Assignment: HW5 [40 points]

Assigned: 2016/11/08
Due: 2016/11/15

P5.1 [6 points]

Find the characteristic frequencies of the coupled circuits shown in Fig. 1 below. Comment on the two modes of oscillation (*Hint: only one mode is damped*). Examine how the damped mode depends on the relation between R^2 and $\frac{L}{C}$.

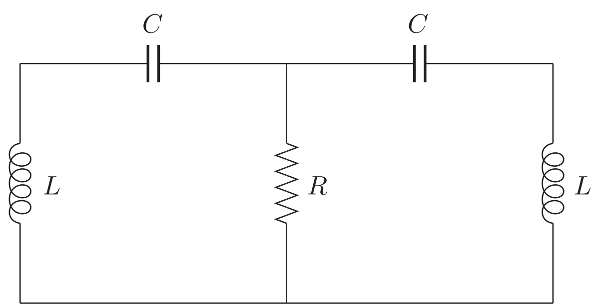

Figure 1: For Problem 5.1.

P5.2 [10 points]

A mass M moves horizontally along a smooth rail. A pendulum of mass m hangs from M by a massless rod of length ℓ in a uniform vertical gravitational field $\mathbf{g}$ as shown in Fig. 2. Ignore all terms of order θ^3 and higher in expansions of trigonometric functions, as well as terms of order $\theta^2\dot{\theta}$ and higher in the Lagrangian. Find the eigenfrequencies and describe the normal modes.

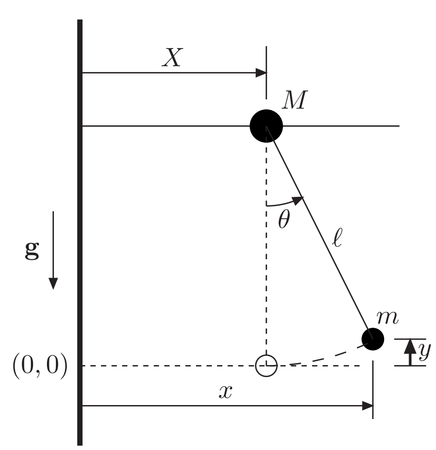

Figure 2: For Problem 5.2.

P5.3 [6 points]
Three oscillators of equal mass m moving in one dimension are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1(x_1^2 + x_3^2) + \kappa_2 x_2^2 + \kappa_3(x_1 x_2 + x_2 x_3)\right] \tag{1}$$

where

$$\kappa_3 = \sqrt{2\kappa_1\kappa_2}. \tag{2}$$

Find the eigenfrequencies by solving the secular equation. What is the physical interpretation fo the zero-frequency mode?

P5.4 [6 points]
Goldstein (3rd Ed): Problem 6.12 (symmetric 3-spring, 2-mass arrangement, P274).
Two particles, each of mass m, move in one dimension attached to three springs of spring constants k, $3k$ and k in the symmetric arrangement shown in Fig. 3. All springs are unstressed at equilibrium. Find the eigenfrequencies and normal modes of the system.

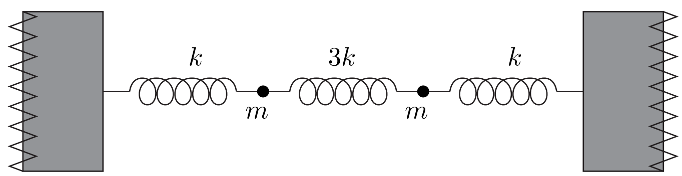

Figure 3: For Problem 5.4.

P5.5 [6 points]
Goldstein (3rd Ed): Problem 6.16 (oscillation on a $y = ax^4$ curve, P275).
A particle of mass m moves in a constant gravitational field $\vec{g} = g\hat{y}$ along the curve $y = ax^4$. Find the equation of motion for small oscillation centered at the equilibrium.
Note: You are only asked to find the equation, not to solve it.

P5.6 [6 points]
A simple plane pendulum has a mass m hanging at the end of a massless string of length ℓ in a field of constant gravitational acceleration $\vec{g}$. While the pendulum is in motion, the length of the string is changed at a constant rate $\dot{\ell} = v_0$. Find the Lagrangian and the Hamiltonian, determine whether or not $T + V$ and H are conserved, and comment on the physical interpretation of your results. This is a rather famous problem discussed by Einstein, Lorentz, and others at the 1911 Solvay Conference.
Hint: Since ℓ is not an independent generalized coordinate, but is constrained to be a simple linear function of time, the system only has one degree of freedom.