Assignment: HW1 [40 points]

Assigned: 2016/08/30
Due: 2016/09/06

P1.1 [$2 + 2 + 2 = 6$ points]

Let G be a multiplicatively-written group with identity element e. Prove each of the following statements:

(a) If $(x, y) \in G$ with either $xy = e$ or $yx = e$, then $y = x^{-1}$.
(b) $(x^{-1})^2 = (x^2)^{-1}$ for all $x \in G$.
(c) $(xy)^{-1} = y^{-1}x^{-1}$ for all $(x, y) \in G$.

P1.2 [$2 + 2 + 2 + 2 + 2 + 2 = 12$ points]

The simplest non-trivial cyclic group is called Z_2. It has only two elements: e (the identity element), and a. The *Cayley table* for Z_2 is

	e	a
e	e	a
a	a	e

The group $Z_2 \times Z_2$ is the smallest non-cyclic group. It has 4 elements and is called the *Klein four-group* or, simply, the *Klein group*, and is denoted by K_4. All non-identity elements of the *Klein group* have order 2.[1]

(a) Write down the *Cayley table* for the cyclic group with 4 elements Z_4.
(b) Give an example representation of Z_4.
(c) What are the orders of the different elements of Z_4?
(d) Write down the *Cayley table* for K_4.
(e) Give an example representation of K_4.
(f) Comment on the difference between K_4 and Z_4.

P1.3 [4 points]

Let G be a multiplicatively-written group with identity element e. Show that G is Abelian if $x^2 = e$ for all $x \in G$.

P1.4 [8 points]

Explicitly show all the identity and commutation relations of the *Pauli matrices*, the generators of the $SU(2)$ group.

P1.5 [10 points]

Show that the Galilei transformations $g(\mathbf{R}(\psi, \hat{\mathbf{n}}), \mathbf{w}, \mathbf{a}, s)$,

$$\begin{pmatrix} \mathbf{r} \\ t \end{pmatrix} \xrightarrow{g} \begin{pmatrix} \mathbf{r}' = \mathbf{R}\mathbf{r} + \mathbf{w}t + \mathbf{a} \\ t' = \lambda t + s \end{pmatrix}, \tag{1}$$

with $\det \mathbf{R} = +1$, $\lambda = +1$, form a group.[2]

[1] The order of an element a of the group is the smallest integer m for which $a^m = 1$.
[2] This is the *proper, orthochronous Galilei group* $G_+^{\uparrow 4}$.