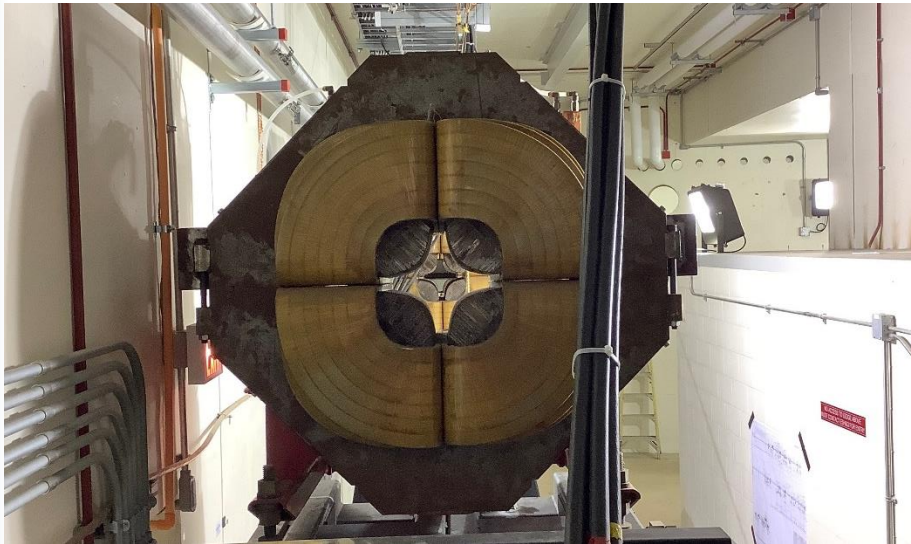
Several examples are provided in the distribution. They are located in the "examples" directory under the install directory. In Linux the install directory is usually in your HOME directory; in Mac OS X it is usually in /Applications; in Windows a Copy of "G4beamline examples" is placed into "Documents". When installing G4beamline, it is helpful to put the G4beamlineExamples directory into your HOME.

Visualizing the system

Un-checking the Visualization box will run the simulation and write the NTuple files; checking it will run the simulation and viewer without writing NTuple files. Each image in the viewer is a single run with the selected number of events.

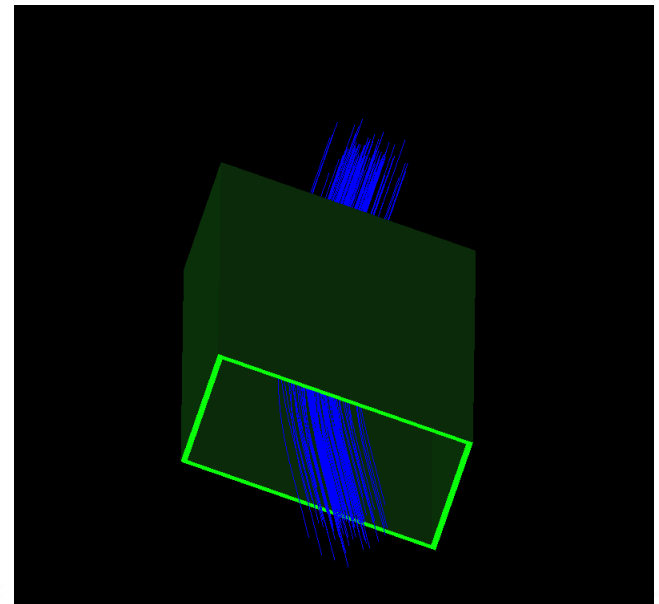
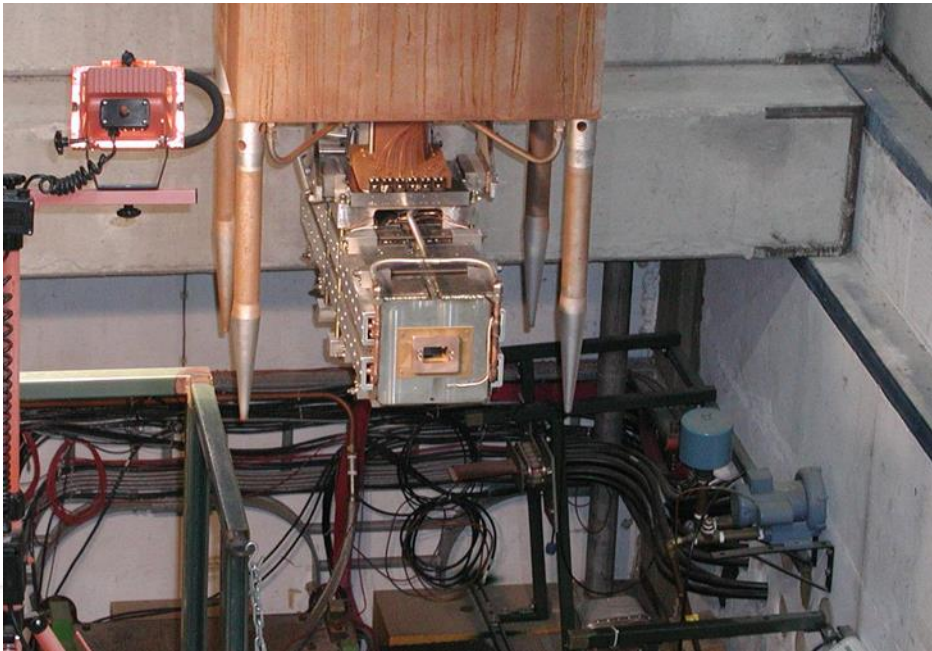
Quadrupole magnet

- Unlike MAD, G4beamline includes not only the “focusing” effects of a magnet but also the aperture effect.



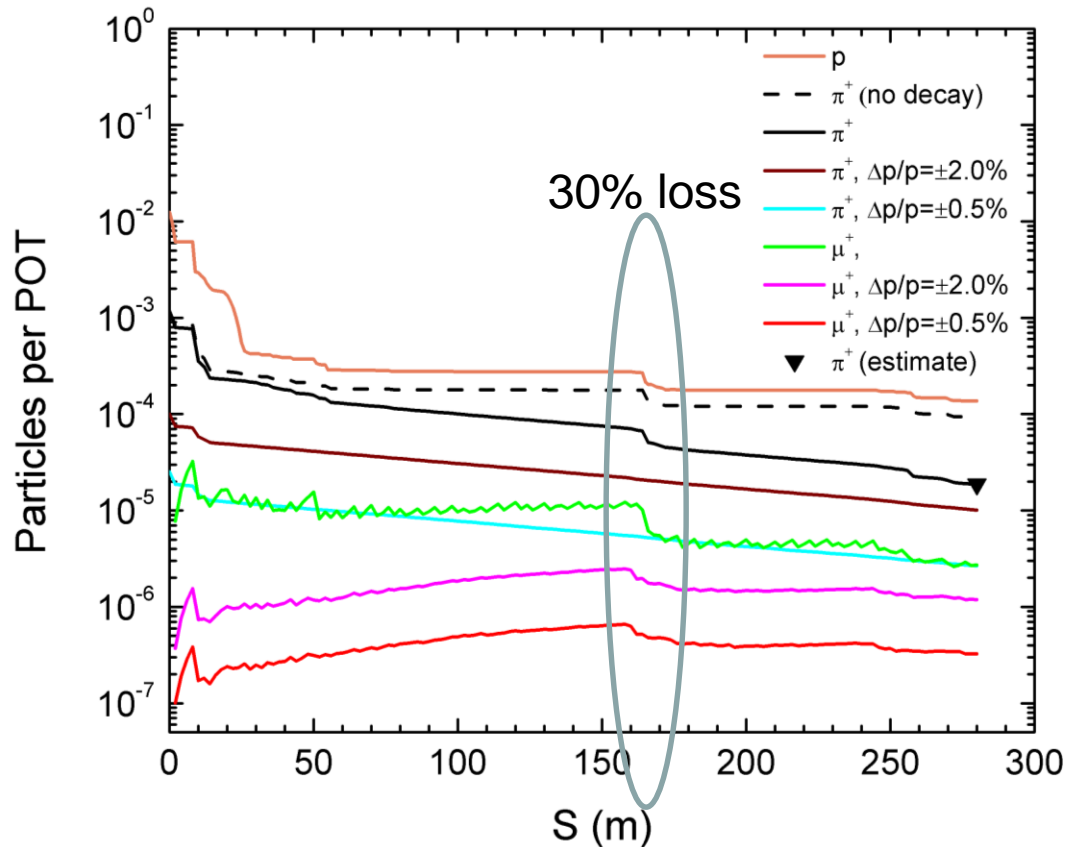
Dipole magnet

- G4beamline includes not only the “bending” effects of a magnet but also the aperture effect.



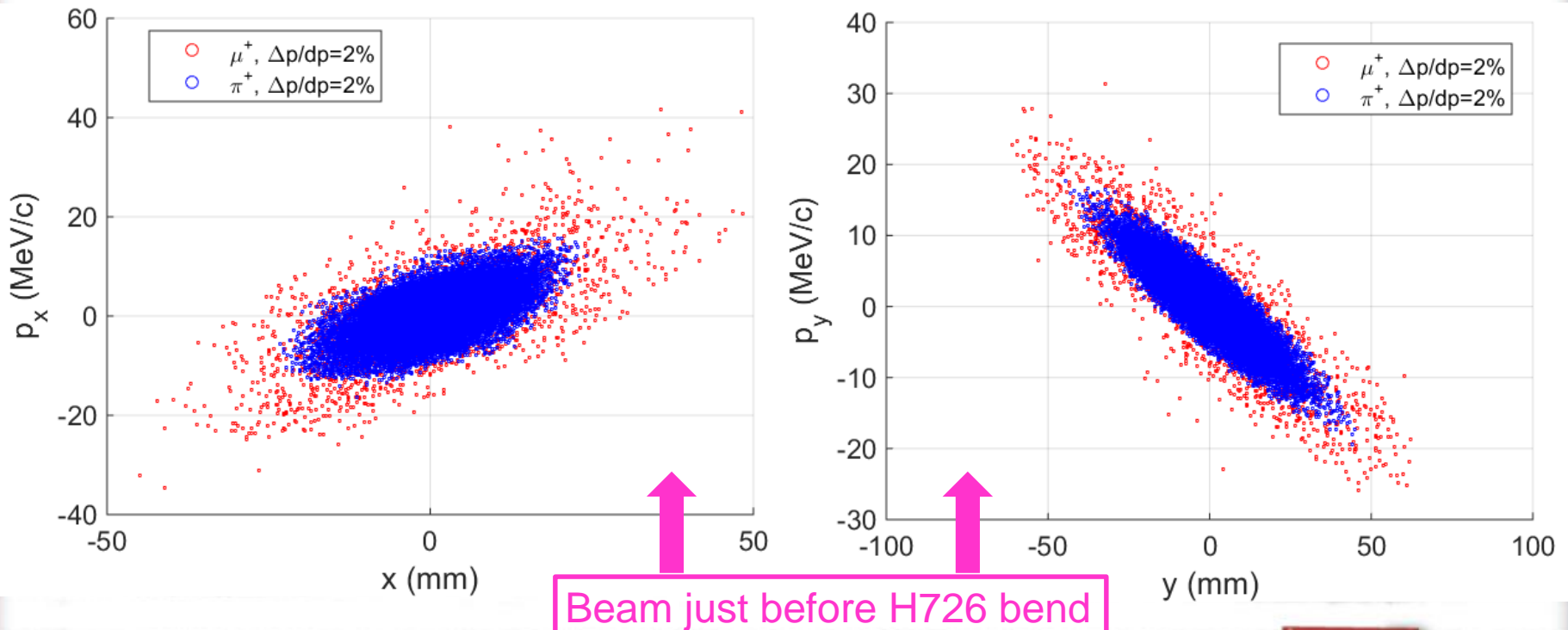
Expected performance

- Secondary beam consist of protons, pions, muons, positrons and deuterons



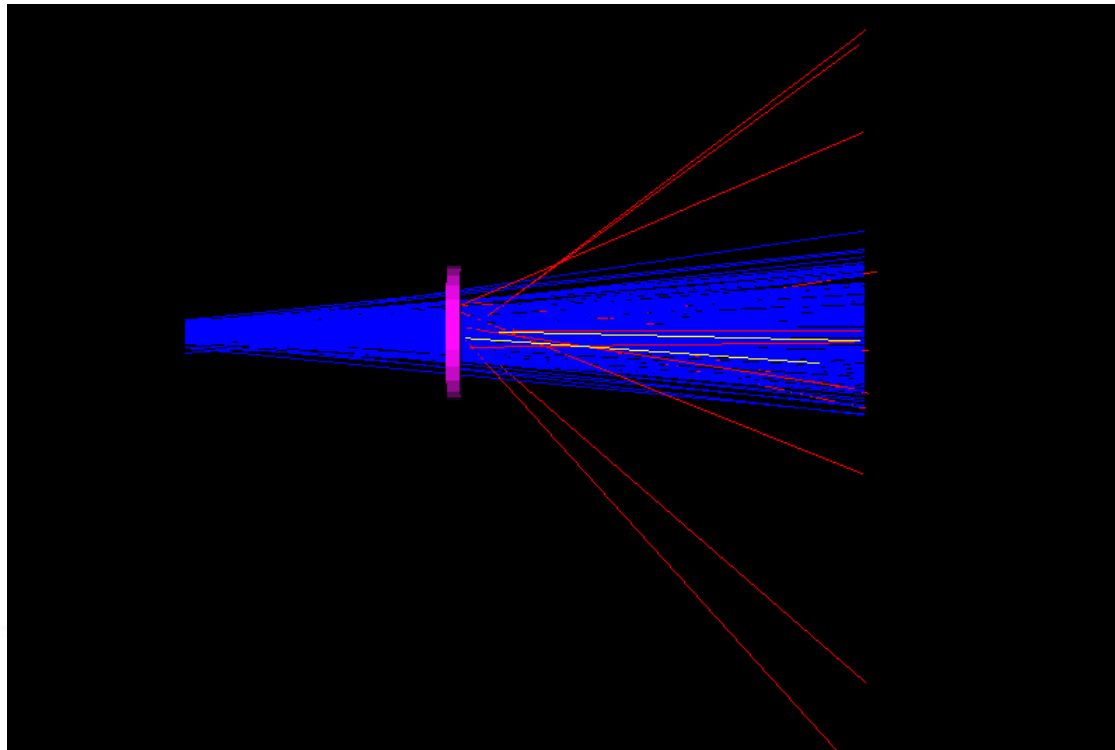
Phase-space analysis: π vs μ

- μ^+ have larger transverse momentum (compared to π^+)
- As a result, muons are lost in apertures between H726 & H729 bends



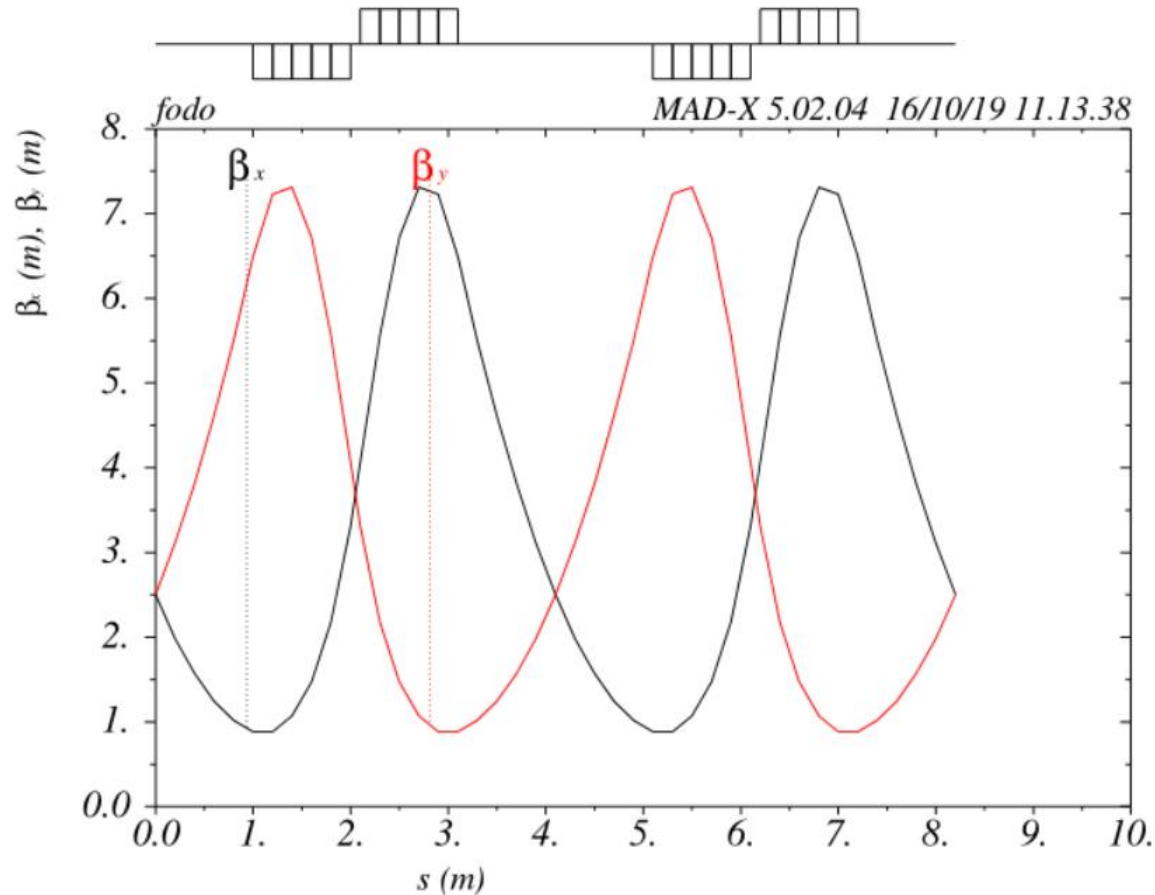
Interaction with materials

- G4beamline includes key physical processes such as ionization, scattering and straggling.



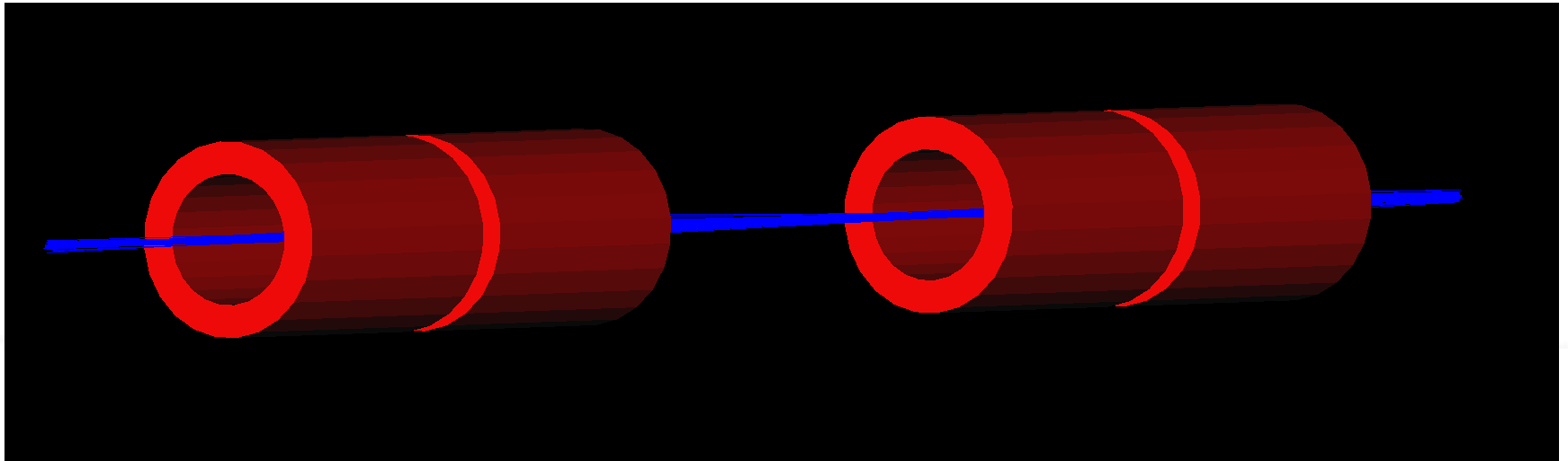
FODO Example (MAD view)

- MAD shows you the elements and exports Twiss functions



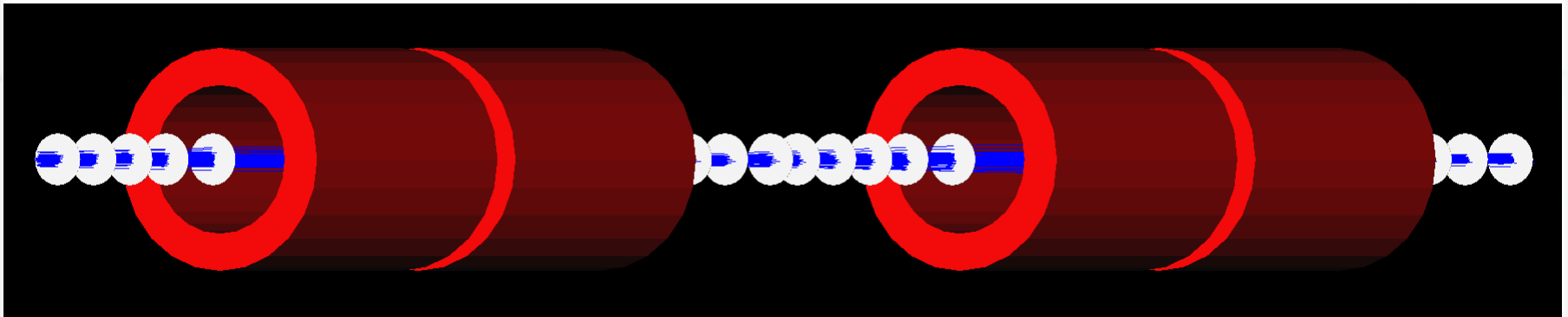
FODO Example (G4beamline view)

- Unlike MAD, G4beamline includes apertures and 3D graphical interfaces
- Does not export Twiss parameters
- You need to “collect the data” and post-process it by your own.



FODO Example (G4beamline view)

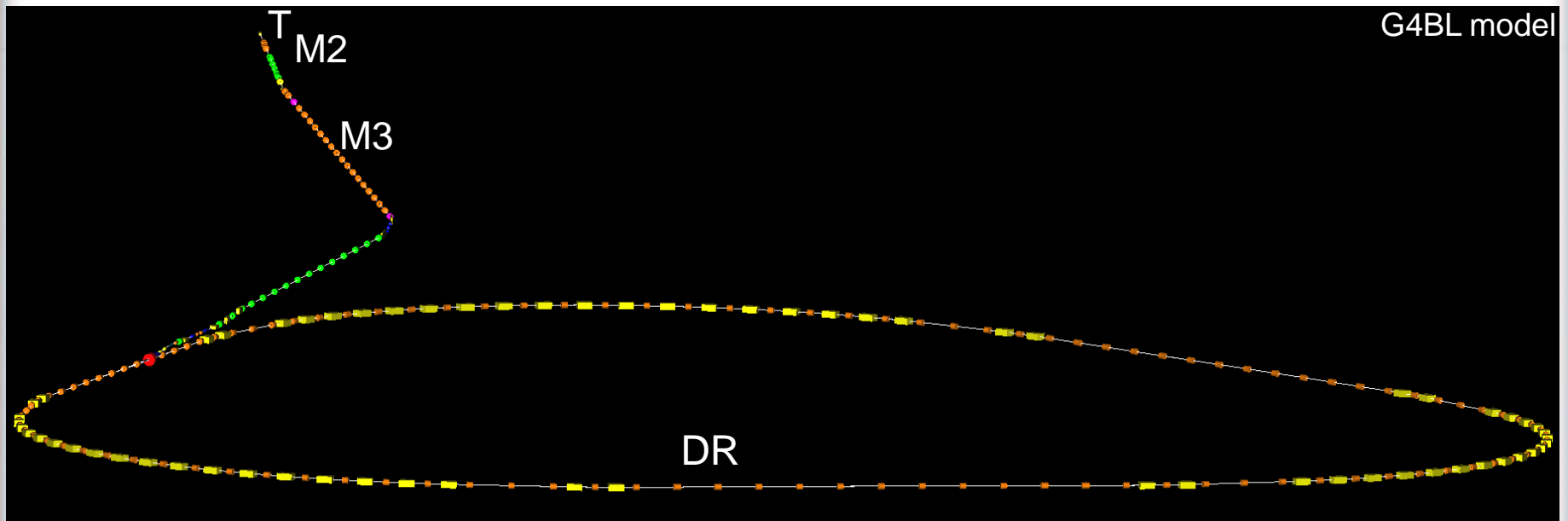
- The trick is to add several Virtual Detectors along the line



Example of an input file

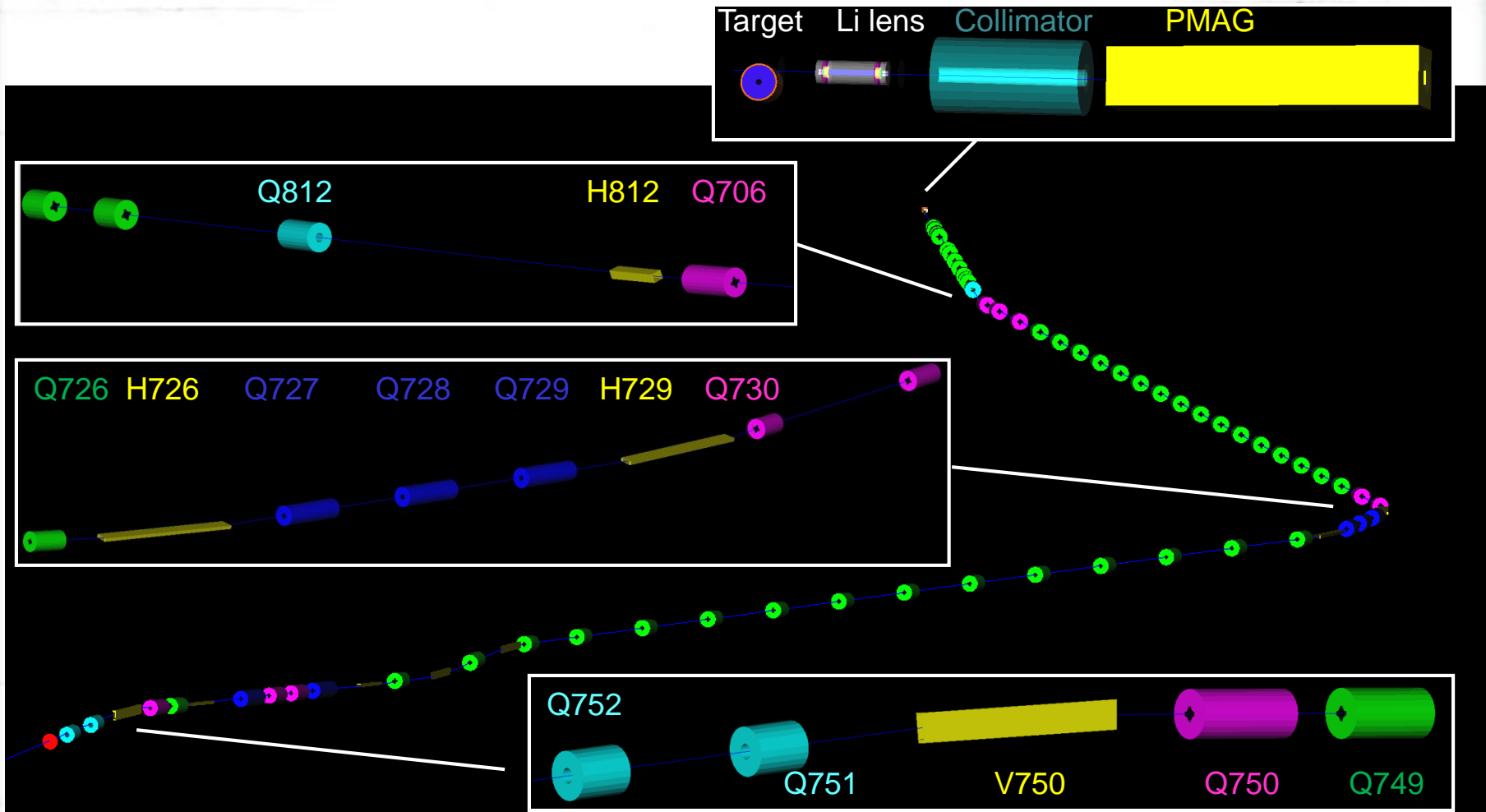
```
#BLTrackFile ... user comment...
#x y z Px Py Pz t PDGid EventID TrackID ParentID Weight
6.802637 -16.598810 0.000000 -6.471170 -9.628622 3084.618282 0.000000 -13 1 1 0 1
-8.731608 -0.329091 0.000000 17.371093 -12.317586 3098.052692 0.000000 -13 2 1 0 1
11.357872 -2.882008 0.000000 -22.424179 1.214885 3080.485061 0.000000 -13 3 1 0 1
6.282338 -6.525666 0.000000 -10.776342 -8.769307 3086.600291 0.000000 -13 4 1 0 1
13.731559 1.035823 0.000000 -25.447105 -10.417208 3096.875303 0.000000 -13 5 1 0 1
-1.340019 1.210609 0.000000 3.728586 11.652227 3105.110773 0.000000 -13 6 1 0 1
2.671119 -1.791614 0.000000 -1.887064 4.404255 3095.704934 0.000000 -13 7 1 0 1
8.360061 -14.253557 0.000000 -21.172346 -10.712157 3105.979796 0.000000 -13 8 1 0 1
2.699270 0.007972 0.000000 -3.382148 -5.192966 3096.813927 0.000000 -13 9 1 0 1
3.976922 0.535405 0.000000 -9.186198 -20.737834 3089.972247 0.000000 -13 10 1 0 1
-12.089317 -3.530778 0.000000 32.293101 4.218068 3092.320501 0.000000 -13 11 1 0 1
-7.213055 -8.445001 0.000000 22.523822 -16.328386 3097.391753 0.000000 -13 12 1 0 1
-12.255426 1.705230 0.000000 22.314921 -1.962983 3077.894842 0.000000 -13 13 1 0 1
7.515872 7.180219 0.000000 0.216456 14.110747 3099.286286 0.000000 -13 14 1 0 1
-2.761276 17.632681 0.000000 10.530404 43.791920 3112.657264 0.000000 -13 15 1 0 1
3.683348 -12.039462 0.000000 -19.962415 -21.619136 3090.065385 0.000000 -13 16 1 0 1
-5.481315 -7.371757 0.000000 0.415553 -26.012904 3086.168634 0.000000 -13 17 1 0 1
-7.878201 -1.261328 0.000000 16.559805 -1.810486 3098.842656 0.000000 -13 18 1 0 1
-12.119890 -7.652256 0.000000 31.626997 -2.644823 3103.336974 0.000000 -13 19 1 0 1
-1.927845 -5.629353 0.000000 14.294566 -12.263660 3099.008430 0.000000 -13 20 1 0 1
12.522456 3.217174 0.000000 -5.083759 -18.604711 3083.352416 0.000000 -13 21 1 0 1
-10.904257 0.336843 0.000000 29.559633 9.167577 3092.676692 0.000000 -13 22 1 0 1
-4.097192 9.127308 0.000000 10.982384 -2.163567 3086.365990 0.000000 -13 23 1 0 1
8.793803 -2.375145 0.000000 -11.081205 -2.620028 3093.044541 0.000000 -13 24 1 0 1
-11.662832 8.796357 0.000000 0.354087 25.917972 3085.448127 0.000000 -13 25 1 0 1
-11.655065 -5.053066 0.000000 26.668211 -8.937786 3093.245065 0.000000 -13 26 1 0 1
2.146562 5.337979 0.000000 -21.439311 9.052843 3095.233273 0.000000 -13 27 1 0 1
3.178387 -12.388542 0.000000 -1.722679 -17.182395 3086.877183 0.000000 -13 28 1 0 1
-14.760259 -8.207974 0.000000 23.605049 -21.167073 3083.212163 0.000000 -13 29 1 0 1
5.521583 3.018658 0.000000 -27.935240 -9.018114 3089.886228 0.000000 -13 30 1 0 1
4.403696 6.439173 0.000000 -3.617263 -13.140424 3082.477198 0.000000 -13 31 1 0 1
-10.261837 5.006774 0.000000 25.921391 6.358151 3075.392099 0.000000 -13 32 1 0 1
3.579964 12.789446 0.000000 -1.505302 24.330217 3107.621438 0.000000 -13 33 1 0 1
0.190570 11.090831 0.000000 9.001475 20.533660 3090.669915 0.000000 -13 34 1 0 1
-0.442682 -8.517654 0.000000 8.362777 -14.946446 3096.318528 0.000000 -13 35 1 0 1
11.947408 -1.558275 0.000000 -14.713339 3.018041 3094.007031 0.000000 -13 36 1 0 1
6.820025 -4.732313 0.000000 -20.410085 -13.130618 3092.305640 0.000000 -13 37 1 0 1
```

Simulation model



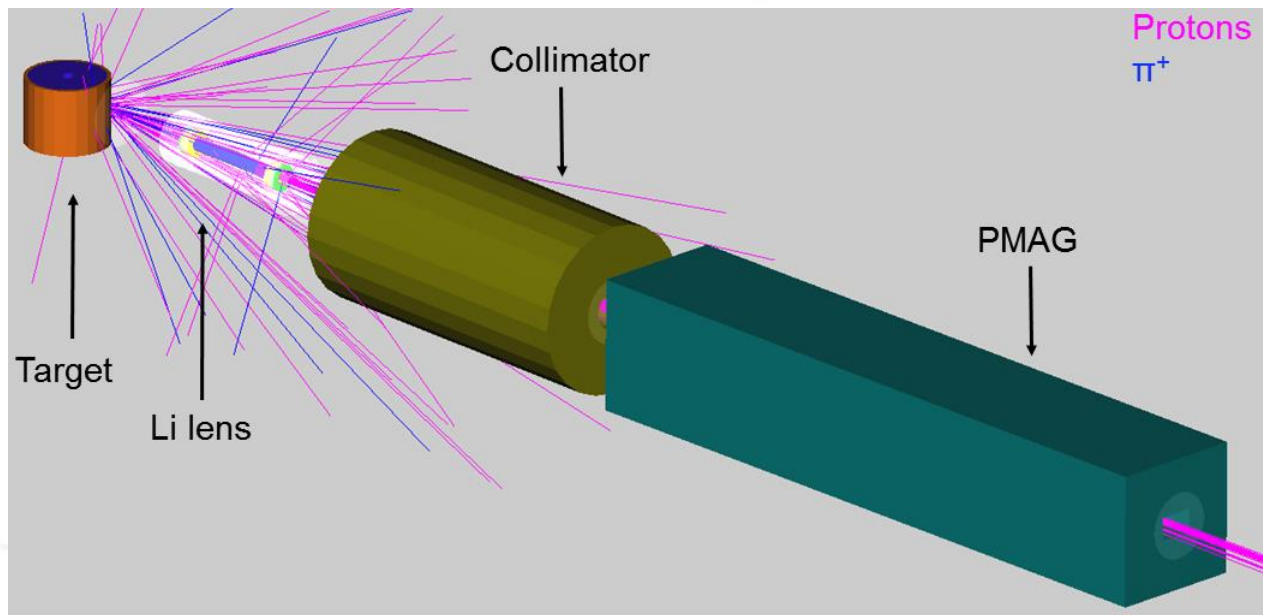
- M2 & M3 lines will carry the secondary beam from the target (T) to the delivery ring (DR)
- Loop four times until μ^+ yield peaks and all p are removed

Model for the M2-M3 beamlines



Target station

- Target station consists of five devices: production target, lithium lens, collimator, pulsed selection magnet & dump
- Muons are produced indirectly: $p \rightarrow \pi^+ (26 \text{ ns}) \rightarrow \mu^+ (2 \mu\text{s})$



Target model for g-2

Blue: Inconel600
Orange: Beryllium

Incoming beam

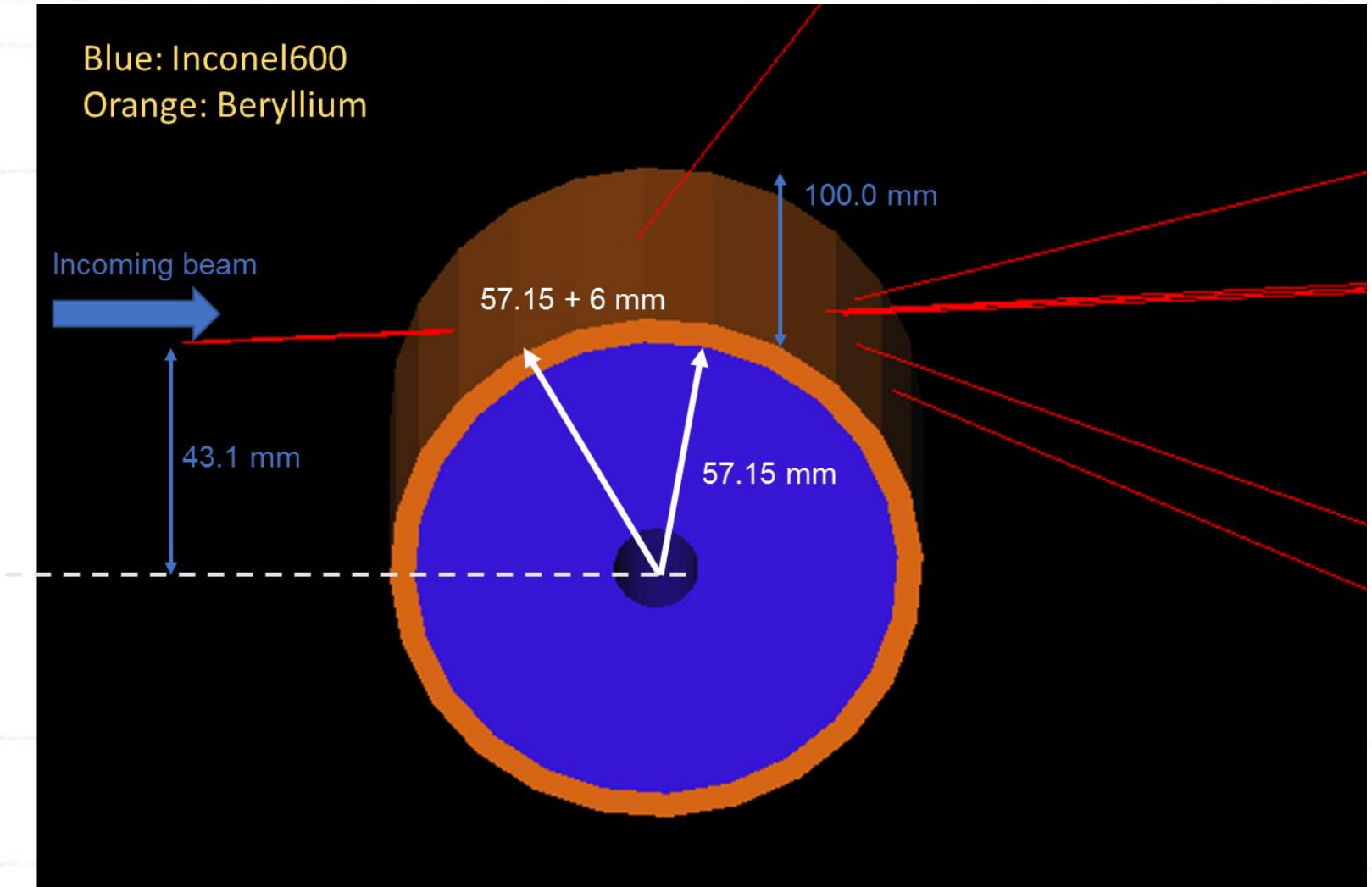


43.1 mm

57.15 + 6 mm

100.0 mm

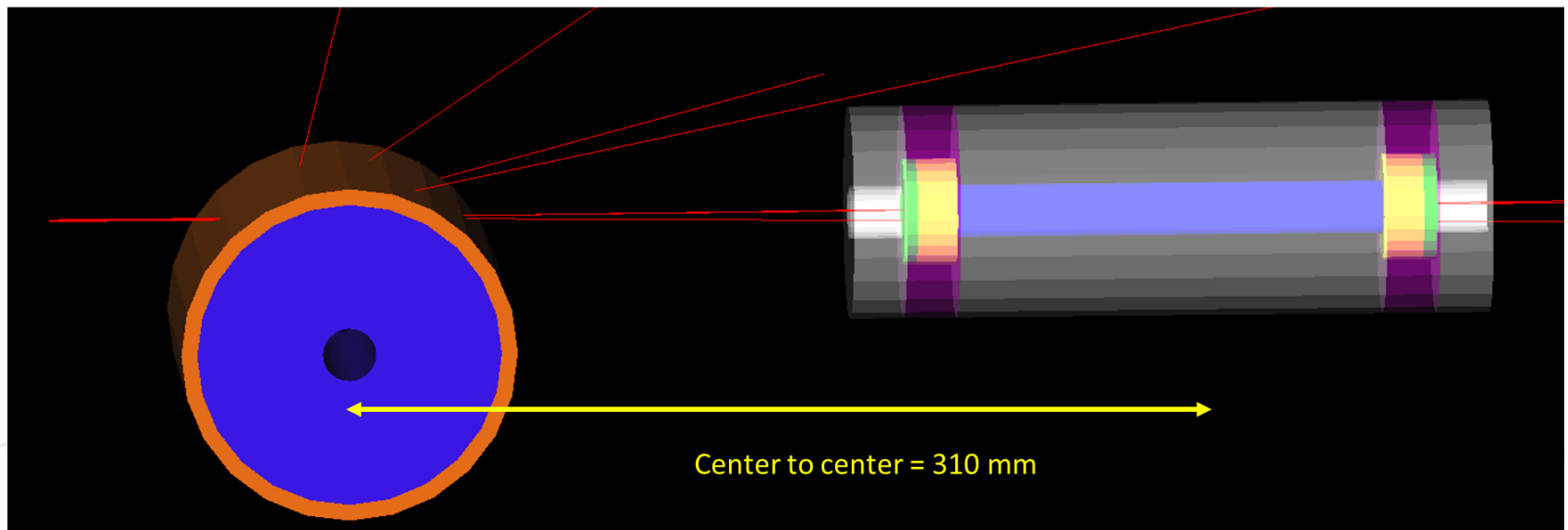
57.15 mm



Lithium Lens

- Lithium rod with beryllium vacuum windows and titanium casing
- 116 kA current through lithium generates focusing magnetic field

Lithium lens



Modeling the B-field of lens

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic permeability of free space = $4 \cdot \pi \cdot 10^{-4} \text{ T} \cdot \text{mm/A}$

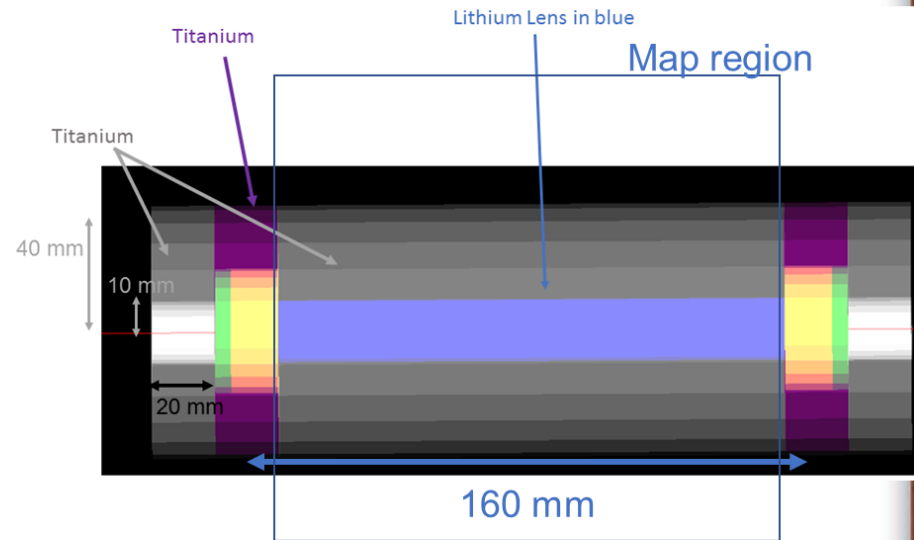
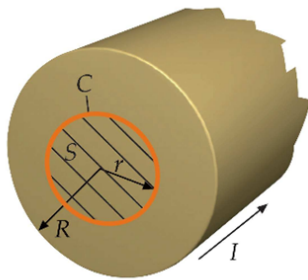
If $r < R$ (loop 2), the current enclosed is proportional to the area, i.e.

$$I_{\text{enclosed}} = I \frac{\pi r^2}{\pi R^2} = I \frac{r^2}{R^2}$$

so that

$$B = \frac{\mu_0 I}{2\pi R^2} r$$

Magnetic permeability of Li
 = $(1 + 1.4 \cdot 10^{-5}) \cdot 4 \cdot \pi \cdot 10^{-4} \text{ T} \cdot \text{mm/A}$

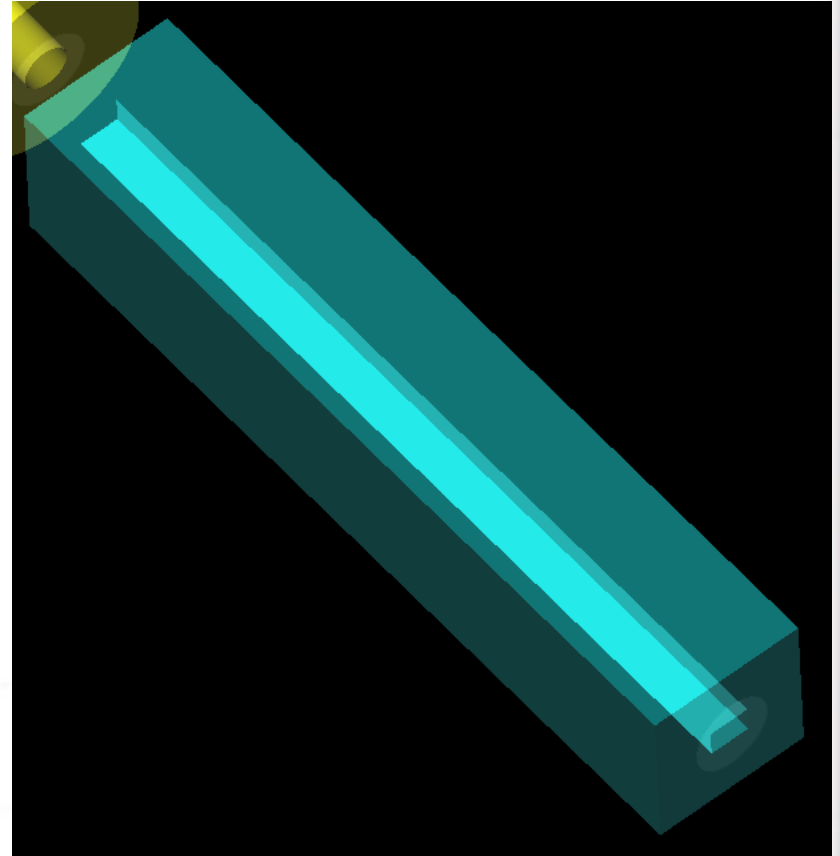


$I = 116000.0 \text{ A}$, $R = 10 \text{ mm}$ (radius of lithium lens)

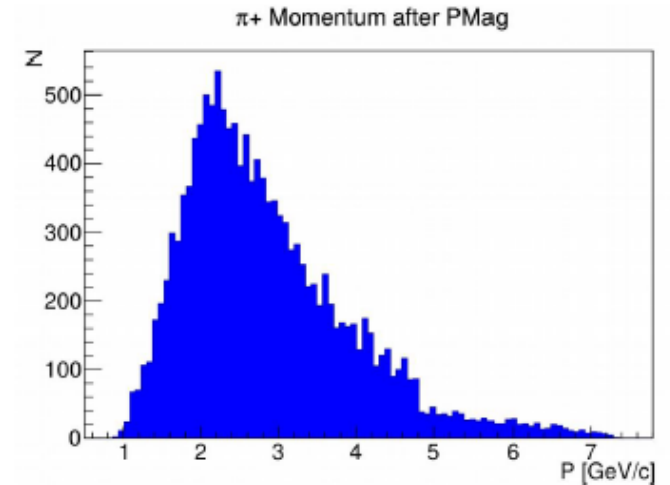
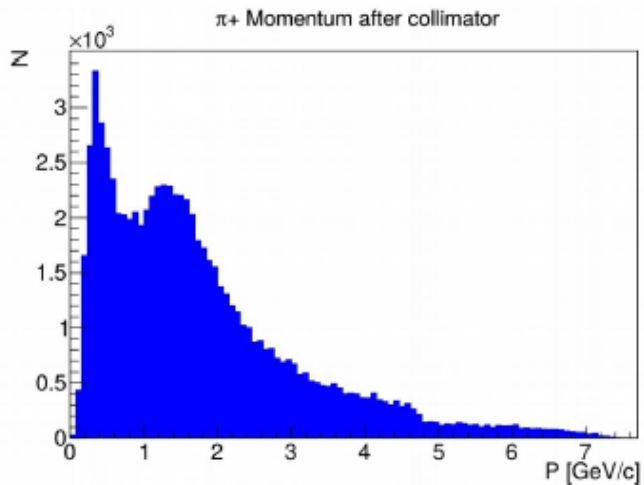
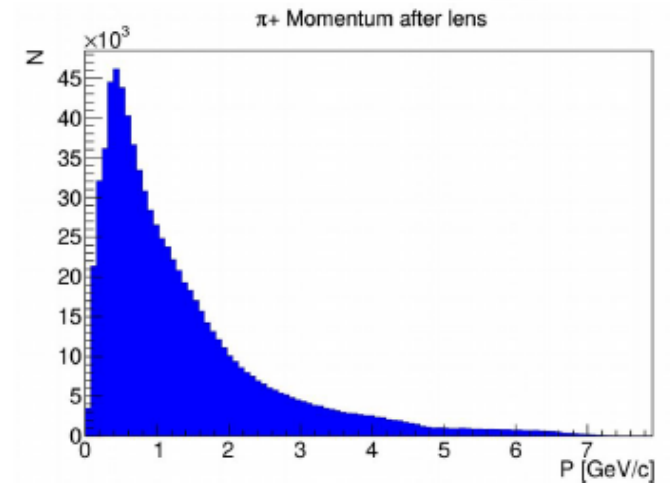
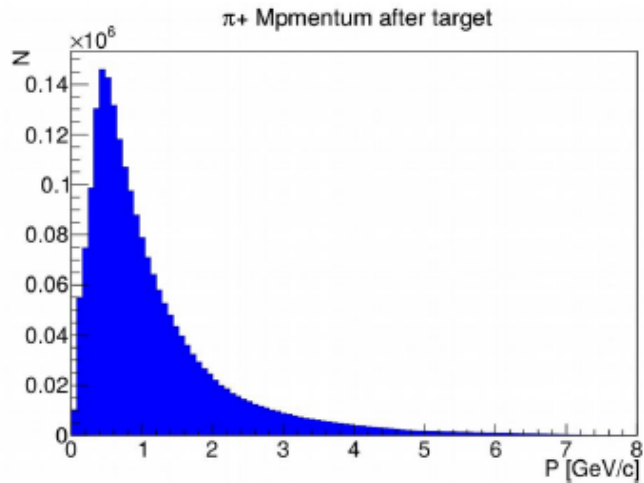
Within the 160 mm length, the following expressions were used: $B_z = 0$, $B_r = 0$, and $B_\phi =$ See above (either inside or outside)

Lithium Lens

- 0.53 T vertical field bends particle paths
- Particles with momentum around 3.1 GeV/c continue to next part of the beamline
- Unbent leftover protons sent to beam dump

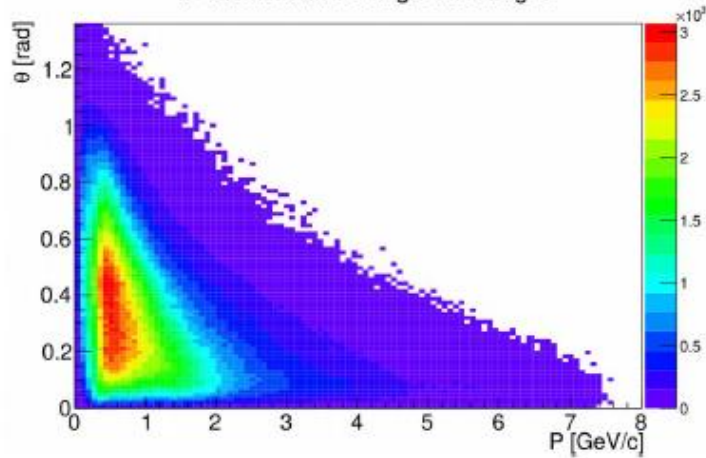


Beam simulation through target (1)

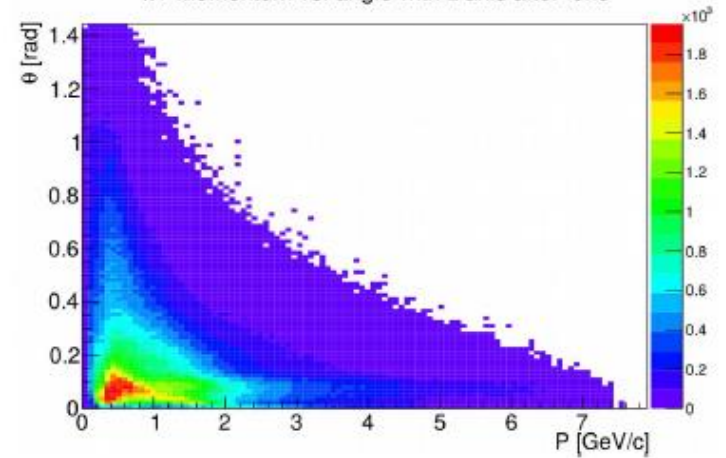


Beam simulation through target (2)

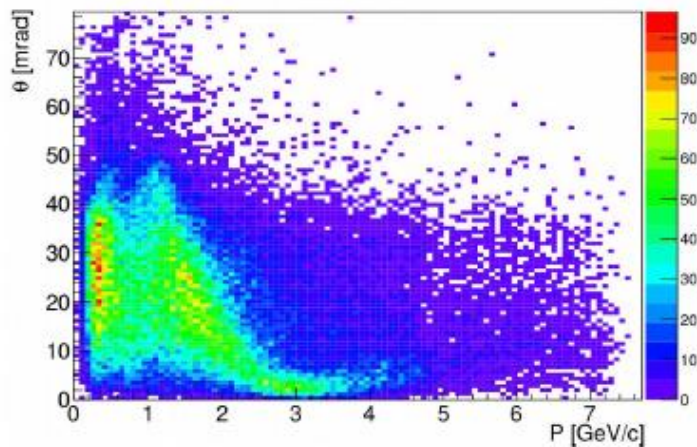
π^+ Momentum vs. angle after target



π^+ Momentum vs. angle with z axis after lens



π^+ Momentum vs. angle with z axis after collimator



π^+ Momentum vs. angle with z axis after PMag

